NEW THEORY OF PATTERN RECOGNITION ON THE BASIS OF STOCHASTIC GEOMETRY

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ABSTRACT

The article offers a new approach towards the construction of recognition features independent of images' displacement or linear deformation. The distinguishing characteristics of the group of features under study is representing each of them as a sequential composition of three functionals acting upon the function of one variable. The process to construct the new features suggested boasts of the advantages as follows: a)a host of new features can be easily constructed; b) the features obtained can be structurized along with parallel computations. Great many new features have been constructed to successfully solve the task of recognizing coloured images in biological systems, for instance, blood cells in gematology.

Keywords: pattern recognition, stochastic geometry, triple functional.

1. INTRODUCTION

In the field of pattern recognition we traditionally distinguish feature construction and decision procedure. In literature on cybernetics a vast majority of works on the pattern recognition have been historically devoted to decision rules, there actually being no works on feature construction. There has been general agreement that it could be explained by the fact that the process of constructing features is empirical and dependent on the intuition of the recognition system designer.

The approach of stochastic geometry, developed in [Fedtv90b], allows us to bridge the gap and create processes to generate great many new features for image recognition, along with a constitutive theory of features. Such a prominent shift of stress from decision procedures to new recognition features gives the approach a strong resemblance to neuro-computing.

In [Fedtv90b], the author suggests using probabilities of geometrical events understood as the result of geometrical objects interaction (intersections, overlapping and so on), as image recognition features. Geometrical objects here are, on the one hand, complex scanning trajectories with random parameters (segments, lines, curves, figures, etc.), and on the other hand, fragments of an image being recognized. The structure of similar recognition systems and examples of particular technical implementations, are considered. Possible extensions of the fundamental recognition process on stochastic geometry are considered as well. One of the extensions deals with a complication of observing a random event (an intersection of a scan trace and an image), i.e. with the application of more complicated recognition features.

The article presents the basics of a new theory to construct recognition features and shows its applicability to the recognition of classes of images, the latter being dependent neither on image motions, nor on affine transformations. The present paper is an extention of [Fedtv90f]. The theory suggested has been devoted on the basis of [Fedtv94c] , [Fedtv94b], [Fedtv94d], [Fedtv94a].

Preservation of lines within the image is known to be a feature common for the affine transformations. An affine transformation could be defined as a one-to-one continual transformation of a plane where lines lengthen into lines. The distinguishing feature of the group of transformations being considered is that each of them can be represented as a sequential composition of three functionals, each acting upon the function of one variable.

As we are interested in images' motions, and here we mean sliding along certain lines, we should be naturally interested in the functionals' reaction to the slid-

ing of the kind.
We can establish two types of reactions, namely: functionals' independence of shifts, and their dependence (or sensitivity), where the shift could be distinguished as a separate addend. For each of the three functionals one could easily find tens of various concretizations satisfying the conditions required (they are to be found further).
Hence, we can immediately obtain thousands of new features which are invariant to motion. To recognize 2^n objects we need about n features, so, there exists a possibility to recognize samples characterized by a multiple alphabet, and, for the practical purposes, to diagnose a large number of diseases.
To diagnose cases, modern medicine uses microphotoes of tissues, and, in particular, those of blood specimen. The peculiarities blood cells popssess make it possible to make a diagnosis. But when lots of photoes are to be studied, a researcher cannot avoid making mistakes for the reason of fatigue. Moreover, the diagnosis is not to be made if you have tested only one cell. It is possible only after you've analysed the statistics of the cell's peculiarities distribution. It means that you are to analyse great many microobjects to build a solid foundation for statistically meaningful samples and tables. It is extremely difficult for a researcher to analyse large amounts of information manually. Hence the task of machine analysis of blood micriphotoes and tissue sections comes topical. Theortical methods of recognition involving the features invariant to images' motion, become especially important. Advantages of new theory are shown when distinguishing features of blood cells.
Let us consider an input retina of a recognizing device. By the retina we mean an image plane section which is being scanned. Within the plane section there is an image, the rest of the plane being background. Thus, the image is finite. Consider a random straight line l, which may intersect the image. Supposing, intersection of l with the image allows us to compute a certain g characterizing their location as to each other. While tossing l randomly onto the plane several times, we could obtain a sample for random value g. Then we could define an empirical characteristic n of the random value g. There whole procedure described could be implemented in a radio-electronic system, which performs image recognition [Fedtv90b].
Mathematical apparatus of the procedure considered has been researched intensively by stochastic geometry. It has been established that under certain conditions n might have an explicit geometrical meaning. It is important for us that, being easily implemented in devices, the idea may provide a starting point to get new features for pattern recognition both in theory and in practice.

