

Automated 3D scan multi-view registration based on rotation estimation

Huaxin Wang, Joris S. M. Vergeest, Yu Song, Tjamme Wiegiers
CADE, DE, Industrial Design Eng.
Delft University of Technology
Landbergstraat 15
2628 CE, Delft

{huaxin.wang, j.s.m.vergeest, y.song, t.wiegiers}@tudelft.nl

ABSTRACT

A new approach is presented for automatic registration of a sequence of 3D scanned range images into a complete model. Provided that the user follows the proposed scanning strategy, the difficulty to register the split views into a complete model is alleviated and success rate of automatic registration is improved by an adaptive forward estimation based on the extracted rotation axes and angles of the object between each scanning steps. The basic idea of this improvement is that when each of the scan differs slightly from its previous one in a small rotation about an approximately constant axis, the rotation axes and angle in the previous scanning step can be used as the first estimation of the rotation in the next step, so as to avoid manual coarse initial alignment. Formulation of the principle is provided and experimental results demonstrated.

Keywords

3D scan, automatic registration, estimation

1. INTRODUCTION

In industrial design, despite of the ubiquitous application of CAD digital modeling, traditional hand-made clay models remain as an intuitive approach to demonstrate and polish design concepts. As digital models are required for further machined prototyping and manufacturing purposes, fast digitalization of these physical models for import into CAD systems helps to build a seamless design process encompassing both physical and virtual modeling approaches.

In our recent project *Synthetic Environments for Design*, digitalization of physical models into the virtual environment for further simulations and operations is also an interesting approach. This is where the 3D scanning technique finds an application among others including reverse engineering of shapes for improvement or adaptation of the shape design of products which are in or out of production.

In the process of 3D digitalization of physical

models, registration (matching and merging) of the different slices of the model surface scanned from different view angles is perceived as a bottleneck, as it requires tedious pre-processing to identify and provide initial matching points, which a product designer is typically unwilling to follow, and even economically impossible for bigger or complexer models that require registration of typically dozens to hundreds of scanned views. In the literature, a lot of attempts to automate the registration process have been reported, yet unfortunately, full automation of the registration of 3D scan data is, in general, still an unsolved problem [Ver01].

Nevertheless, supplementary data provided either by human assistance or additional devices, can help the registration algorithm -- normally an improved variant of ICP (iterative closest point) -- to succeed.

Besides manual selection of the initial matching points, other possible sources of supplementary data for automated registration were explored, the most significant approaches among which are: making use of the shape feature or geometric characteristics (spin image, normal vectors, etc.) [Ben01] [Joh01] [Pin01] [Sch01] [Sha01]; exploiting the scanning sequence information [Pin01]; global registration making use of overall consistency of all matches [Hub01]; improving the algorithm to find corresponding points, and exclude outlier points of the two shells to be registered [Che01] [Mas01] [Liu01] [Liu02] [Zha01].

Permission to make digital or hard copies of all or part of this work for personal or classroom use is granted without fee provided that copies are not made or distributed for profit or commercial advantage and that copies bear this notice and the full citation on the first page. To copy otherwise, or republish, to post on servers or to redistribute to lists, requires prior specific permission and/or a fee.
Copyright UNION Agency – Science Press, Plzen, Czech Republic.

Summarizing all the available approaches, [Pla01] attempted to unify the terminologies used in this field and build a theoretical framework based on the taxonomy of the variety of methods, for which a thorough survey and comparison can be found in [Mat01]. Most of the above mentioned methods tried to attack the problem from two aspects: point matching and/or shell pair matching. Our proposed method won't try to replace the available registration algorithms, but to make use of the information acquired with the registration algorithms to improve their rate of success and thus can be combined with them.

The previous studies suggested that data about the initial relative or absolute poses can be introduced to reduce the number of wrong pair-wise matches (hence the space of solution to seek in), improve the robustness of the whole process and speed up the calculation for real-time interactive operation. Especially, a new approach in which a large series of 3D scans is made, where subsequent views differ in a small rotation about an approximately constant axis is proposed [Ver01]. Based on this assumption and statistical analysis of extracted rotation axis in previous scans, estimation of further rotation and translation can be used as a hint to rotate the next view into a better position that nearly snaps on the previous registered views.

