An algorithm for finding multiple DC operating points using the concept of restart homotopy

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Abstract **This paper deals with BJT and MOS transistor circuits having multiple DC operating points. To find the operating points an algorithm is developed, based on the idea of restart homotopy and the Adams-Bashforth method for solving differential homotopy equation. A numerical example illustrates the proposed approach.**

Keywords **DC analysis, multiple solutions, restart homotopy.**

I. INTRODUCTION

Circuit having multiple DC operating points are frequently encountered in electronic devices. Finding the operating points is a basic and difficult task in circuit simulation. The methods for finding all the DC operating points are very time consuming and capable of analysing only small size circuits. In those circumstances research works have been recently focused on the problem of finding the multiple operating points, but not necessarily all of them [1]-[4]. They do not require a high computing power and enable us to analyse much larger size circuits. This paper brings a method in this area based on the concept of the restart homotopy [6] which has been successfully applied to fault diagnosis of analogue circuits [5]. The method is dedicated to BJT and MOS transistor circuits manufactured in micrometer technology. The transistors are characterized by the Gummel-Poon and the Shichman-Hodges models, respectively.

II. MAIN RESULTS

The main idea of the method is as follows. On the basis of the nodal equation $\boldsymbol{g}(\boldsymbol{e}) = \boldsymbol{0}$ describing the circuit, the homotopy equation is written

$$\boldsymbol{h}(\boldsymbol{e}, \lambda) = \boldsymbol{g}(\boldsymbol{e}) - (1-\lambda)\boldsymbol{g}\left(\boldsymbol{e}^{(0)}\right) = \boldsymbol{0} \ , \qquad (1)$$

where $\boldsymbol{e}^{(0)}$ is an arbitrary point. For this equation the homotopy differential equation is formulated

$$\frac{\mathrm{d}\boldsymbol{e}}{\mathrm{d}\lambda} = \boldsymbol{f}(\boldsymbol{e}), \text{ where } \boldsymbol{f}(\boldsymbol{e}) = -\left(\frac{\mathrm{d}\boldsymbol{g}}{\mathrm{d}\boldsymbol{e}}(\boldsymbol{e})\right)^{-1} \boldsymbol{g}\left(\boldsymbol{e}^{(0)}\right) \qquad (2)$$

and solved using the second order Adams-Bashforth algorithm

$$\boldsymbol{e}^{(k+1)} = \boldsymbol{e}^{(k)} + \left(\frac{3}{2}\boldsymbol{f}\left(\boldsymbol{e}^{(k)}\right) - \frac{1}{2}\boldsymbol{f}\left(\boldsymbol{e}^{(k-1)}\right)\right)(\lambda_{k+1} - \lambda_k) \ . \qquad (3)$$

If at some index $k = l$ the constraint $\left\| \boldsymbol{h}\left(\boldsymbol{e}^{(l)}, \lambda_l\right) \right\| < \varepsilon$ is violated, then the restart homotopy [6] is created

$$\boldsymbol{h}_l(\boldsymbol{e}, \lambda) = \boldsymbol{g}(\boldsymbol{e}) - \frac{1-\lambda}{1-\lambda_l}\boldsymbol{g}\left(\boldsymbol{e}^{(l)}\right) \qquad (4)$$

and the procedure of solving the homotopy differential equation continued. Any intersection of the homotopy path with the $\alpha = 1$ plane gives a DC operating point.

III. NUMERICAL EXAMPLE

Let us consider the CMOS circuit shown in Fig. 1. The proposed method gives three DC operating points. Voltages at nodes A, B, and C, all in volts, corresponding to these points are as follows:

$$\left(v_A^*\right)_1 = 0.872, \quad \left(v_A^*\right)_2 = 2.750, \quad \left(v_A^*\right)_3 = 1.952,$$
$$\left(v_B^*\right)_1 = 2.407, \quad \left(v_B^*\right)_2 = 0.891, \quad \left(v_B^*\right)_3 = 1.928,$$
$$\left(v_C^*\right)_1 = 1.244, \quad \left(v_C^*\right)_2 = 2.406, \quad \left(v_C^*\right)_3 = 1.948.$$

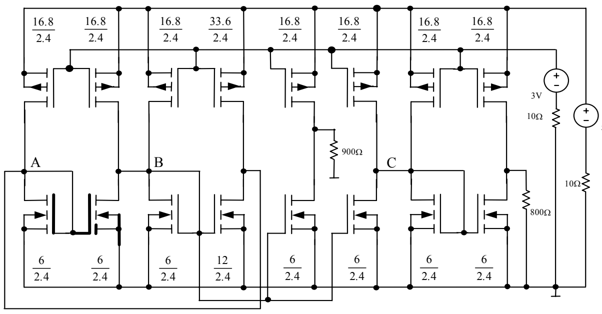

Fig. 1. An exemplary CMOS circuit

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V. REFERENCES

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