SHAPE MATCHING USING SET OF CURVE GEOMETRIC INVARIANT POINT

C. Pintavirooj*, P. Nantivatana***, P. Putjarupong*,
W. Withayachumnankul**, M. Sangworasil*
*Department of Electronics
**Department of Information Engineering,
Faculty of Engineering and Research Center for Communication and Information Technology (ReCCIT)
King Mongkut's Institute of Technology Ladkrabang, Thailand.
***Department of Electrical Engineering, Faculty of Engineering, Sripatum University, Thailand.
kpchucha@kmitl.ac.th

ABSTRACT

We introduce a non-iterative geometric-based method for shape matching using a novel set of geometric landmarks residing on a 2D contours. These landmarks are intrinsic and are computed from the differential geometry of the curve. We exploit the invariant properties of geometric landmarks that are local and preserved under the affine and some perspective transformation. Geometric invariant exploits coplanar five-point invariant and ration of area constructed from a sequence of consecutive landmarks. These invariants are preserved not only in affine map but weak perspective map as well. To reduce the sensitivity of the landmarks to noise, we use a B-Spline surface representation that smoothes out the curve prior to the computation of the landmarks. The matching is achieved by establishing correspondences between the landmarks after a conformal sorting based on derived absolute invariant and registering the contours. The experiments have shown that the purposed methods are robust and promising even in the presence of noise.

Keywords Geometric Invariant, Affine Invariant, B-Spline

1. INTRODUCTION

Shape matching is a central problem in visual information system, computer vision, pattern recognition image registration, and robotics. Application of shape matching includes image retrieval, industrial inspection, stereo vision, and fingerprint matching. The term shape is referred to the invariant geometrical properties of the relative distance among a set of static spatial features of an object. These static spatial features are known as shape features of the object. After extracting the shape features for a model and a scene, a similarity may be used to compare the shape features. The similarity measure is referred to as a shape measure.

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The shape measure should be invariant under certain class of geometric transformation of the object. In the simple scenario, shape measures are invariant to translation rotation and scale. In this case, the shape measures are invariant under similarity transformation. When included the invariance of shape measures to shear effect, the shape measures are said to be invariant under affine transformation. Finally in the complicated case, shape measures are invariant under perspective transformation when included the effect caused by perspective projection.

There are many techniques available to shape matching which can be classified mainly into two main categories; a global method and a local method. The global method works on an object as a whole; while the local method on a partially visible object or occlusion. The well-known global method includes Wavelet transform [Job95a, Wan97a], Moment-based approach [Hu62a], Fourier descriptors [Bri68a, Zah72a, Ott92a] and Median Axis Transformation [Blu67a, Blu78b, Pel81a]; while the well-known local method exploits intrinsic properties of the shape which includes geometric invariant [Bes88b, Gov99a, Mun92a], curvature extrema point [Bol82a, Mil89a, Chi89a, Rao94a], zero-curvature points

[Ali98a], line intersections [Sto82a], and [Kan81a], centroid of closed-boundary region [Gos86a], knot points [Coh98a], etc. In this paper study local geometric intrinsic features from which we derive a set of fiducial points which are related to curve derivative. Our shape matching is based on image registration. We use the set of fiducial points as a landmark. We then establish the correspondence between two sets of fiducially points by exploiting a local geometric projective (affine and weak perspective) invariants which are extracted from the five-point coplanar [Mun92a] spanned by the landmark points. Once the corresponding landmark is found, linear transformation parameter is estimated and the shape are aligned.

This paper is organized as follows. Section 2 introduces the local geometric curve features used as landmarks on the curve. The registration process is thoroughly discussed in section 3. Experimental results are shown in section 4. Discussion and conclusion are presented in section 5.