In this paper, following and extending the above ideas, we try to further improve the success rate of automatic registration based on current available registration algorithms, starting with the attempt to acquire a quantitative impression of how far the assumption of nearly constant rotation axis of the user operation can stand, and how efficient this method can be.

Detailed description of the problem and geometric formulation of the solution are provided in section 2. After that, numerical results of some experiment examples are presented and analyzed. Our current comments and suggestions of possible extensions and improvements to this approach can be found at the end.

2. Theory

2.1. Principle

Because of the relative movement between the scanned object and the scanner caused by rotating the object or moving the scanner around the object, there is always relative displacement of the next scanned view (as point cloud and polygon shell data after processing) from where it should be registered with the previously scanned pieces. If there is a way to restore the position of the next scanned view to where it should be, the registration is done.

This relative movement can be decomposed into a pure rotation along an axis and a translation of the rotation axis. First, we consider the case in which the object is rotated without translation.

At the beginning, the object is scanned at certain position to get the first view. Then it's rotated by a certain angle θ along a rotation axis a and scanned to get a second view. If the second scanned view is rotated by angle θ along a in the opposite direction, theoretically it is registered with the first scanned view. So if there is a way to know the exact value of θ and a , registration can be done. As the general practice, these data can be acquired by using a digitally controlled rotation table or certain controlling or sensing apparatus with 3D position measurement. As the project is targeted at Small to Medium Business (SMB) applications, we're interested in less expensive automated solutions suitable for routine freehand operations by novice users.

On the other hand, it's a common experience that when the position difference of two scanned views are sufficiently small, currently available ICP code and the registration algorithms in commercial 3D CAD software (such as RapidForm of INUS) can present good result, even without manual selection of matching points. Yet scanning with very small position steps is very time consuming and things are even worse when it comes to the registration of the large number of shell data acquired.

So we now propose to aim at a balance between some efforts required from the inexperienced users, versus the chance to achieve automated registration.

The user is asked to rotate the object about a roughly constant axis, first by relatively small angles. Since the overlap of two subsequent views is big, the chance to obtain a successful automatic registration of the first two views with the current available CAD software is expected to be high. The rotation angle and axis of the object between these 2 scans can be extracted from the registration process. Then the user is guided to rotate the object further in the same direction with a steadily bigger angle step by the almost fixed rotation axis. Based the previously extracted rotation angle and axis, the next rotation can be estimated by extrapolation to rotate the shells to be registered into a better initial position for the registration algorithm to succeed.

By proceeding the scanning in sequence, 360 degree views of the object can be obtained and at the same time automatically registered. Normally it's still insufficient to obtain a digitization of the entire surface of the object, so optionally another (or even more) sequence of views rotated about a different

axis should be taken. The user should rotate the object in roughly the same direction between each consecutive scan, unless a change of direction is necessary to cover the whole surface of the object.

The method to extract rotation angles and axes is further developed in the next section.

2.2. Geometric formulation of registration

The scanning and registration procedure is illustrated in Fig 1. For simplicity of illustration, a simple triangle surface representing one of the many surfaces that match between two consecutive scan views on a digitized shell of certain 3D shape is used. In the process of registering different scan views, this basic relationship still applies, though some of the surfaces on the two shells to be registered won't match with each other.

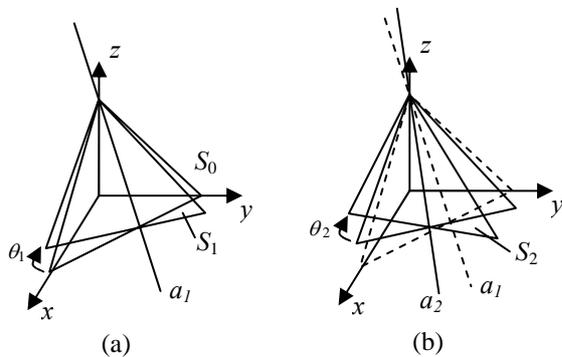


Fig 1. The consecutive scan and registration of surfaces

S_0 is one the scanned surface of the object in the starting position. Then, the object is rotated with an angle θ_1 about the axis a_1 in the direction shown as the arrow labeled θ_1 , and a second scan is made in which the original surface S_0 is rotated to the position of surface S_1 , as shown in Fig 1 (a).

For an ideal registration of S_0 to S_1 , the resulting S_0 should be rotated to the exact position of S_1 . The transform matrix \mathbf{T}_1 got from the registration process can thus be taken as the same for rotation θ_1 .