Formulas are presented in [Fedtv90b] to serve the basis for recognition criteria. Only binary images (black figures against a white background) are being considered.

1. Consider an image as a piecewise smooth curve, which may be a boundary of a figure. Let g the number of intersections of the curve with a random l. Then mathematical expectation Mg is proportional to the length of the curve.
2. Consider an image as a convex figure. It may be a convex hull of another figure. Let g be the length of the convex figure intersection with l. Then the average values Mg^0, Mg^1 and Mg^2 are proportional to the perimeter, square and eigenpotential of the uniform domain, respectively.

2. TRACE-TRANSFORMATION

The above-considered formulas and their multiple analogues possess the following limitations as to pattern recognition: 1) their number proves limited, for explicit geometrical characteristics are few, and we need thousands of, and even more, features; 2) the formulas apply to binary images only. Possibilities of parallel computation (with several straight lines being processed simultaneously) and those of stochastic implementation, should be considered advantageous. Stochastic implementation makes it possible to cut the process, a required accuracy having been reached. Features are normally known to be strongly dependent on object rotation and shift, the latter being totally uninformative for a host of recognition problems.
Within the article, we put forward a generalization of the approach mentioned, to cope with its limitations and preserve the advantages, the generalization being complete in a certain aspect.
Let F denote a finite image. Given straight line l, g characterizing the location of l and the image as to each other, is to be computed according to a certain rule T: $g = \mathrm{T}(l, F)$; map T is called a functional.
Just like in stochastic geometry, random value $g = \mathrm{T}(l, F)$ is defined, its distribution being independent of image shifts and rotations. Therefore, numeric characteristics of the random value may again serve as image features, which are to be established with the help of special engineering devices and systems. The limitation of the new family of features is that they originally lack an explicit geometrical meaning, and their differentiating capability is a priori unknown. However for pattern recognition, it proves not very important, experimental testing being decisive.
Let us note yet another property of a totally invariant

functional T (Trace): it is not necessarily to be defined by a cross-section of a straight line with an image only. Other information, say, characteristics of the cross-section vicinity could be used for computation as well.

To understand that the generalization proposed in a certain aspect exhausts its own possibilities, we are going to state the theory of Trace-transformation (or Tr-transformations). Polar coordinates introduced to the plane, l is characterized by distance p from the origin to l, and by angle θ (up to 2π) of its directional vector:

$$l = \{(x, y): x\cos\theta + y\sin\theta = p\}, l = l(\theta, p),$$

where x, y are Cartesian coordinates on the plane. If we allow p to take negative values, too, then

$$l(\theta, p) = l(\theta + \pi, -p).$$

Thus, a set of all directed straight lines intersecting a circle of radius R with the center in the origin (the «retina»), is unambiguously parameterized by set

$$\Lambda = \{(\theta, p): 0 \le \theta \le \pi, -R \le p \le R\}$$

which provided parameters $(0, p)$ and $(\pi, -p)$ define the same straight line. The set of straight lines The set of straight lines on the retina are clearly seen to be topologically nothing but a Möbius band. Thus, the set of numbers $\mathrm{T}(l(\theta, p), F)$, depending on a point on Möbius band Λ, is a certain image transform, which we may call a Trace-transform. If, for instance, a matrix represents a Trace-transform in numerical analysis, we may call it a Trace-matrix. If axis 0θ is directed horizontally, and axis $0p$ vertically, matrix element, indicated (i, j), i.e. value $\mathrm{T}(l(\theta_j, p_i), F)$, is in point θ_j, p_i. θ_j and p_i are here certain values of uniform discrete grids on the axes mentioned. Along the horizontal axis, matrix is 2π-periodic, its columns rotating within each interval of length π.

