

# Fractal modeling of vacuum arc cathode spots

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## ABSTRACT

This paper introduces a modeling algorithm of spots movement along a working surface of metal cathode during vacuum arc evaporation process with a use of fractal geometry. The proposed spot dynamics model reflects spot self-similar chaotic motion, retrograde rotation due to tangent magnetic field, spontaneous spot splitting and extinction. Several fractal generation techniques are considered: modified Diffusion Limited Aggregation, modified Dielectric Breakdown Method. The advantage of the fractal approach is that unlike computational modeling, it describes the spot movement in a more natural way. Modeling of vacuum arc cathode spots behavior allows reducing production costs of physical vapour deposition coated samples by increasing the cathode utilization efficiency.

## Keywords

Fractals, DLA, DBM, cathode spots, box counting dimension.

## 1. INTRODUCTION

The production of thin coatings in the micrometer or submicrometer range relies on physical vapour deposition (PVD) technologies. One variation of PVD methods utilizes an arc discharge burning under high vacuum conditions in the medium delivered from cathode spots. These are small bright clouds of dense highly ionized surface plasma that move over the cathode surface. According to [Mes95a] cathode spots are observed as short bursts or avalanches of electrons, which he named "ectons". Cathode spots concentrate the discharge energy in areas of only a few hundred micrometers in diameter. The plasma flow condenses on a substrate that is placed in front of a cathode and forms a functional or decorative layer. Functional coatings made of ultrathin diamond-like carbon are used widely on read-write heads of hard drives in the high-tech electronics industry, while decorative coatings are cost-effective substitution for samples intended to be covered with gold.

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In practice, as the arc is basically a current carrying conductor, an electromagnetic field is used to control the arc movement over the entire surface of the target, so that a cathode surface is evenly eroded over the time.

Many laws in physics are linear and periodic and show invariance to additive translation. However, not all physical phenomena can be described in this manner, in fact, a great number of phenomena are nonlinear, aperiodic, chaotic. In the 1980s, a branch of mathematics and physics started to flourish: the science of deterministic chaos and self-similar structures, dubbed "fractals" by Benoit Mandelbrot [Man83a]. Fractals are invariant to scaling, which makes them "self-similar" to multiplicative changes of scale. A self-similar object appears, generally, unchanged after increasing or decreasing the scale of measurement and observation. Self-similarity may be discrete or continuous, deterministic or probabilistic. Cathodic arcs show many features that suggest to model spot phenomena using the theory of fractals [Fed88a].

"The fractal approach is both effective and natural. Not only should it not be resisted, but one ought to wonder how one could have gone so long without it" [Man83a].

The paper is organized as follows. In section 2 we define the problem in general and present grounds of the problem analysis. Section 3 reviews recent

researches of the problem available in the literature. Section 4 presents evidences of fractal nature of cathode spots. In Section 5 we give a description of our new algorithm, considering peculiarities of implementation. Section 6 presents modeled images and their comparison with original samples. Finally, section 7 presents some conclusions and suggestions for future work.

## 2. PROBLEM DEFINITION

Modeling cathodic arcs has been a challenge for decades [Kes64a, Bei95a], which is due to the small scale and the short times of arc spot ignition processes (usually nanoseconds). Studies of the fluctuations aimed to identify characteristic scale and times yield general result that the greater the resolution the finer is the structure.

To understand the vacuum arc it is mandatory to know the processes occurring in these microscopic plasmas. The research of spot movement control is crucial to important evaporation process characteristics. Random arc can spend too much time on burning a big crater on energy efficient cathode surface peak producing high level of emission that causes droplets on the coated surface. Uniform distribution of arc residence along the cathode surface made with steering facilities results in a good homogeneity of the layer thickness on large coated samples with substantial decrease of droplets in the coating and helps to avoid dangerous cathode thinning.

Insight of the problem through a prism of computer graphics is the one of many solutions, taking into consideration an increase in computer graphics processing power during recent years.

## 3. RELATED WORK

As found in the experiments [Jut99a], the spot moves due to cyclical ignition and extinction of the fragments on a nanosecond timescale. Apparent fragment merging into one spot is due to the extinction of all of them except one, while apparent spot splitting is due to the formation of a new fragment outside the spot center.

The work [Rac99a] proposes off-lattice computational method for calculation of target erosion patterns during cathode arc evaporation process. For simulation, the magnetic field was been measured at 48x14 knots. The data were interpolated by bivariate spline approximation. Ten approximations of 25 seconds discharge time were calculated by evaluation of 500k time steps. The method yields calculated dependence of the spot residence time from the simulated tracks of cathode spots.