In case the registration algorithm fails, the user can be requested to revert and do a smaller rotation to retry. We assume that given a sufficiently small rotation, the general registration algorithms should succeed, thus a \mathbf{T}_1 can always be got.

Then the user is asked to do the next scan with the object rotated in the same way as in the previous step. The next rotation angle θ_2 and axis a_2 might be different from θ_1 and a_1 , because a manual operation can't be accurate, as shown in Fig 1 (b).

If we can know the rotation θ_2 about a_2 done by the user and apply it to the scanned data S_1 , it can be

registered to S_2 . Normally this information can be acquired by position tracking hardware facilities such as a digitally controlled / readout turn-table. Nevertheless, we can take the approximation that θ_2 is repeating the similar angle of θ_1 , and a_2 is the similar as a_1 . If S_1 is rotated about axis a_1 with angle θ_1 , it will be roughly aligned with S_2 , whereafter within a tolerable error range for the automatic registration to work thus the requirement of a complex position tracking hardware facility is released. If the registration fails, possibly it's because the rotation angle θ_1 is too big or too small as a guess value for θ_2 . First we try to increase the guess of rotation angle by multiplying a ratio $R > 1$ to the previous angle repeatedly, until registration succeeds, or an upper limit is reached. Then we try to decrease the guess of rotation angle by multiplying a ratio $R < 1$ to the previous angle repeatedly, until registration succeeds, or a lower limit is reached. If registration can't be done in this way, then the rotation axis a_2 might be too different from a_1 , so the process reverts to how \mathbf{T}_1 was decided.

2.3. Extraction of rotation angle and axis

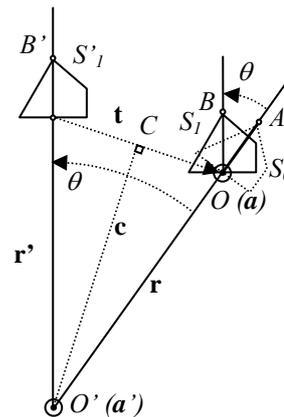


Fig 2. Effective rotation and translation in the registration procedure

Fig 2 shows a 2D projected view of two 3D scanned shells S_0 and S_1 , where S_0 is to be registered to S_1 . Shell S_0 at position A is rotated about axis a (located at point O , pointing out of the paper surface) with an angle θ to S_1 at position B . The vector \mathbf{r} points from the world coordinate origin O' to O , which is one point on the rotation axis a such that \mathbf{r} is perpendicular to a .

The process of registration is equal to the problem to seek for a transform matrix \mathbf{T} , such that for the corresponding points on S_1 and S_0 , in the least square error sense, $\mathbf{T}S_0 = S_1$, where

$$\mathbf{T} = \begin{bmatrix} \mathbf{R} & \mathbf{t} \\ \mathbf{0} & 1 \end{bmatrix}, \mathbf{R} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \quad (1)$$

where \mathbf{R} is the 3x3 rotation matrix, and \mathbf{t} is the translation vector $[t_x, t_y, t_z]^T$. In general, \mathbf{T} specifies a rotation \mathbf{R} around an axis passing the coordinate origin followed by a translation along vector \mathbf{t} .

\mathbf{T} is a pure rotation if and only if the rotation axis \mathbf{a} passes through O' , the origin of the world coordinate system. The transform matrix \mathbf{T} of the registration process in effect rotates S_0 about the axis \mathbf{a}' with the same angle θ , to S'_1 at position B' , then translate it along vector \mathbf{t} to position B . \mathbf{a}' is parallel with \mathbf{a} and passes through the world coordinate origin O' .

The rotation angle θ and direction vector \mathbf{k} of rotation axis \mathbf{a}' can be extracted from \mathbf{T} with the following formulae [Cra01]:

$$\theta = \arccos\left(\frac{r_{11} + r_{22} + r_{33} - 1}{2}\right) \quad (2a,b)$$

$$\mathbf{k} = \frac{1}{2 \sin \theta} \begin{bmatrix} r_{32} - r_{23} \\ r_{13} - r_{31} \\ r_{21} - r_{12} \end{bmatrix}$$

Vector \mathbf{t} is perpendicular to vector axis \mathbf{a}' and its length $|\mathbf{t}| = 2|\mathbf{r}|\sin(\theta/2)$.

