A Subdivision Scheme Based on Vertex Normals for Triangular Patches

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ABSTRACT

In order to achieve a smooth surface while rendering a triangle-based mesh, we need to eliminate the mismatch between the smoothness of the shading and the non-smoothness of the geometry that is particularly visible at silhouettes. To eliminate these artifacts, we substitute the geometry of a cubic triangular Bézier curved patch for a triangular flat geometry. Meanwhile a subdivision algorithm is proposed by using the degree elevation to approximate the triangular cubic Bézier patch only with little cost. The proposed algorithm can be processed without further knowledge of neighboring triangles, and can be operated as a process prior to a traditional rendering pipeline and required little change to existing authoring tools.

Keywords

Smoothing, geometric modeling, subdivision, surface, triangular Bézier patch.

Figure 1. (a) shading of initial model, (b) shading of our scheme

1. INTRODUCTION

Interpolated shading, such as Gouraud shading and Phong shading, has been used in computer graphics widely because of its effectiveness and simplicity. But in these two methods there is an inherent mismatch between the smoothness of the shading and the non-smoothness of the geometry that is particularly visible at silhouettes (see Figure 1a). In this paper, we study the problem for removing straight silhouette edges and develop a subdivision algorithm based on vertex normals for triangular patches (see Figure 2), which can be taken as an inexpensive means to improve the visual quality (see Figure 1b).

The paper is organized as follows. In section 2 we review the related work on this topic. Section 3 discusses how to construct a triangular cubic Bézier control net. Section 4 analyzes the degree elevation algorithm and adapts it to our subdivision scheme. Section 5 summarizes our work.

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2. RELATED WORK

Since Doo and Sabin [Doo78a] gave a corner-cutting algorithm to generate bi-quadratic uniform B-spline patches, Subdivision schemes defining smooth surfaces have been widely studied in the field of computer graphics [Cat78a, Loo87a, Aym99a, Jör98a, Den00a]. The basic idea behind subdivision is to create a smooth limit function by infinite refinement of an initial piecewise linear function. However, they rely on the availability of vertices of adjacent patches rather than vertices and normal vectors of the patch.

Recently local subdivision over a patch to eliminate the artifacts at silhouettes has received much attention. Alex Vlachos and Overveld [Vla01a, van97a] introduced curved point-normal (PN) triangle that is re-triangulated into a number of sub-triangles to replace the flat triangle. Stefan Karbacher [Ste00a, Ste98a, Ste98b] present a non-linear subdivision scheme for the refinement of triangle meshes that generates smooth surfaces with minimum curvature variations. Different to these schemes our scheme takes the information of each subdivision step into account. Our method is more simple and efficient. The algorithm is described in details in the following sections.

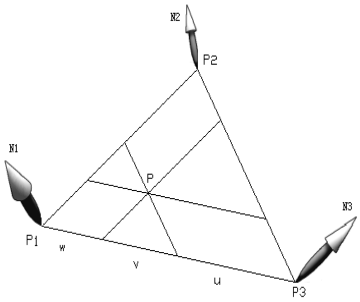

Figure 2. Input Triangle

3. CONTROL NET OF THE PATCHE

The bi-cubic triangular Bézier patch can be written in terms of Bernstein polynomials:

$$P(u, v, w) = \sum_{i=0}^{3} \sum_{j=0}^{3-i} b_{i,j,k} B^3_{i,j,k}(u, v, w);$$
$$0 \le u, v, w \le 1; \; u + v + w = 1; \quad i + j + k = 3; \quad i, j, k \ge 0 \tag{3.1}$$

$$B^3_{i,j,k}(u,v,w) = \frac{3!}{i!\,j!\,k!} u^i v^j w^k \tag{3.2}$$

For the patch, its end tangents must be perpendicular to the given normal vectors that attached to the three end points. We group the $b_{i,j,k}$ together as:

End vertices: b_{300}, b_{030}, $b_{003;}$ Center vertex: $b_{111;}$

Boundary vertices: b_{210}, b_{120}, b_{021}, b_{012}, b_{102}, $b_{201.}$

3.1 Construct the boundary curves

Here we construct a cubic curve leading from P_i to P_j that is normal to N_i at P_i and to N_j at P_j. In Bezier form, we already have $b_{3i} = P_i$ and $b_{3i+3} = P_j$, We still need to find the two inner Bézier points b_{3i+1} and b_{3i+2}. For each inner Bézier point, we have a one-parameter family of solutions: We only have to ensure that each triple b_{3i-1}, b_{3i}, b_{3i+1} is collinear on the tangent directors I_i at b_{3i}:

$$b_{3i+1} = b_{3i} + \alpha_i I_i;$$
$$b_{3i-1} = b_{3i} - \beta_{i-1} I_i \tag{3.3}$$

Where the tangent directors I_i have been normalized to be of unit length: $\| I_i \| = 1$.