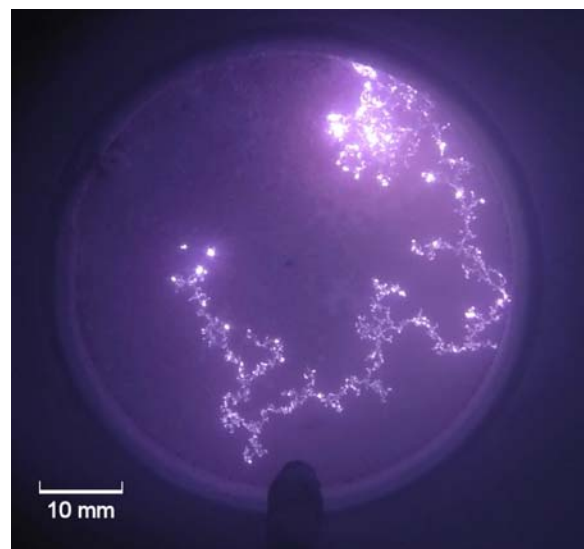
In the work [And05a], a brief review of spot properties is given, touching the differences between behaviour of spot type 1 (on cathodes surfaces with dielectric layers) and spot type 2 (on metallic, clean surfaces) as well as the known spot fragment and cell structure. Several points of evidence for the fractal nature of spots are provided. It is shown that fractal properties can be observed down to the cutoff by measurement resolution or occurrence of elementary steps in physical processes.

## 4. FRACTAL NATURE OF CATHODE SPOTS

Many objects in nature grow by the random addition of subunits. Examples include snow flakes, lightning, crack formation along a geological fault, and the growth of bacterial colonies. Although it might seem unlikely that such phenomena have much in common, the behavior observed in many models that have been developed in recent years gives us clues that these and many other natural phenomena can be understood in terms of a few unifying principles.

It is commonplace that a model of any physical process is defined in Euclidian geometry. The main disadvantage is that Euclidian geometry recreates the perceived visual characteristic of the process, but not the variety that actually builds its structure.

Figures 1a,b show spots movement over the working surface of a cylindrical shape cathode made of Al and Ti metal respectively.



**Figure 1a. Photograph of arc spots moving over cylindrical Al cathode. Exposure time 8 ms.**

Fractal geometry objects, unlike Euclidian geometry objects, whose dimension is integer only, are infinitely complex and have fractal dimension. The more closely they are examined, the more detail is revealed. For example, a tree is a fractal form. Using

Euclidian geometry calculations we can create an approximation of a tree, but it always looks artificial. Underlying the perceived visual characteristic is a controlled randomness, and increasing complexity at finer levels of resolution. This self-similar quality is the defining characteristic of fractals. Most natural structures have this characteristic.



**Figure 1b. Photograph of arc spots moving over cylindrical Ti cathode. Exposure time 8 ms.**

One model that can provide much insight into physics of cathode spot movement is known as diffusion limited aggregation or DLA [San00a]. While the DLA fractal growth rules are simple, they are highly nonlocal and give rise to complex branching structures that cannot be described easily by any small perturbation of a smooth surface.

A useful generalization of the DLA model is the Dielectric Breakdown Model (DBM) [Has01a], which includes an additional free parameter  $\eta$ . In the DBM model the growth probabilities at a specific site of the cluster is determined by the value of the electric field raised to a power  $\eta$ . With varying values of  $\eta$  clusters of different geometry are grown each with their own characteristic properties of the multifractal spectrum.

Self-similarity and associated power laws are abundant in cathode spot phenomena [And05a, And05b], including visual appearance, the trajectory of arc traces, the power laws in macroparticle distributions, the power laws found in noise distributions of the fluctuating parameters. For example, the noise of arc burning voltage shows a characteristic  $1/f^2$  dependence, where  $f$  is frequency, which is known as "Brownian noise". With this interpretation one can recognize that spot characteristics should not be expressed by a single number but described as a fractal model. This

approach allows to consolidate various other models such as a cathode layer model [Bei95a] and the explosive emission model [Mes95a]. Cathode spot phenomena are fractals in space and time.

Cathode spot in the presence of a magnetic field demonstrates the motion in a direction opposite to that predicted by Ampère's law and depends on the arc current, gas pressure, kind of gas, and magnetic field strength. Studies of the effects of these variables have been made [Jut00a], including measurements of velocity and the critical pressure at which reversal of motion occurs.

## 5. PROPOSED METHOD

Physical processes that give rise to the dielectric breakdown are known as Laplacian growth phenomena. There are several techniques for simulating Laplacian growth: Diffusion Limited Aggregation, the Dielectric Breakdown Model.