The absolute position of axis \mathbf{a} can be derived as follows. Let $\mathbf{r}' = \mathbf{T}\mathbf{r}$, then $\mathbf{t} = \mathbf{r} - \mathbf{r}'$. Let point C be defined as the mid-point of OB' , then the line $O'C$ is perpendicular to \mathbf{t} . The direction vector of $O'C$ can be represented by $\mathbf{k} \times \mathbf{t}$. Therefore we get:

$$O'C = \mathbf{c} = \frac{1}{2} |\mathbf{t}| / \tan(\theta/2) (\mathbf{k} \times \mathbf{t}) / |\mathbf{k} \times \mathbf{t}| \quad (3a)$$

$$\mathbf{r} = \mathbf{c} + \mathbf{t}/2 \quad (3b)$$

Since \mathbf{a} is parallel to \mathbf{k} , the points on the rotation axis \mathbf{a} can be represented as $\mathbf{r} + p\mathbf{k}$ where p is any real number. Any points \mathbf{a}_i on the rotation axis \mathbf{a} will satisfy the equation $\mathbf{T}\mathbf{a}_i = \mathbf{a}_i$, i.e. \mathbf{a}_i are the eigenvectors of \mathbf{T} corresponding to the eigenvalue 1.

Besides rotation, there can also be a small translation \mathbf{t}_a between two consecutive scans. The translation vector \mathbf{t}_a can be decomposed into 2 components:

$$\mathbf{t}_a = \mathbf{t}_n + \mathbf{t}_t \quad (4)$$

where \mathbf{t}_t is perpendicular to the rotation axis \mathbf{a} , and \mathbf{t}_n is parallel with \mathbf{a} . \mathbf{t}_t is in effect contained in the translation vector \mathbf{t} in the above formulation as a shift of the rotation axis location, while the effect of \mathbf{t}_n is currently ignored. If it's beyond certain limit that defeats the registration algorithm, the user can be

prompted to redo the scanning with a smaller translation movement.

2.4. Algorithm of rotation estimation

Based on the approach described in section 2.1-2.2, an adaptive automatic registration algorithm utilizing rotation estimation can be devised. Among the several possible sequences to pre-rotate the scanned shells to provide a roughly registered initial position for the easiness of automatic registration, we chose for simplicity to transform the previously registered scanned views to be aligned with the newly scanned data in each step. The detailed procedures are listed below:

1. Import scan data as shell S_i .
2. Rotate the object with a small angle to ensure that the rotation amount fits in the tolerance range of the registration algorithm.
3. Import the next scan data as shell S_{i+1} .
4. Fix S_{i+1} in place and register S_i to S_{i+1} . The transform matrix T_i of the registration operation is got.
5. Check if the registration is OK.
 - 5.1. Not OK: Ask the user to rotate back the object then redo the rotation with a smaller angle and goto 3.
 - 5.2. OK: Merge S_i with S_{i+1} and name it as S_{i+1} .
6. Extract the rotation angle θ_i and axis \mathbf{a}_i from T_i .
7. Ask the user to rotate the object in the same direction as the previous rotation in step 2.

Although the same rotation angle as the previous step makes the automatic registration more reliable, for most cases a gradually bigger angle is preferred to speed up the scan process, while a slowing down preparation with gradually smaller angles is suggested (Fig 3) if the user plans to rotate the object about another distinct rotation axis after the next few scan steps in order to cover the complete surface of the object. In Fig 3, the positive and negative half axis is used to denote rotation angle θ about different rotation axes.

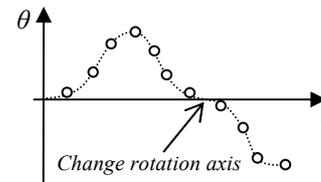


Fig 3. Change of rotation angle and rotation axis

8. Import scan data as S_{i+2} .

9. Assuming the user rotated the object roughly about the same axis as he did in the previous step, θ_i can be used as the initial guess value of the rotation angle in step 7. Use θ_i and \mathbf{a}_i as initial estimation of the rotation, set $\theta_g = \theta_i$.

10. Set magnification ratio $R = 1$;

11. Rotate S_{i+1} about axis \mathbf{a}_i with angle $\theta_g \cdot R$ to be S'_{i+1} . Shell S'_{i+1} is now expected to be roughly aligned with S_{i+2} after this operation.