In addition, let us consider, that if l does not intersect the image, $\mathrm{T}(l, F)$ is a given number (say, 0) or another fixed element, if T is nonnumeric. In this case, a new image Tr (F) corresponds to the original image F, $\mathrm{T}(l(\theta, p), F)$ may be treated as an image which characteristics at (θ, p) are its Tr-image).

Fig. 1,a explains the computation of a Trace - transformation. It shows how to obtain a binary function $f(\theta, p, t)$ of a real variable for a scanning line l. Function $f(\theta, p, \bullet)$ equals 1 within the interval (t_1, t_2) and (t_3, t_4). Within other precise it equals 0. Let T stand for a functional applied to the function , its independent variable being designated by t. Thus we get $g(\theta, p) = \mathrm{T}f(\theta, p, t)$. We call function g result of trace – transformation (trace – transform).

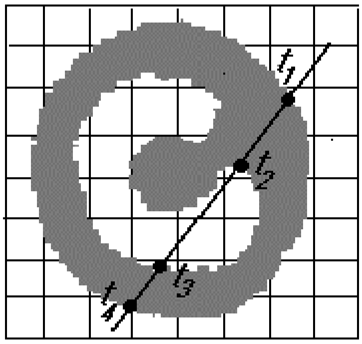

Fig. 1,a

For instance, let $\mathrm{T}f(\theta, p, t)$ be maximum interval within function $f(\theta, p, \bullet)$ domain. In Fig.1,a it is the value of $t_2 - t_1$ (max *G*).If we determine a similar $\mathrm{T}f(\theta, p, \bullet)$ for an aggregate of scanning lines intersecting the image of a erythrocyte in Fig. 1,a, at various angles θ and various distances p, we can get its Trace – image in Fig.1,b.

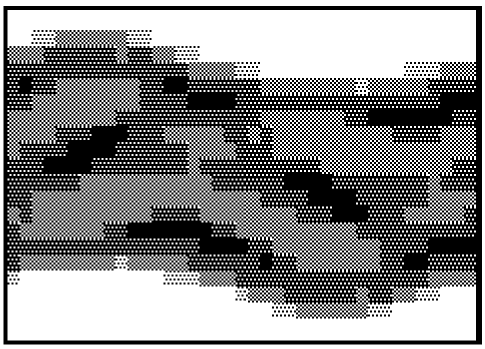

Fig. 1,b

Note that the famous Radon transformation can be viewed as an example of a Trace – transformation. For a two–level image, such a Trace–transformation could be obtained in case $\mathrm{T}f(\theta, p, \bullet)$ is the total of all the intervals from the domain of the function to be defined. For Fig.1,a it is the value of $t_2 - t_1$ and $t_4 - t_3$ segments' total.

3. TRIPLE FEATURES

Let us consider formation of triple features which are a consecutive composition of three functionals:

$$\Pi(F) = \Theta \circ \mathrm{P} \circ \mathrm{T}(F \circ L(\theta, p, t)).$$

Each functional (Θ, P and T) effects the function of one variable (θ, p and t) correspondingly.

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We call functional T invariant to all shifts (or just invariant, to be short) if $\mathrm{T}(h(t+a)) = \mathrm{T}(h(t))$ for any and an admissible $a \in R$ function h. An average value of function h could serve as an example here.

We call functional T sensitive to all shifts, if $\mathrm{T}(h(t+a)) = \mathrm{T}(h(t)) + a$ for each $a \in R$. The functional of the value of the argument of maximum function h could be taken as an example (in case function h has actual values).