We introduce the modeling algorithm of spots movement along a working surface of metal cathode during vacuum arc evaporation process based on a random walker technique unifying DLA and DBM models.

During past several years General Processing Unit (*GPU*) underwent dramatic increase in terms of programmability and speed. The latest generations of programmable graphics processors can render to and read from floating-point precision render target textures [www04a]. This allows the iterative calculations, which give rise to deterministic fractal images, to be performed on the GPU hence decreasing performance lag in scores of times.

Despite a long way passed, GPU architecture still lacks friendly interface for general purpose processing exposed to public. While GPU power is unrivalled during computation of deterministic fractals [Gre05], e.g., Mandelbrot set, iterated function systems, it is rather helpless regarding stochastic fractals [Hil00]. Generation of stochastic fractals requires constant read-write processor capability on a large data array before single fractal element can be computed.

The algorithm utilizes central processing unit (*CPU*) for non-deterministic fractal generation in combination with GPU for per-pixel fractal image post processing.

The method takes as its input physical properties of cathode and evaporation process parameters (Figure 2), converting them into three basic parameters:  $\eta, \theta, \omega$ . At the next step CPU in combination with GPU generates the final fractal image. Basing on a time residence of an arc over evaporator cathode surface erosion statistics is generated.

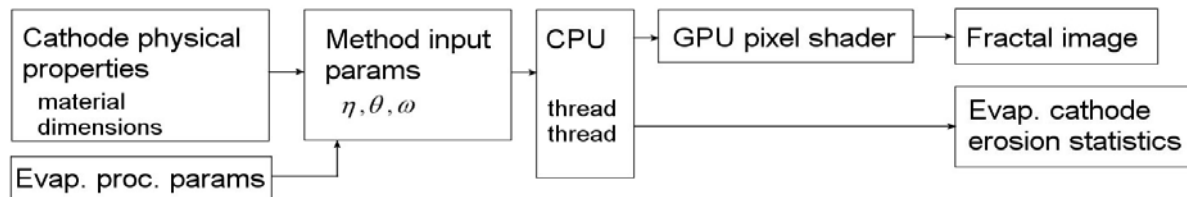


Figure 2. Data flowchart.

### CPU

On the first step a 2D grid is defined with an occupied cell by a seed particle. The grid is mapped onto a working surface of a cathode. Next, a particle is released from the perimeter of an arc  $C$  with a central angle  $\theta$  of a large circle whose center coincides with the seed. The particle undergoes a random walk, that is, diffuses, until it reaches a perimeter site of the seed and sticks. To respect the size of the cathode, random walker can not step outside cathode working surface boundary. Another random walker is released and allowed to walk until it reaches a perimeter site of one of the two particles in the cluster and sticks. The process is repeated many times until a cluster of appropriate size is formed.

Random walk movement is implemented by a Brownian motion that has a fractal dimension of 2 [Lee95a]. One of its occurrences is in microscopic particles world and is the result of random jostling of water molecules. Direction of a Brownian particle movement is a uniformly distributed random variable. So in moving from a given location in space to any other, the path taken by the particle is almost certain to fill the whole space before it reaches the destination.

Parameter  $\eta$  controls a dimension of the fractal. In a walk model it is equivalent to simultaneously releasing  $\eta$  random walkers and requiring that they all hit a given point for growth to occur. For large  $\eta$ , growth occurs largely at the tips of the cluster, and its branches become very long and thin, with a fractal dimension approaching 1 as  $\eta \rightarrow \infty$ . For small  $\eta$ , the cluster becomes less branched, with a dimension approaching 2 in the limit  $\eta \rightarrow 0$ .

Taking into consideration spot circular movement behaviour in the presence of tangent magnetic field the emitting arc  $C$  is rotated with an angular frequency  $\omega$ .

The algorithm is organized in a way to exercise CPU Hyper Threading ability working in several separate threads. Every thread works on computation of position of single particle contributing to the overall final image. An influence of the particle computed in the different thread is neglected.

### GPU

We implement the final step of the algorithm on the GPU using standard techniques in OpenGL. A fragment program applies Gaussian filter with a kernel size 3x3 to blur the final fractal image. To replicate the finite size of the craters that constitute the spot walk the filter selectively darkens pixels of fractal down to the color of the cathode basing on a texture with random values. Since not all state-of-art graphics hardware support random number functions, the texture that contains pseudo-random numbers is pre-computed on CPU, and the fragment program looks up in it guided with the fragment position and current time value.