12. Try to register S'_{i+1} to S_{i+2} . Get the rotation angle θ'_{i+1} and axis \mathbf{a}'_{i+1} from the total transform matrix including pre-rotation in step 11 and registration transform.

13. Check if registration is OK?

13.1. Not OK:

13.1.1. If $R \geq 1$, enlarge R .

13.1.2. If $R < 1$, shrink R .

13.1.3. If $R >$ preset upper limit, set $R < 1$.

13.1.4. If $R <$ preset lower limit, set $i = i+1$, goto step 5.1.

13.2. OK: Set $\theta_{i+1} = \theta'_{i+1}$ $\mathbf{a}_{i+1} = \mathbf{a}'_{i+1}$, $i = i+1$, repeat from step 7.

3. EXPERIMENTS

3.1. Setup

The experiment setup consists of a dark color supporting surface (a table with black covering) or holder (dark color gloves for hand-held scanning or a pedestal to fix the object on the supporting surface for easier single person operation), a Minolta Vivid 700 3D laser scanner, a desktop computer running Vivid scanner controlling and data acquisition software Polygon Editing Tools, and INUS Tech RapidForm 2006. The object to be scanned was simply placed on the supporting surface or holder (as shown in Fig 10) in a moderate distance to the scanner to fit the object image inside the view finder frame. A set of C++ code is written to drive RapidForm 2006 through its COM automation API to implement the proposed automatic registration approach.

3.2. Scan

Because of the current software automation limitations, a simplified procedure similar to that described in 2.4 was followed in the experiments, in which all the scans were done in a batch before registration and stored in a sequence of data files. Then these data files were imported into one RapidForm model file and automatic registration was

carried on by the user code in the same sequence of the scan. This simplified procedure differs from the proposed full one in that it either succeeds, or fails at certain step, without backtracking and re-doing feasibility. Nevertheless, the efficiency of the rotation estimation algorithm can still be demonstrated despite the limitations of the simplifications.

Time used for preparations like hardware / software adjustment, test of scan parameters for best output, etc. ranges from 0.5 to 1 hour for an experienced operator familiar with the hardware system and scan procedures.

Time used for each scan averages in 40 to 70 seconds for an experience operator.

As a comparison, general manual registration process for each pair of scanned shells takes about 1 to 3 extra minutes to select 3 matching points, which are once too often difficult to choose thus a new scan with proper markers attached to the object is required to provide matching points, otherwise completely impossible for objects with few visible feature points.

3.3. Registration

RapidForm API Function *RFMeshTools.RegisterShells()* was called in step 4 of section 2.4. The 4x4 transform matrix *RFTMatrix.m* returned by this API was taken as T_i .

3.4. Results

Test 1: Verification of the axis extraction

Several steps of rotation are simulated by rotating one shell in RapidForm several times to create a model file containing a set of purely rotated shells and the axis extraction algorithm was verified from registration result of this model file. The errors of the extracted rotation angles are acceptable. For ideal registrations, the extracted rotation axes should be theoretically the same, while those as results of non-ideal registrations are shown in the following Tab 1, and illustrated in Fig 4.

Tab 1. Rotation angle and axis extraction results

a) Angle

Step	Ideal (deg)	Extracted (deg)
1	5	5.002890
2	10	10.003064
3	10	10.001035

b) Axis

Step	Ideal	Extracted
1	(0, 1, 0)	(0.000492, 1.000000, -0.000316)
2	(0, 1, 0)	(0.000061, 1.000000, -0.000422)
3	(0, 1, 0)	(0.001556, 0.999999, -0.000375)

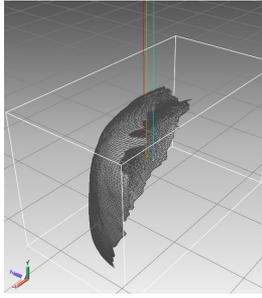
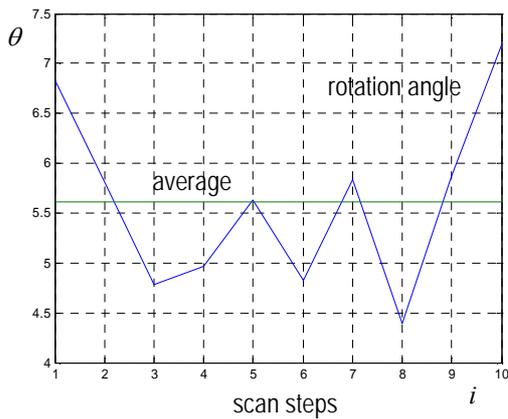


