# A numerical method for magnetic field determination in three-phase bus-bars of rectangular cross section

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*Abstract* - This paper presents a new numerical computation method for determining the magnetic field distributions in highcurrent busducts of rectangular busbars. This method is based on the integral equation method and the Partial Element Equivalent Circuit (PEEC) method. It takes into account the skin effect and proximity effects, as well as the complete electromagnetic coupling between phase bars and the neutral bar. In particular, the magnetic fields in busbars of unshielded three-phase systems with rectangular phase and neutral busbars, and the use of the method are described. Finally, two applications to three-phase unshielded systems busbars are presented.

Keywords: Rectangular busbar, high-current bus duct, magnetic field, numerical method.

# I. INTRODUCTION

High-current air-insulated bus duct systems with rectangular busbars are often used in power generation and substation, due to their easy installation and utilization.

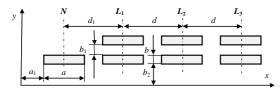


Fig. 1. Three phase high-current bus duct of rectangular cross-section with two busbars per phase and one neutral busbar

The distribution of AC magnetic field in the region surrounding the busbars can be found exactly only for simple geometries like round wires and tubes, or very long and thin rectangular busbars (tapes or strips). For more complex cross-sections analytical-numerical and numerical methods must be used to find the magnetic field distributions, which is further modified by the proximity of other conductors – "proximity effect". Both the skin effect and proximity effect will generally cause the magnetic field distribution differs considerably from the expected one without taking into account both effects.

### II. MULTICONDUCTOR MODEL OF THE BUSBARS

In this model, each phase, neutral busbars and each plate of enclosure is divided in several thin subbars. For the  $m^{\text{th}}$  subbar the integral equation can be written as follows

$$\frac{\underline{J}_{i,k}^{(m)}(X)}{\sigma_i} + \frac{j\omega\mu_0}{4\pi} \sum_{j=1}^{N_c} \sum_{l=1}^{N_j} \sum_{n=1}^{N_{j,l}} \int_{v_{l,l}} \frac{\underline{J}_{j,l}^{(n)}(Y)}{\rho_{XY}} dv_{j,l}^{(n)} = \underline{u}_i \quad (1)$$

From Eq. (1) we have

$$R_{i,k}^{(m)} \underline{I}_{i,k}^{(m)} + j\omega \sum_{j=1}^{N_c} \sum_{l=1}^{N_j} \sum_{n=1}^{N_{j,l}} M_{(i,k)(j,l)}^{(m,n)} \underline{I}_{j,l}^{(n)} = \underline{U}_i$$
(2)

where  $R_{i,k}^{(m)}$  is the resistance of of the  $m^{\text{th}}$  subbar and  $M_{(i,k)(j,l)}^{(m,n)}$  is the self or the mutual inductance. Then, we can find the admittance matrix  $\underline{\hat{Y}}$ , which is the inverse matrix of the impedance matrix  $\underline{\hat{Z}}$ , and it is expressed as

and the current

$$\underline{I}_{i,k}^{(m)} = \sum_{j=1}^{N_c} \sum_{l=1}^{N_j} \sum_{n=1}^{N_{j,l}} \underline{Y}_{(i,k)(j,l)}^{(m,n)} \underline{U}_j$$
(4)

(3)

## III. MAGNETIC FIELD

 $\hat{\boldsymbol{Y}} = \boldsymbol{Y}_{(i,k)(i,l)}^{(m,n)} = \hat{\boldsymbol{Z}}^{-1}$ 

Knowing the current distribution (4) in each subbar, the evaluation of the magnetic field can be performed. The vector magnetic potential  $\underline{A}_{i,k}^{(m)}(X)$  induced by the  $m^{\text{th}}$  subbar is given by

$$\underline{A}_{i,k}^{(m)}(X) = \frac{\mu_0}{4\pi} \iiint_{V_{i,k}^{(m)}} \frac{\underline{J}_{i,k}^{(m)}(Y)}{\rho_{XY}} dv = \mathbf{a}_z \underline{A}_{i,k}^{(m)}(x, y, z)$$
(5)

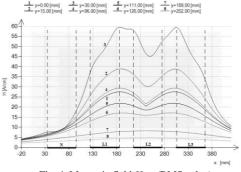


Fig. 4. Magnetic field  $H_{\text{max}}$  (RMS value)

#### IV. CONCLUSION

The proposed approached combines the Partial Element Equivalent Circuit (PEEC) method with the exact closed formulae for AC self and mutual inductances of rectangular conductors of any dimensions, which allows the precise accounting for the skin and proximity effects. Complete electromagnetic coupling between the phase busbars and the neutral busbar is taken into account as well.

### V. REFERENCES

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