## 6. RESULTS

Figures 3a,b show cathode spot walk models with the different values of  $\eta$  parameter. The spot movement trajectories modeled with fractal approach demonstrate close similarity to the original ones shown in Figures 1a,b. Generation speed of fractal cluster depends on number of particles involved and  $\eta$  parameter. Fractals can be generated from scratch or continuously grow towards generating arc.

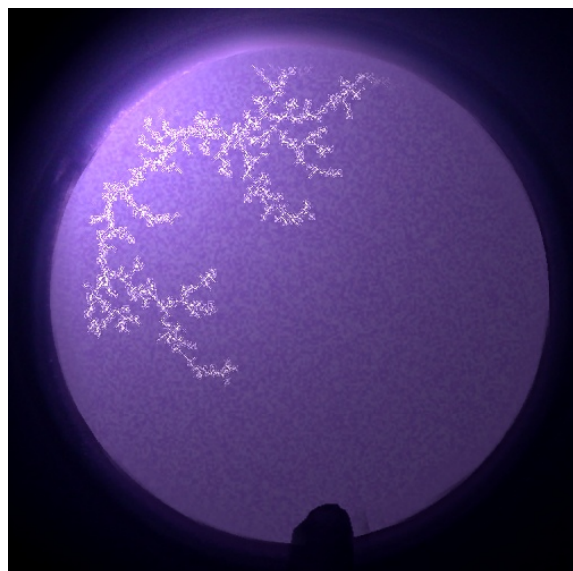
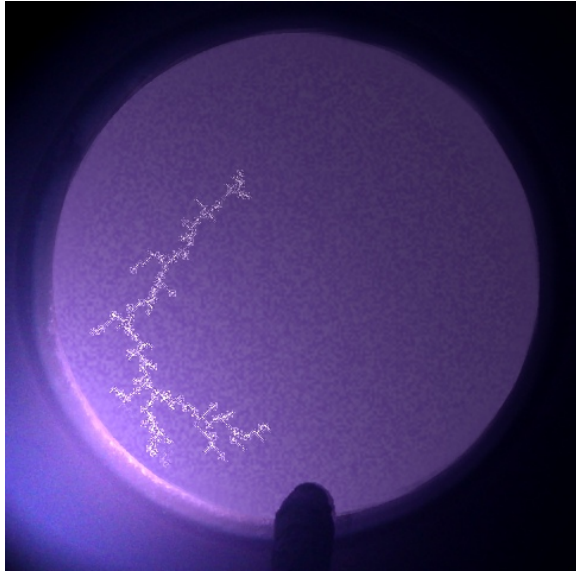


Figure 3a. Cathode spot walk fractal model containing 1200 particles ( $\eta = 1$ ).



**Figure 3b. Cathode spot walk fractal model containing 600 particles ( $\eta = 2$ ).**

One basic quantitative measure of a fractal structure is the dimension  $D$ . To calculate the fractal dimension, we use the box counting method in which space is divided into  $d$ -dimensional boxes of size  $l$ . Let  $N(l)$  equal the number of boxes that contain a piece of the trajectory. The fractal dimension is defined by the relation:

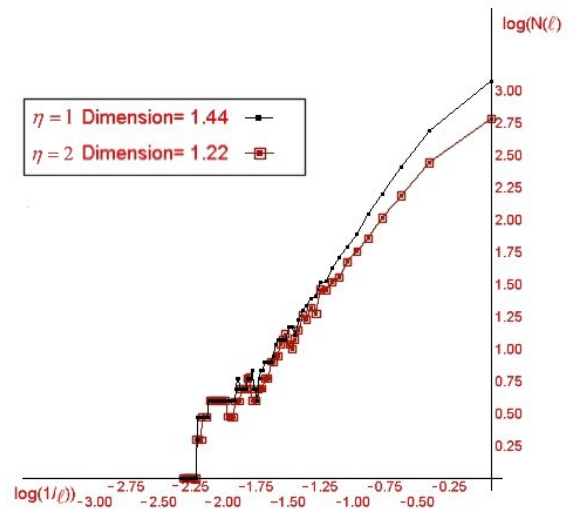
$$D = \lim_{l \rightarrow 0} \frac{\log N(l)}{\log 1/l}. \quad (1)$$

Equation (1) is accurate only when the number of boxes is much larger than  $N(l)$ .

The box counting algorithm starts from the box size approaching the size of the fractal object and decreases down to a single image pixel. An appropriate region for fractal dimension estimation lies within the center of the plot. When the box covers the whole image i.e.  $N(s) = 1$ , the curve slope equals to zero and when the box approaches image resolution the method starts to calculate fractal object area.