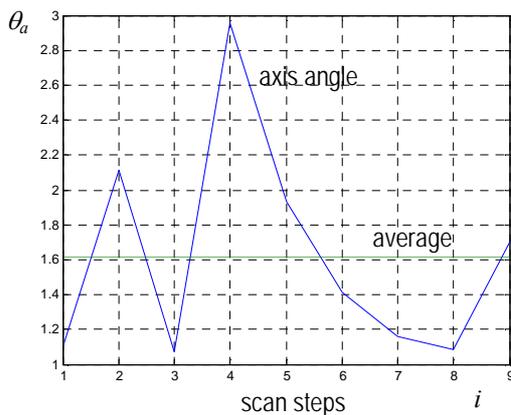
Fig 4. The directions of the rotation axes are almost the same, but the origins of rotation axes have small vertical shifts (along the rotation axis) and apparent horizontal shifts (in xoz plane).

Test 2: Accuracy of manual rotation

To investigate the accuracy of the user’s manual rotation to maintain the same rotation axis and angle between each scan, 10 scans were made with manual rotation of almost the same axis and angle on the table. The rotation axes and angles were extracted as approximated estimation of the rotation motion done by the user. Results are illustrated in Fig 5 and Fig 6, which provide an intuitive verification of how well the presumption of constant rotation axis can be achieved manually on a supporting surface.



a) Rotation angle and average angle (degree)



b) The angle (degree) between the estimated approximate rotation axis and the first axis, and the average

Fig 5. Manual rotation on the table

3D illustration of the approximate rotation axes extracted from the registration process of a scanned model rotated by hand on the table is presented to give a visual impression of the variation of location and direction of the rotation axes as an empirical indication of the stability of the estimation method.

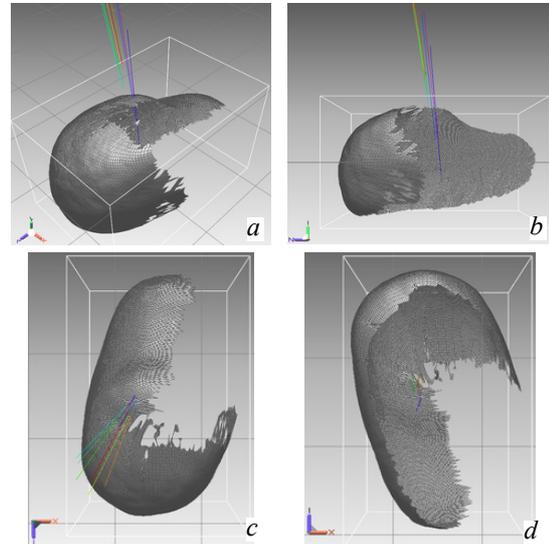


Fig 6. Approximate rotation axes for manual rotation on table a) isometric b) back c) top d) bottom view

Test 3: Comparison of rotation angles of registrations with and without pre-rotation

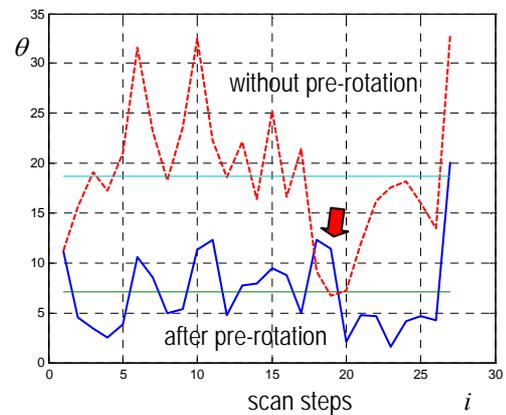


Fig 7. Rotation angle of registration without pre-rotation (upper line) **and after pre-rotation** (lower line), (arrow points to region of rotation axis change)

The rotation angles as returned results of registrations are compared in Fig 7, for the case without pre-rotation and the one with pre-rotation based on estimated approximate rotation motion in the previous step. It shows that the rotation angles of registrations are apparently decreased by the pre-rotation. This means the estimation and pre-rotation

did help to make the registration easier by remarkably reducing the necessary amount of rotation to do the registration.

