Parallel Edge Finite Element Method to Solve Eddy Current Field Problems

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***Abstract*—The paper presents and compares two non-overlapping domain decomposition methods (DDMs), which can be used to the parallelization of the finite element method (FEM) with edge element approximation. In this case, the methods under investigation are the Schur complement method and the Lagrange multiplier based Finite Element Tearing and Interconnecting (FETI) method, and their solvers. The performance of these methods has been investigated in detail for eddy current field problems as case studies.**

***Index Terms*—Parallel finite element method, Domain decomposition, Eddy current field**

I. Introduction

The finite element method [1] is an important technique for the solution of a wide range of problems in science and engineering. The most time consuming part of finite element computations is the solution of the large sparse system of equations. Therefore, the solution of a large system of equations must be parallelized in order to speedup the numerical computations.

The reason for employing the domain decomposition technique was the small memory of computers. To solve large scale problems, a domain has been divided into sub-domains that fit into the computer memory. However, the computer memory grow, the demand for solution of large real life problems is always ahead of computer capabilities. The large scale computations and simulations performed with finite element method (FEM) often require very long computation time. While limited progress can be reached with improvement of numerical algorithms, a radical time reduction can be made with multiprocessor computation. In order to perform finite element analysis a computer with parallel processors, computations should be distributed across processors.

The Schur complement method [3], [4], as sequential algorithm was started to use many decades ago, when computer RAM was extremely small. Nevertheless, nowadays, this method is a very popular parallel domain decomposition technique among engineers [4].

In the last decade, the Finite Element Tearing and Interconnecting (FETI) method [2], [3], [5] has seemed as one of the most powerful and one of the most popular solvers for numerical computation. The FETI requires fewer interprocess communication, than the Schur complement method, while is still offers the same amount of parallelism [3].

This paper presents a parallel approach for the solution of two-dimensional eddy current field problems by parallel finite element method. These problems are benchmarks to show the steps of the DDMs with parallel finite element technique. The comparison focused on the time, speedup and memory efficiency of solvers of methods. Furthermore, the nodal and edge element based parallel FEM method have also been compared.

II. Parallel Finite Element Method

The general form of a linear algebraic problem arising from the discretization of the problem defined on the domain Ω can be written as [1], [2]

$$\left.\begin{array}{r}\nabla(\nu_0\nu_r\nabla\times\vec{A})=\nabla\times\vec{T}_0\\ \nabla\times(\nu_0\nabla\times\vec{A}^*)+\sigma\dfrac{\partial\vec{A}^*}{\partial t}=\vec{0}\\ \text{Appropriate boundary condition}\end{array}\right\}\quad\rightarrow\quad\mathbf{K}\mathbf{a}=\mathbf{b},\qquad(1)$$

where ν_0, ν_r are the reluctivity of vacuum and the relative reluctivity, $\vec{A}$, $\vec{A}^*$ are the magnetic vector potential in the conducting part Ω_c and in the nonconducting part Ω_n, $\vec{T}_0$ is the impressed current vector potential, σ is the conductivity, **K** is the symmetric positive definite matrix, **b** on the right hand side of the equations represents the excitation, and **a** contains the unknown potentials. The $\vec{T}_0$ is known quantity, i.e. it is calculated before the numerical simulation, also in parallel.

The application of domain decomposition methods to discretised problems is based on the split of FEM mesh into several groups while additional conditions assuring continuity are introduced.

A. Schur Complement Method

After the problem is partitioned into a set of N_S disconnected sub-domains, (1) has been split into N_S particular blocks [3], [4]

$$\begin{bmatrix}\mathbf{K}_{jj} & \mathbf{K}_{j\Gamma_j}\\ \mathbf{K}_{j\Gamma_j} & \mathbf{K}_{\Gamma_j\Gamma_j}\end{bmatrix}\begin{bmatrix}\mathbf{a}_j\\ \mathbf{a}_{\Gamma_j}\end{bmatrix}=\begin{bmatrix}\mathbf{b}_j\\ \mathbf{b}_{\Gamma_j}\end{bmatrix}\qquad(2)$$

where *j*=1, ..., N_S, $\mathbf{K}_{jj}$ is the symmetric positive definite sub-matrix of the j^{th} sub-domain, $\mathbf{a}_j$ is the vector of the right hand side defined inside the sub-domain. The sub-matrix $\mathbf{K}_{j\Gamma_j}=\mathbf{K}^{\mathrm{T}}_{\Gamma_j j}$ contains the value of j^{th} sub-domain, which connect to the interface boundary unknowns of that region. The $\mathbf{K}_{\Gamma_j \Gamma_j}$ and $\mathbf{a}_{\Gamma_j}$ expresses the coupling of the interface unknowns.