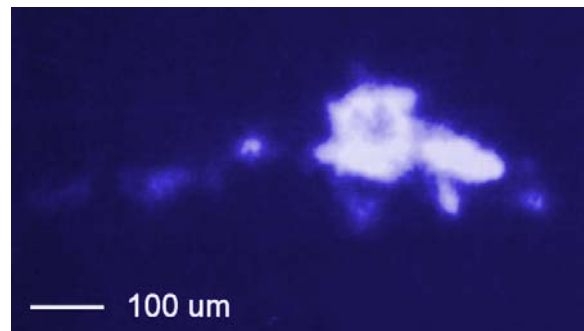
Figure 4 shows box counting plot calculated on the fractal structures generated on a square lattice with the proposed algorithm with  $\eta = 1$  and  $\eta = 2$ . A least squares fit to the box counting data yields a slope of approximately 1.44 and 1.22 respectively. The best estimates of  $D$  for the square lattices are  $D \approx 1.5$  and  $D \approx 1.3$  respectively [Gou95a].

The finite resolution of fractal image results in an underestimation of counts for smaller boxes, resulting in a convex log-log plot and an underestimate of dimension value itself.

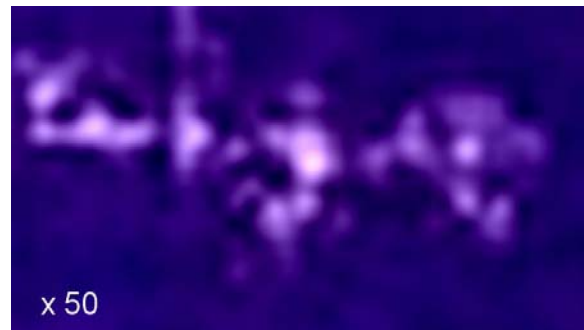


**Figure 4. Fractal dimension box counting plot. Resolution scale in dependence on number of boxes occupied.**

Cathode spot movement self-similarity is observed until one reaches cutoff limits that are imposed by elementary processes of spot fragment formation and extinction. Figure 5a illustrates complex structure of the spot that is consisted of fragments. Comparing original spot trace with a modeled one (Figure 5b) it can be seen that original fragments show much more resolution than got from 256x256 modeling grid.



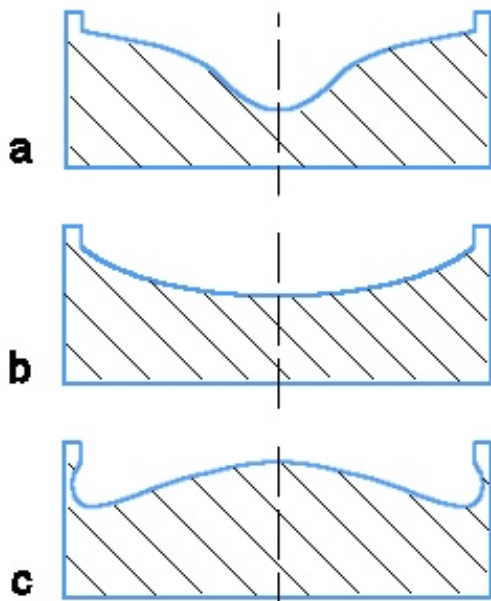
**Figure 5a. Photograph of the spot trace. Exposure time 0.1 ms.**



**Figure 5b. Fractal model of the spot trace.**

During continuous run of a cathode its working surface takes the form perpendicular at the every

point to the direction of magnetic field lines of stabilizing coil. It can result in dangerous cathode thinning in case of water-cooled system (Figure 6).



**Figure 6. Evaporator cathode erosion: a- without magnetic field; b- with optimal magnetic field strength; c- with excessive value of magnetic field strength.**

Modifying modeling method basic parameters  $\eta, \theta, \omega$  and analyzing cathode erosion perpendicular cross-sections the optimum value of a current in stabilizing coil can be defined indirectly.

## 7. CONCLUSIONS AND FUTURE WORK

Fractal geometry demonstrates that local randomness and global determinism can coexist to create a stable, self-similar structure of a pattern. The simulation model for the spot dynamics we introduced here could be shown to point out the significant features of the actual spot dynamics in magnetic field of a stabilizing coil.

The proposed method is capable of generating fractal clusters consisting of several hundred particles to several thousands in real time.

The set of parameters  $\eta, \theta, \omega$  defines the cluster structure and dimension, thus providing calculation basis for finding of optimum field strength and, consequently, prolong the life of a cathode.

## 8. ACKNOWLEDGMENTS

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