Test 4: Comparison of registration failure without and with the help of rotation estimation

One of the examples is shown in Fig 8 in which the automatic registration is successful with rotation estimation based pre-rotation, while both the RapidForm global registration and the sequential pairwise registration failed. The failure seems caused by the geometrical similarity of different parts of the shape.

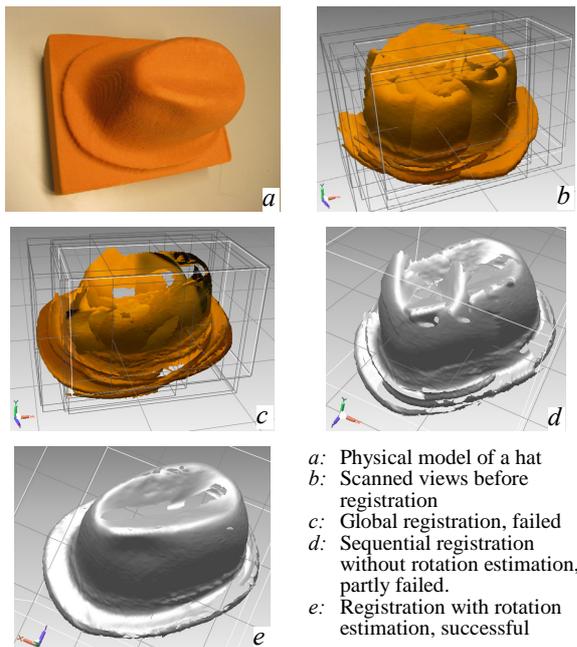


Fig 8. With and without rotation estimation

Test 5: Complete surface scan

An example of successful registration of complete surface scans of an object is demonstrated in Fig 9. The rotation axes and angles were changed as suggested in section 2.4, step 7, in order to cover the complete surface of the object, as shown in Fig 10. Totally 28 scans were done with rotation angles varying from 5 to 35 degrees, while the rotation angles for registration after pre-rotation estimation ranges from as low as 1.6 degree up to 20 degrees (Fig 7).

The extracted rotation axes group in two different direction clusters which correspond to those in the scanning process. But note that the relative angles between the vertical rotation axes (the first rotation direction) and the object is not correctly drawn because of relative movement of the shells later in another different direction in the registration process.

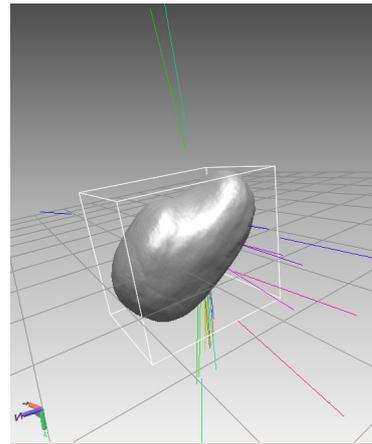


Fig 9. Complete surface registration and rotation axes

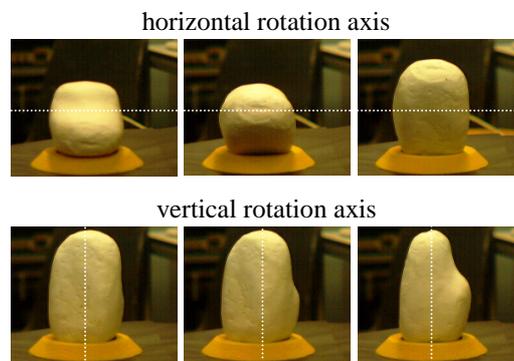


Fig 10. Rotation axis direction change

4. CONCLUSION

4.1. Advantages

The following advantages of the proposed approach can be observed in the theoretical formulation and experiment results:

- a. Automatic batch registration is successful for more cases, especially for those objects with many similar geometric features for which automatic registration algorithms are doomed to fail without help of rotation information.
- b. No complex hardware device is required to record the rotation angles. For the same reason it's also suitable for cases where such device is not applicable, e.g. scanning of non-movable objects.
- c. No sticker marker is required to be placed on the object to help choosing of matching points for registration. Thus the scanned texture images are not disturbed by the markers and can be retained for later texture mapping of the 3D model.