The same definitions hold for functionals P and Θ. The Theorem 1 that follows is proved: if functionals T, P and Θ are invariant, then value F is independent of the image motions and affine transformations. (To make it short, we have put the problem of Domain (T), etc., aside).

Functional T, corresponding to a Tr-transformation, has been above considered in detail. In a discrete variant of computation the result of the transformation, or the Tr-transform, $\mathrm{T}(F \circ L(\theta, p, t))$ is a matrix, which elements are, say, values of brightness parameter for image F at the intersections with the scanning line $l(\theta, p)$.

Parameters of the scanning line p specify the position of the element within the matrix. Computation of feature to follow involves a consecutive processing of the matrix columns with the help of functional P, which we call diametrical.

Functional "Variation" $Ph(p) = \sigma h(p) / Mh(p)$ has been used as functional P, other instances of diametrical functionals applied may be the functional called "Min", which is the minimum value of the function in a trace–matrix column.

The result of applying P ("Variation") functional to a trace – matrix (Fig.1,b) is a 2π - periodic curve shown in Fig.1,c.

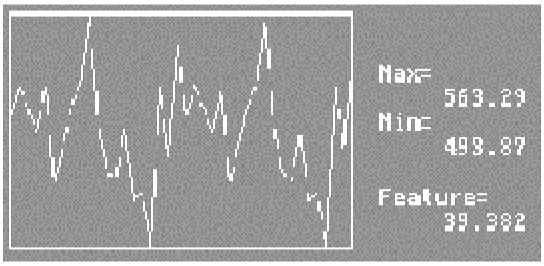

Fig. 1,c

Next stage is to perform transformations on the curve with the help of Θ functional which we call a circus within N. The functional " Norm", a standard Euclidean norm, has been used as a variant of Θ functional, being computed through

$$\Theta h(\theta) = \sqrt{\int h^2(\theta) d\theta}.$$

The triple recognition features considered may be computed through a highly parallel process. Like features formed by neuron nets, the given features have no pre-assigned meanings, their selection being realized during a machine experiment, considering their being useful for classification only.

4. THE FEATURES USED IN NUMERIC EXPERIMENTS

Trace-functional has been used in six variations. Their numbers are 1, 2, 3, 5, 6, 7 (number 4 missing). It is connected with the fact that the working program disposes of a standard classification of functionals. It has been empirically established that the above listed trace-functionals are applicable for the recognition of blood cells, or eritrocites, better than the rest of them within the program. Some of the below listed functionals could be computed for binary images only (functionals 2, 3, 5, namely). That is why the image is first to be transformed into a binary one according to the following rule.

We are having 16 shades of colour (from black to white) in the image. A binary image could be obtained if we use threshold Colour Triger=5,5.

Here are the Trace-functionals:

1. Integral taken along the line (such a transformation results in Radon transformation.

2. The length of the maximum segment where line l intersects the image F, the intersection being considered if only the image shade excesses number Colour.
3. The number of segments where the line intersects the image (Colour Triger is used).
5. The length of the segment between image first and last tangency. Mathematically, it is the length of the function support's (or bearer's) convex shell (Colour Triger is used).
6. Variance of the function, which has been normalized on its integral. The normalization is performed to apply the notion of variance itself, which is specified for nonnegative function only, its integral being 1. If the function has been identical zero, we consider variance equals 0. It is necessary to provide for the trace-transformation continuity (though normally a zero function variance is considered to qual infinity.)
7. Function variance computed as above and then multiplied by the function integral. It is performed to consider the function infinitesimal, and to assign a small weight to random noise or distortions.
The matrix to deduce transform (or trace-matrix) has the following dimensions. Along variable θ axis is horizontal, range of variable is 0, ..., 2π; number of discret being 70. Along variable p axis is vertically spaced, range of variable is - 100, ..., 100, number of discret being 50.
Diametrical functionals. For a diametrical functional five options have been used:
1. Gilbert norm of function $\mathrm{P}h(p) = \sqrt{\int h^2(p)dp}$;
2. Maximum value of function;
5. Measure of the function carrier (for the functions assigned in the table it is the number of nonzero components multiplied by the step of discretization;
6. Maximum of the first variable absolute value;
9. Variation of function.
Circus functionals. For a circus functional four options have been used:
7. Amplitude of second harmonic of Fourrier function divided by function maximum;
10. Euclidean norm (i. e. Gilbert norm of space L^2);
12. Euclidean norm divided by the function variation;
14. Amplitude of fourth harmonic of Fourrier function divided by the function Euclidean norm.
In total, we dispose of 6 * 5* 4 = 120 different functionals. The number of pictures to be processed is 35. Thus, there shoud be 120 * 35 = 4200 numbers which are features. Some of the features are not informative, others, little informative to be applied to he problem of recognizing pathological erythrocytes. Still when used in the aggregate, they solved the recognition task completely.
Thus, each image is characterized by a set of 120 numbers which constitute a vector. Then we have to compute the average distance in a 120D space to the representatives of each class of images. The class distance to, which is minimum, is to be considered the result of the process to recognize the image under study.

5. DESCRIPTION OF THE MATERIAL FOR COMPUTATION. ERITHROCYTES

Fig. 2 shows erythrocytes and other blood cells. Five classes of images have been selected here: a, b, e, g, h. Each class is represented by seven samples, i. e. we get images:
a1, a2, a3, a4, a5, a6, a7 - class a,
b1, b2, b3, b4, b5, b6, b7- b,
e1, e2, e3, e4, e5, e6, e7 - e,
g1, g2, g3, g4, g5, g6, g7 - g,
h1, h2, h3, h4, h5, h6, h7 - h.
Moreover, we distinguish sets. These are images:
a1, b1, e1, g1, h1 - set 1,
a2, b2, e2, g2, h2 - 2,
a3, b3, e3, g3, h3 - 3,
a4, b4, e4, g4, h4 - 4,
a5, b5, e5, g5, h5 - 5,
a6, b6, e6, g6, h6 - 6,
a7, b7, e7, g7, h7 - 7.
The task is to assign each image to one of the classes.

6. THE RESULT OF COMPARING THE SETS

For comparing the images sets we calculate distances for recognition images a, b, e, g, h different sets (1..7). Table 1 demonstrates the exsample of competition the images features of set 1and set 5 used only one Trace functional.

Table 1.

Distances for recognition images a, b, e, g, h set 5 using patterns a, b, e, g, h set 1					
	a1	b1	e1	g1	h1
a5	0,32	0,28	0,64	0,34	0,36
b5	0,37	0,32	0,64	0,29	0,25
e5	0,39	0,38	0,65	0,26	0,38
g5	0,62	0,89	0,40	0,71	0,61
h5	0,35	0,40	0,59	0,32	0,26

In the reseach computations, we used 1 * 5 * 4 = 20 features (a fixed Trace-functional, 5 Diametrical functionals and 4 Circus functionals). The results' correlation makes about 80% when using 20 features.

7. THE RESULT OF RECOGNITIONS OF SEPARATE IMAGES CLASSES

For the determination of classes each set is compare to all other (different from it) sets, which act as standards. We can achieve it a simple finding of root-mean-square all results, which are refer to the recognition of given set. After we calculate sums of distances for recognition images a, b, e, g, h different sets using all other patterns of clases a, b, e, g, h. The results of comparing the clases are shown in Table 2 and Table3.

Table 2.