The assembly of the sub-matrices can be performed parallel by independent processors. However, for the solution of $\mathbf{a}_{\Gamma_i}$ use the sub-matrices from the independent processors. After obtaining the unknowns of interface boundary, it must be sent back to the independent processors to calculate the sub-solutions.

B. *Finite Element Tearing and Interconnecting Method*

After mesh partitioning, the FETI method consists in transforming the original problem (1) with the equivalent system of sub-domain equations [2], [3], [5]

$$\mathbf{K}_j \mathbf{a}_j = \mathbf{f}_j - \mathbf{B}_j^{\mathrm{T}} \mathbf{\Lambda}, \tag{3}$$

with the compatibility of the magnetic vector potentials at the sub-domain interface [2], [3], [5]

$$\sum_{j=1}^{N_S} \mathbf{B}_j \mathbf{a}_j = \mathbf{0}, \tag{4}$$

where j=1, ..., N_S, the number of sub-domains, $\mathbf{K}_j$,$\mathbf{b}_j$ and $\mathbf{a}_j$ are respectively the system matrix, the representation of the excitation and the unknown potentials of j^{th} sub-domain. The vector of Lagrange multipliers $\mathbf{\Lambda}$ introduced for enforcing the constraints (4) on the sub-domain interface, and $\mathbf{B}_j$ is a signed ($\pm$) Boolean mapping matrix, which is used to express the compatibility condition at the j^{th} sub-domain interface.

Usually, the partitioned problem may contain $N_f \leq N_S$ floating sub-domains, where matrices $\mathbf{K}_j$ being singular [5]. Because of the floating sub-domain, a robust direct solver or a preconditioned iterative solver, here the preconditioned conjugate gradient (PCG) is needed to handle the singular matrices.

III. Comparison

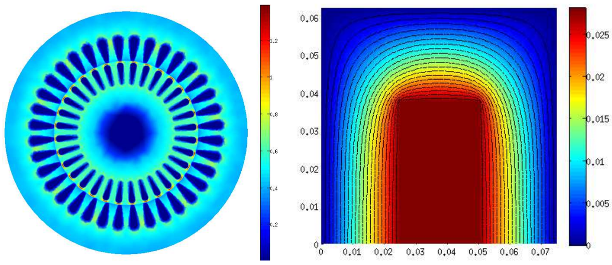
Two problems have been used for comparison, the induction motor problem (Fig. (1a)) and the quarter of the transformer (Fig. (1b)), because in this case the problem contains floating sub-domain.

Fig. 2 shows the time of the function of the number of the applied processors. The number of processors is equal the number of sub-domains at all simulations. The Schur solver is little bit slower (Fig. (2a)) or much faster (Fig. (2b)), than the FETI method. It seems to be, the Schur complement method is the faster at nodal finite elements, but it depends on the problem.

IV. Conclusions

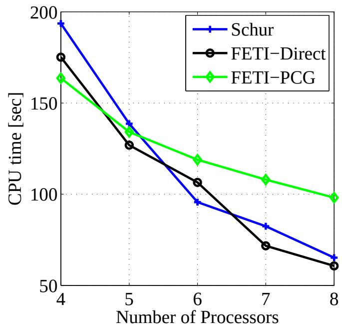
It can be concluded that the Schur complement method is faster than the FETI solvers, but the memory efficiency of the FETI solvers, mostly the iterative solver are better in the case of nodal element based finite element method.

The full paper will present the solvers of domain decomposition methods, and the comparison of the edge element and the nodal element based parallel finite element method through the two benchmark problem.

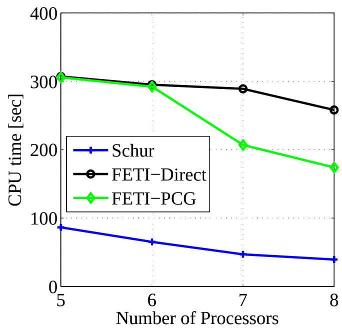


(a) Induction motor. (b) Transformer.

Figure 1: The magnetic flux density distribution of induction motor and the magnetic vector potential and equipotential lines of transformer.



(a) Motor problem.



(b) Transformer problem.

Figure 2: The time of the nodal element based finite element method.

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