4.2. Limitations

- a. Can't work well with hand-held rotation raised above the supporting surface.
- b. Scanning should be kept in sequence thus inserting scans to patch a region is difficult for automatic registration. To insert a scan in to the sequence, the user must rotate step by step back to the position where the desired scan is to be made.
- c. Cumulative registration error in such sequential pair-wise registration approaches may degrade the global registration result and should be taken into consideration. An extra global fine registration step should improve the result further.

4.3. Possible future extensions

Registration result for the hand-held rotation case needs to be improved for more casual applications of 3D scan. The proposed method may be combined with methods like feature point extraction and matching to improve the rotation axis estimation.

Currently, the judgment of whether the registration of each step is successful is done by visual inspection of the user. Fully automation of the registration process requires a method to evaluate the registration validity and accuracy. It may also help in the rotation angle estimation step (section 2.4, step 10) to choose the best magnification ratio R .

5. ACKNOWLEDGMENTS

The authors gratefully acknowledge the support of the Dutch Innovation Oriented Research Program 'Integrated Product Creation and Realization (IOP-IPCR)' of the Dutch Ministry of Economic Affairs.

6. REFERENCES

- [Ben01] Bendels, G. H., Degener, P., Wahl, R., Koertgen, M., Klein, R. Image-Based Registration of 3D-Range Data Using Feature Surface Elements, The 5th Int. Symp. Virtual Reality, Archaeology and Cultural Heritage (VAST 2004).
- [Che01] Chen, Y., Medioni, G. G. Object modelling by registration of multiple range images, *Image and Vision Comp.* 10(3), pp 145-155, 1992.
- [Cra01] Craig, J. J. Introduction to robotics: mechanics and control. Addison-Wesley Publishing Company, Inc., 1989, Edition 2, p52.
- [Hub01] Huber, D. F., Hebert, M. Fully Automatic Registration of Multiple 3D Data Sets, *Image and Vision Computing* Vol. 23, 2003, pp. 637 – 650.
- [Joh01] Johnson, A. E., Hebert, M. Using spin images for efficient object recognition in cluttered 3D scenes. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 21, 5, 1999, pp433- 449.
- [Liu01] Liu Y. Improving ICP with easy implementation for free-form surface matching, *Pattern Recognition*, Vol. 37, No. 2, February 2004, pp. 211-226, [http://dx.doi.org/10.1016/S0031-3203\(03\)00239-5](http://dx.doi.org/10.1016/S0031-3203(03)00239-5).
- [Liu02] Liu, Y. Automatic 3d free form shape matching using the graduated assignment algorithm, *Pattern Recognition*, Vol. 38, Issue 10, October 2005, Pages 1615-1631, <http://dx.doi.org/10.1016/j.patcog.2005.01.008>.
- [Mas01] Masuda, T., Yokoya, N. A robust method for registration and segmentation of multiple range images. *Comp. Vision and Image Under.* 61(3), pp 295-307, May 1995.
- [Mat01] Matabosch, C., Salvi, J., Fofi, D., Meriaudeau, F. Range Image Fusion for Industrial Inspection, *SPIE Electronic Imaging-Machine Vision Applications in Industrial Inspection XIII*, San José, USA, January 2005 http://vision.u-bourgogne.fr/le2i/user_data/publications/matabosch05b.pdf
- [Pin01] Pingi, P., Fasano, A., Cignoni, P., Montani, C., Scopigno, R. Exploiting the scanning sequence for automatic registration of large sets of range maps, *Computer Graphics Forum*, 24:3, Sept. 2005, p. 517.
- [Pla01] Planitz, B.M. et al. The correspondence framework for 3D surface matching algorithms. *Computer Vision and Image Understanding*, 2005.
- [Sha01] Sharp, G. C., Lee, S. W., Wehe, D. K. Invariant features and the registration of rigid bodies. *Proc. IEEE Int. Conf. on Robotics and Autom.*, pp 932-937, 1999.
- [Sch01] Schuetz, C. J., Huegeli, H. T. Multi-Feature Matching Algorithm for Free-Form 3D Surface Registration, *International Conference on Pattern Recognition*, Vol. 14 No. 2, 1998, pages 982-984.
- [Ver01] Vergeest, J., Song, Y. ICP fitness analysis for 3D shape data matching. Technical Report, Delft University of Technology, 19 January 2006.
- [Zha01] Zhang, Z. Y. Iterative point matching for registration of free-form curves and surfaces. *Int. J. of Computer Vision*, 13(2), pp 119-15, Oct. 1994