UNIVERSITY
OF WEST BOHEMIA

Applied and Computational Mechanics 3 (2009) 17–26

Applied and Computational Mechanics

Magnetostatic analysis of a rotor system supported by radial active magnetic bearings

P. Ferfecki[a,*]

[a] *Center of Advanced Innovation Technologies, VSB-TU of Ostrava, 17. listopadu 15, 708 33 Ostrava-Poruba, Czech Republic*

Received 31 August 2008; received in revised form 11 March 2009

Abstract

The development and the design of a radial active magnetic bearing (AMB) reflects a complex process of the multidisciplinary rotor dynamics, electromagnetism and automatic control analysis. Modelling is performed by application of the physical laws from different areas, e.g. Newton's laws of motion and Maxwell's equations. The new approach in the numerical modelling of radial AMB and design methodology allowing automatic generation of primary dimensions of the radial AMB is proposed. Instead of the common way of computation of electromagnetic forces by linearizing at the centre position of the rotor with respect to rotor displacement and coil current, the finite element computation of electromagnetic forces is used. The heteropolar radial AMB consisting of eight pole shoes was designed by means of the built up algorithms for rotor system with two discs fixed on the cantilever shaft. A study of the influence of the nonlinear magnetization characteristics of a rotor and stator material on the equilibrium position of a rotor system is carried out. The performed numerical study shows that results obtained from the analytical nonlinear relation for electromagnetic forces can be considerably different from forces computed with magnetostatic finite element analysis.

Keywords: radial active magnetic bearing, nonlinear magnetostatic analysis, finite element method

1. Introduction

In the field of rotating machinery the number of applications of rotating systems running in radial active magnetic bearing is expanding. Radial AMB supports moving shaft by means of magnetic levitation without mechanical contact. The magnetic bearing have several advantages and disadvantages in contrast with rolling element bearings and fluid film bearings. Two primary advantages of magnetic bearings are the very low power consumption and very long life because there is no mechanical friction or wear. Other advantages of magnetic bearings are related to their use in extreme conditions, e.g. high speed revolutions, low temperatures, vacuum or in environments where lubricants are undesirable. Magnetic bearings have a number of disadvantages including a high cost, a relatively large size and a requirement for auxiliary bearing systems for start up and shut down purposes.

Rotor systems supported by means of the radial AMB are complicated due to the composite action of the mechanical, the electrical and the electronic parts. The finite element method can be a powerful technique for magnetic field analysis. Some engineers and scientists examining a radial AMB use this method at the design stage. The geometrical design of a radial AMB combined with the magnetostatic finite element analysis and numerical optimization algorithm is performed in Ph.D. thesis [3]. The analysis of the radial AMB is done using a static model

*Corresponding author. Tel.: +420 597 325 752, e-mail: petr.ferfecki@vsb.cz.

with distributed parameters and a dynamic model with lumped parameters, which is based on matrices of the flux linkage determined by the finite element method differentiated with respect to current and position. The stabilization of the radial AMB is achieved with a decentralised closed-loop structure consisting of a cascade connection of PD and PI position controllers. Obtained numerical results concerning the designed control system, the iron core saturation and the magnetic couplings between electromagnets are verified by performed experimental results.

The calculation of the electromagnetic forces has been very popular research topic during the last decades because the computational speed, the accuracy and the reliability of used methods have significant differences. The summary of methods for computation of electromagnetic forces is stated and a new method for rotor performing eccentric motions with respect to the stator is proposed in the doctoral thesis [5]. The impulse method to calculate the frequency response of the force is determined. The force computed by the impulse method and a conventional computation shows very good agreement.

This paper is focused on a multiphysics finite element analysis of the rotor system supported by means of radial AMBs. First, the automatic computational procedure to design radial AMB in the COMSOL Multiphysics software and the computer system MATLAB was proposed. Created algorithms based on the finite element magnetostatic analysis were used to design a radial AMB. The objective of this technique is to allow more accurate computation of the magnetic field properties and verify the influence of the geometry of bearing, the material properties (i.e. the nonlinear magnetisation curve of bearing parts) and the rotor movement on the bearing parameters. Numerical simulations show an influence of some bearing parameters on the equilibrium position of a rotor system and several possibilities of computations of the electromagnetic forces have also been introduced.

2. The motion equation of a rotor system supported by means of the radial AMBs

In order to achieve a realistic model of a rotor system the finite element analysis is used. The motion equation of examined rotor system is build up in the stationary co-ordinate system and was derived under following assumptions: (i) the shaft is flexible, linear elastic and in the computation model the shaft is represented by a flexible beam-like body that is discretized into finite elements, (ii) the shaft element is modelled on the basis of Bernouli-Euler beam theory, (iii) the stationary (i.e. non-rotating) part is considered to be absolutely rigid and fixed, (iv) the rotor is coupled with the stationary part through a radial AMB and a massless axial bearing, (v) the discs are considered to be absolutely rigid axisymmetric bodies, (vi) inertia and gyroscopic effects of the rotating parts are taken into account, (vii) material damping of the shaft and other kinds of damping are regarded as linear, (viii) the rotor is loaded by a gravitational force and forces with periodic time histories (ix) the rotor rotates at constant angular speed and can be expressed as follows:

$$\mathbf{M}\ddot{\mathbf{q}}(t) + (\mathbf{B}_R + \eta_V \mathbf{K}_{SH} + \omega\mathbf{G})\dot{\mathbf{q}}(t) + (\mathbf{K} + \omega\mathbf{K}_C)\mathbf{q}(t) = \mathbf{f}_M(\mathbf{q}, \mathbf{i}) + \mathbf{f}_A(t) + \mathbf{f}_V, \quad (1)$$