Sums of distances for recognition images a, b, e, g, h set 1 using all other patterns a, b, e, g, h and all trace functionals					
	a	b	e	g	h
a1	1,03	1,18	1,83	1,34	1,24
b1	0,98	0,82	2,17	1,17	1,04
e1	1,67	1,69	1,21	1,86	1,66
g1	0,94	1,06	2,36	0,8	1,1
h1	1,32	1,22	1,87	1,24	0,93
Sums of distances for recognition images a, b, e, g, h set 2 using all other patterns a, b, e, g, h and all trace functionals					
	a	b	e	g	h
a2	0,83	0,85	2,29	0,9	1,11
b2	1,01	0,82	2,19	1,18	1,04
e2	1,67	1,83	1,17	1,92	1,6
g2	1,16	1,16	2,29	0,99	1,21
h2	1,13	1,15	2,01	1,2	0,97
Sums of distances for recognition images a, b, e, g, h set 3 using all other patterns a, b, e, g, h and all trace functionals					
	a	b	e	g	h
a3	0,79	0,9	1,84	1,02	1,08
b3	0,83	0,78	2,2	0,96	1,03
e3	3,29	3,22	1,72	3,29	2,9
g3	0,94	1,06	2,21	0,8	1,1
h3	1,29	1,1	1,8	1,32	0,86

Table3.

Sums of distances for recognition images a, b, e, g, h set 4 using all other patterns a, b, e, g, h and all trace functionals					
	a	b	e	g	h
a4	0,82	0,86	2,08	0,93	1,1
b4	1,07	0,91	2,37	1,12	1,22
e4	1,93	2,05	0,95	2,13	1,78
g4	0,99	1,04	2,3	0,81	1,19
h4	1,17	0,98	2,05	1,07	0,97
Sums of distances for recognition images a, b, e, g, h set 5 using all other patterns a, b, e, g, h and all trace functionals					
	a	b	e	g	h
a5	0,76	0,89	2,06	1	1,11
b5	0,87	0,72	2,16	1	0,98
e5	2,11	2,27	1,11	2,21	1,9
g5	0,92	1	2,37	0,79	1,14
h5	1,07	1,12	1,97	1,15	0,91
Sums of distances for recognition images a, b, e, g, h set 6 using all other patterns a, b, e, g, h and all trace functionals					
	a	b	e	g	h
a6	0,79	0,93	2,28	0,92	1,24
b6	0,89	0,82	2,08	1,13	1,12
e6	2,65	2,81	1,34	2,91	2,46
g6	1,19	1,21	2,25	1	1,29
h6	1,16	1,03	1,98	1,1	0,9
Sums of distances for recognition images a, b, e, g, h set 7 using all other patterns a, b, e, g, h and all trace functionals					
	a	b	e	g	h
a7	0,82	0,9	2,17	0,96	1,12
b7	0,88	0,83	2,1	1,03	1,08
e7	2,06	2,15	1,07	2,27	1,91
g7	1,06	1,2	2,18	0,92	1,16
h7	1,07	1,05	1,8	1,14	0,92

8. ANALYSIS OF THE EXPERIMENT RESULTS

A conclusion can be made that most features listed in the present paper can successfully solve the set problem of distinguishing erythrocytes. Certain features work successfully even if the researcher is unable to note evident distinctions. It has been demonstrated, besides, that there exist features which prove finer than a given concrete task requires. The recognition system suggested, thus, proves promising. As the result of the experiment, having recognized classes 35 of images, we got no errors.

9. CONCLUSION

It has been established that he features suggested can successfully distinguish the classes of erythrocytes suggested for the analysis. It proves the usefulness of the theory in question for medical practice.

We consider essential that great many new good features, actually about a hundred of them, have been introduced at once. Certain vector components, characteristic of the image, may be independent of certain transformations of the image, others may depend upon such transformations in a simple way, which makes it possible to establish the image parameters. Hence, the theory developed helps not only recognize a great number of standard images, but to establish similar parts of the image fractal structure. The process suggested could be easily transferred to gray-tone and full-colour images.

The theory is not sensitive to the quality of image outline. Computations can be performed in parallel. The results of work prove that the theory developed could be used for recognition in biological systems for self-acting or computer – aided recognition of biological microobjects.