2. INTRINSIC GEOMETRIC FEATURE OF CURVE

Let r(t) =[x(t), y(t)], where t is a parameter, represent a shape (or curve) C in Cartesian coordinate system. The Inflection points are the point at which their curvatures are zero, i.e. The points at which

$$k(t) = \frac{r^{(1)}(t)\times r^{(2)}(t)}{|r^{(1)}(t)|^3} = \frac{\dot{x}(t)\ddot{y}(t)-\ddot{x}(t)\dot{y}(t)}{(\dot{x}(t)+\dot{y}(t)^2)^{3/2}} = 0$$

or the points at which

$$r^{(1)}(t)\times r^{(2)}(t) = \dot{x}(t)\ddot{y}(t)-\ddot{x}(t)\dot{y}(t) = 0 \qquad (2.1)$$

As a results, $r^{(1)}(t)$ and $r^{(2)}(t)$ are parallel at the inflection point.

Inflection points on the affine transformed curve are the points at which

$$k_a(t) = \frac{r_a^{(1)}(t)\times r_a^{(2)}(t)}{|r_a^{(1)}(t)|^3} = \frac{\dot{u}(t)\ddot{v}(t)-\ddot{u}(t)\dot{v}(t)}{(\dot{u}(t)+\dot{v}(t)^2)^{3/2}} = 0$$

or the points at which

$$\dot{u}(t)\ddot{v}(t)-\ddot{u}(t)\dot{v}(t) = \det[T]\left(\dot{x}(t)\ddot{y}(t)-\ddot{x}(t)\dot{y}(t)\right) = 0$$
$$\dot{x}(t)\ddot{y}(t)-\ddot{x}(t)\dot{y}(t) = 0 \qquad (2.2)$$

where [T] is the transformation matrix. Equation (2.2) is the same as (2.1). As a consequence, at the inflection points, the curvature or affine curvature is zero and $r^{(1)}(t)$ and $r^{(2)}(t)$ are parallel. Since the affine map preserve parallelism, we have shown that $r_a^{(1)}(t)$ and $r_a^{(2)}(t)$ are also parallel. Therefore the inflection points of the affine transformed curve are the transformed inflection points of the original curve and hence are affine invariant. Inflections have been widely used as the candidates for curve matching [Wal98a].

If we take derivative of (2.1), we have

$$\dot{x}(t)\dddot{y}(t)+\ddot{x}(t)\ddot{y}(t)-\ddot{x}(t)\ddot{y}(t)-\dddot{x}(t)\dot{y}(t) =$$
$$\dot{x}(t)\dddot{y}(t)-\dddot{x}(t)\dot{y}(t) = r^{(1)}(t)\times r^{(3)}(t) = 0 \qquad (2.3)$$

which is the point at which $r^{(1)}(t)$ and $r^{(3)}(t)$ are parallel and hence affine invariant. This point is the point at which the affine curvature is maximum. We call this point the maximum affine curvature point. Compared with zero affine curvature points, the maximum affine curvature points are more robust to noise. Moreover, threshold of affine curvature can be set such that only the maximum affine curvature point of which its affine curvature exceeding the threshold is selected. As a result, maximum affine curvature points caused by local disturbance are excluded.

The same concept can be applied to derive a set of derivative of curve. Table 1 shows a set of geometric feature. The maximum order of derivative of curve used in the computation of relative affine invariant is limited at two. In such case, we have 6 features; three of which associate with points of minimum relative affine invariant and the other three features associated with points of maximum relative affine.

Relative affine Invariant	Feature related to the zero relative affine Invariant	Feature related to the maximum of Relative affine Invariant
$k_1(t) = r(t)\times r^{(1)}(t)$	$f_1(t) \equiv$ $k_1(t)=0$ $r(t)\times r^{(1)}(t)=0$	$f_2(t) \equiv$ $k_1^{(1)}(t)=0$ $r(t)\times r^{(2)}(t)=0$
$k_2(t) = r(t)\times r^{(2)}(t)$	$f_3(t) \equiv$ $k_2(t)=0$ $r(t)\times r^{(2)}(t)=0$ same as $f_2(t)$	$f_4(t) \equiv$ $k_2^{(1)}(t)=0$ $[r(t)\times r^{(3)}(t)]+[r^{(1)}(t)\times r^{(2}(t)]=0$
$k_3(t) = r^{(1)}(t)\times r^{(2)}(t$	$f_5(t) \equiv$ $k_3(t)=0$ $r^{(1)}(t)\times r^{(2)}(t)=0$	$f_6(t) \equiv$ $k_3^{(1)}(t)=0$ $r^{(1)}(t)\times r^{(3)}(t)=0$