So that the problem become to find reasonable values for α_i and β_i. Values for α_i and β_i that are too small cause the curve to have a corner at b_{3i}, while values that are too large can create loops. An optimal choice must depend on the desired application. Here, we set

$$\alpha_i = \beta_i = 0.33 \| \Delta X_i \|,$$
$$\| \Delta X_i \| = \| b_{3i+3} - b_{3i} \|$$

So using the upper method, we can decide the six boundary vertices: b_{210}, b_{120}, b_{021}, b_{012}, b_{102}, $b_{201.}$ Three end vertices defined as follows:

$$b_{300} = P_1, \; b_{030} = P_2, \; b_{003} = P_{3.}$$

3.2 Control net of triangular patches

The center vertex b_{111} corresponds to the center of the triangle and is decided by six boundary vertices and three end vertices. We move the center vertex from the position V to the average of the six boundary vertices and continue its motion in the same direction for 0.5 the distance already traveled [Vla01a, Far88a].

$$E = (b_{210}+b_{120}+b_{021}+b_{012}+b_{102}+b_{201})/6$$

$$V = (P_1+P_2+P_3)/3, \; b_{111} = E + (E-V)/2.$$

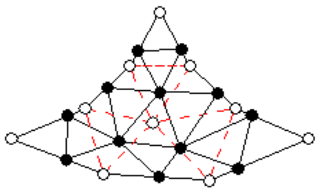

Figure 3. Bézier patch (from cubic to the fourth order)

4. SUBDIVISION SCHEMES

4.1 Degree Elevation

It is possible to represent a degree n triangular Bézier patch in the form of degree n+1:

$$P(u,v,w) = \sum_{i=0}^{n} \sum_{j=0}^{n-i} b_{i,j,k} B^n_{i,j,k}(u,v,w)$$

$$= \sum_{i=0}^{n+1} \sum_{j=0}^{n+1-i} b^*{}_{i,j,k} B^{n+1}{}_{i,j,k}(u,v,w) \quad (4.1)$$

For a triangular Bézier patch, we have u + v + w = 1 (see 3.1). We multiply the left hand of (4.1) by (u + v + w):

$$\sum_{i=0}^{n} \sum_{j=0}^{n-i} [b_{i,j,k} B^n{}_{i,j,k}(u,v,w)u + b_{i,j,k}$$

$$B^n{}_{i,j,k}(u,v,w)v + b_{i,j,k} B^n{}_{i,j,k}(u,v,w)w]$$

$$= \sum_{i=0}^{n+1} \sum_{j=0}^{n+1-i} b^*{}_{i,j,k} B^{n+1}{}_{i,j,k}(u,v,w) \quad (4.2)$$

Use (3.2) to substitute (4.2) and compare coefficients of the terms having the same degree, we can get:

$$b^*{}_{i,j,k} = \frac{1}{n+1}(ib_{i-1,j,k} + jb_{i,j-1,k} + kb_{i,j,k-1}) \quad (4.3)$$

4.2 Subdivision algorithm

Degree elevation process generates a sequence of control nets that have a cubic triangular Bézier surface as the limit after a sequence of successive refinements (see Figure 3). From (4.3) we can find the following three properties:

1. For three vertices of the input flat triangle, two of indices i, j, k equals 0. So we have $b_{n00} = P_1$, $b_{0n0} = P_2$, $b_{00n} = P_3$.
2. For boundary vertices, one of indices i, j, k equals 0, the boundary vertices are only decided by the two edge end points. Using this method, we can get C^0 continuity at the boundary of two surfaces and no hole occurs between two adjacent triangles.
3. Vertices inside the triangle are decided only by three end points.

Finally we also use the degree elevation equation to calculate the normal vector of each vertex for shading. Figure 4 illustrates the choice of normal. The equation defines as follows.

$$N^*{}_{i,j,k} = \frac{1}{n+1}(iN_{i-1,j,k} + jN_{i,j-1,k} + kN_{i,j,k-1}) \quad (4.4)$$

Where $N_{1,0,0} = N_1$, $N_{0,1,0} = N_2$, $N_{0,0,1} = N_3$

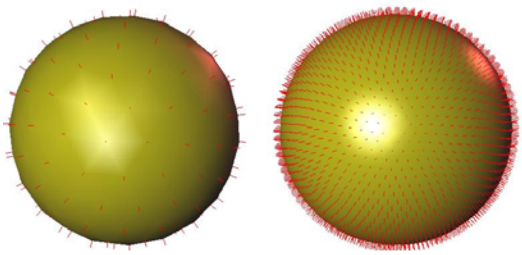

Figure 4: normal vector attached on each vertex (a) initial mesh; (b) after degree elevation

5. SUMMARY AND CONCLUSION

A method has been given for removing the artifacts at silhouettes, based on the vertex normals. Figure 5, 6 and 7 demonstrate that this algorithm can produce a visually smooth effect from an initial coarse mesh model. To some extent, the method eliminates the mismatch between the smoothness of the shading and the non-smoothness of the geometry. The proposed algorithm makes good use of the theory of degree elevation and simplifies the computation. Moreover it can be processed without further knowledge of neighbor triangles. Several questions still remain to be investigated. For example, these surfaces are still not C^1, the theoretical background of computation of the triangular cubic Bézier control net is still not satisfying. Adaptive subdivision is not discussed in this paper.

6. ACKNOWLEDGMENTS

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Figure 5,6,7,. (from left to right) (a) Initial mesh; (b) Mesh with our method; (c) shading model (initial mesh); (d) Shading with our method.