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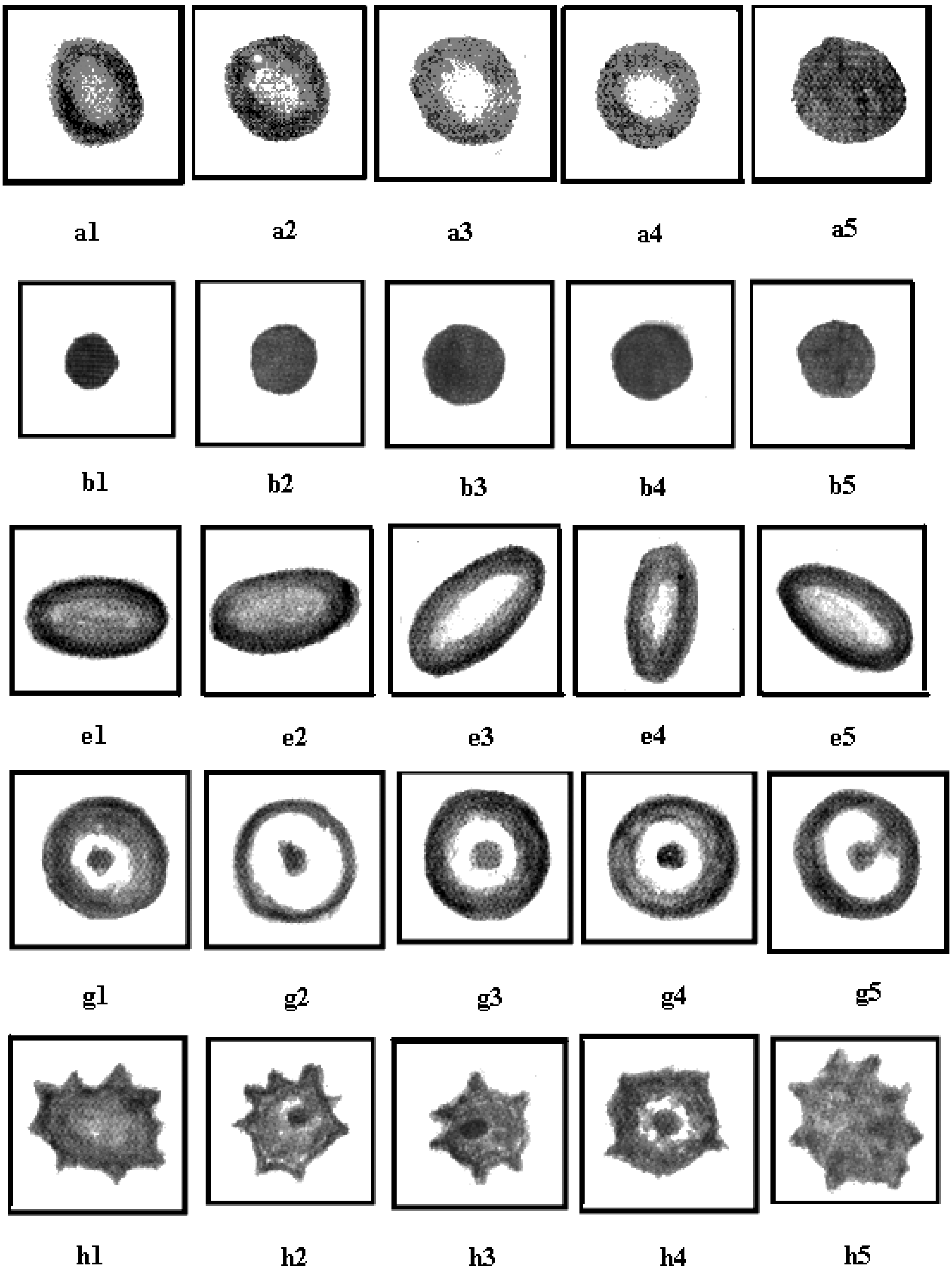
a1
a2
a3
a4
a5
b1
b2
b3
b4
b5
e1
e2
e3
e4
e5
g1
g2
g3
g4
g5
h1
h2
h3
h4
h5

Fig.2.