Table 1. Set of Geometric Feature of Curve

3. CONSTRUCT ABSOLUTE INVARIANT

Geometric invariants are shape descriptors that remain unchanged under geometric transformations such as perspective and affine transformation that will be used mainly as object recognition and/or image registration. A new approach of obtaining the invariants has been developed and used in the curve matching and recognition. A feature point explained in section 3 which is the local and invariant intrinsic property of the curve can be uniquely identified both before and after transformation. The identified feature points then are used to compute the geometric invariants which serve as the curve signature for the curve matching and recognition. The well-known perspective invariant is five-point coplanar.

Any five nonlinear points in the plane, namely $P_1,...,P_5$ can also form perspective invariant- the so-called five-point coplanar [Eam91a] with their image, $P_1',...P_5'$,

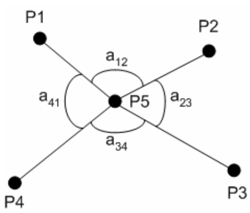

Figure 1. Five-point Coplanar Invaraint

$$\frac{|m'_{431}||m'_{521}|}{|m'_{421}||m'_{531}|} = \frac{|m_{431}||m_{521}|}{|m_{421}||m_{531}|} \qquad (3.1)$$

where $m_{ijk} = (P_i \; P_j \; P_k)$ with $P_i = (x_i \; y_i \; 1)^t$, $m'_{ijk} = (P_i' \; P_j' \; P_k')$ with $P_i' = (x_i' \; y_i' \; 1)^t$ and $|m|$ is the determinant of m.

Since the invariant relationship in equation (3.1) holds under a perspective transformation, a set of perspective invariant can be constructed by considering 4 consecutive maximum-curvature points and a centroid which is also preserved under an affine and perspective transformation. For a curve with n maximum-curvature points, there are n set of geometric invariants denoted as $I(k)$ for $k = 1,2,...,n$

In the absence of noise and/or occlusion, each of $I_a(j)$ in the affine-transformed curve (3.1) will have a counter part *I(i)* on the original curve with $I_a(j) = I(j)$, with that counterpart easily determined through a circular shift involving n comparison where n is the number of invariant. In the presence of noise, occlusion and non-linear transformation, we allow smaller error percentage between counterpart invariant. We adopted a run length method to decide on the correspondence between the two ordered set of maximum curvature point. For every starting point on the transformed, this run length method computes a sequence of consecutive invariant that satisfies $|I(i) - I_a(j)| < 0.05|I(i)|$ and declares a match based on the longest string. Once this correspondence is found, these matched landmarks are used to estimate the transformation matrix

4. EXPERIMENT

To demonstrate the capabilities of the shape matching exploiting a set of geometric invariance curve, we applied the algorithm for shape classification thirteen contours of fish which have the various patterns. Each contour has approximately 400 points of coordinates. The third order B-Spline with control points of 30 is employed to fit the contours of fish. To find the fish which matches with the 0^{th} fish, a feature point f1 is determined. Figure 2 shows the results of shape alignment

Figure 2. The alignment of contour of fish o^{th} (dark contour) against thirteen contours of fish (light contour) using feature f1 and five-point coplanar invariance

5. DISCUSSION AND CONCLUSION

This paper we explore the geometric invariant points. The fact that the geometric invariant points are local and preserved under an affine transformation makes them feasible to use for matching and classification of curve. By exploiting the absolute affine and perspective invariant using the feature points, the correspondence between the feature points on the original and the transformed curve is established, the transformation parameters are computed and the transformed curve is aligned against an original.

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