where $\mathbf{M}, \mathbf{B}_R, \mathbf{G}, \mathbf{K}_C, \mathbf{K}_{SH}, \mathbf{K}$ are the mass matrix, the damping matrix (external damping and damping of material), the gyroscopic effects matrix, the circulation matrix, the stiffness matrix of the shaft and the stiffness matrix of the rotor system respectively, $\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}$ are vectors of generalized nodal displacements, velocities and accelerations respectively, $\mathbf{f}_M, \mathbf{f}_A, \mathbf{f}_V, \mathbf{i}$ are vectors of electromagnetic forces, generalized forces exerting on the rotor system (external and constraint forces) and control currents passing by core of electromagnets respectively, while ω

is the angular speed of a shaft, η_V is the coefficient of viscous damping of a material and t is the time.

If a rotor system is loaded only by static forces (e.g. self-weight, constant forces, etc.) and does not vibrate, the motion equation (1) is transformed into the equation of equilibrium position

$$(\mathbf{K} + \omega \mathbf{K}_C)\mathbf{q}_S = \mathbf{f}_M(\mathbf{q}_S, \mathbf{i}_S) + \mathbf{f}_{ST} + \mathbf{f}_V, \qquad (2)$$

where $\mathbf{q}_S$ is the vector of generalized static nodal displacements, $\mathbf{f}_{ST}$, $\mathbf{i}_S$ are vectors of generalized static forces exerting on the rotor system and static control currents passing by core of electromagnets respectively.

3. The electromagnetic field and force calculation of a radial AMB

The calculation of forces in the radial AMB is based on Maxwell's equations of electromagnetism. For general quasi-static fields, these equations in the differential form can be written according to [4] as

$$\nabla \times \mathbf{H} = \mathbf{J}, \qquad (3)$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \qquad (4)$$

$$\nabla \cdot \mathbf{J} = 0, \qquad (5)$$

$$\nabla \cdot \mathbf{D} = \rho, \qquad (6)$$

$$\nabla \cdot \mathbf{B} = 0, \qquad (7)$$

where $\mathbf{B}$ is the magnetic flux density, $\mathbf{D}$ is the electric flux density, $\mathbf{E}$ is the electric field intensity, $\mathbf{H}$ is the magnetic field intensity, $\mathbf{J}$ is the current density, ρ is the electric charge density and ∇ is the Nabla operator.

The equations (3), (4) and (5) are referred to as Maxwell-Ampére's law, Faraday's law and the equation of continuity respectively. The electric and magnetic forms of Gauss' law are described by equations (6) and (7) respectively.

To obtain a closed system, the constitutive relations has to be included

$$\mathbf{B} = \mu \mathbf{H}, \qquad (8)$$

$$\mathbf{D} = \varepsilon \mathbf{E}, \qquad (9)$$

$$\mathbf{J} = \sigma(\mathbf{E} + \mathbf{v} \times \mathbf{B}) + \mathbf{J}^{\mathrm{e}}, \qquad (10)$$

where $\mathbf{J}^{\mathrm{e}}$ is the external generated current, $\mathbf{v}$ is the velocity of the conductor, ε is the permittivity, μ is the permeability and σ is the electrical conductivity.

In the finite element formulation of electromagnetic field problems, magnetic potential $\mathbf{A}$ is commonly used in the solution of two-dimensional magnetic fields. The magnetic flux density and the electric field intensity are given by the equalities

$$\mathbf{B} = \nabla \times \mathbf{A}, \qquad (11)$$

$$\mathbf{E} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t}, \qquad (12)$$

where V is the electric scalar potential.

Ampère's law (3) by means of equalities (8) and (10) and using the definitions of potentials (11) and (12) can be rewritten as

$$\sigma\frac{\partial \mathbf{A}}{\partial t} + \nabla \times (\mu^{-1}\nabla \times \mathbf{A}) - \sigma \mathbf{v} \times (\nabla \times \mathbf{A}) + \sigma \nabla V = \mathbf{J}^{\mathrm{e}}. \tag{13}$$

The equation of continuity (5) combined with equalities (10), (11) and (12) gives the equation

$$-\nabla \cdot \left[\sigma\frac{\partial \mathbf{A}}{\partial t} - \sigma \mathbf{v} \times (\nabla \times \mathbf{A}) + \sigma \nabla V - \mathbf{J}^{\mathrm{e}}\right] = 0. \tag{14}$$

The set of two equations (13) and (14) is required for the uniqueness of solution of the potentials $\mathbf{A}$ and V.

In the magnetostatics analysis the above equations (13) and (14) are simplified into one equation

$$\nabla \times (\mu^{-1}\nabla \times \mathbf{A}) = \mathbf{J}^{\mathrm{e}}. \tag{15}$$

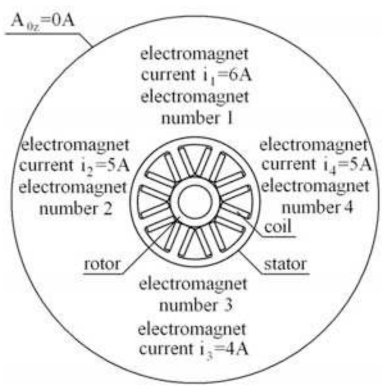

Fig. 1. A sketch of the radial AMB

The analysis of a radial AMB based on equation (15) is assumed to be two dimensional, stationary, no current generated by an external static electric and magnetic field, the currents passing through electromagnets are constant and there is no hysteresis of a nonlinear magnetization curve of the ferromagnetic material.

In order to be able to solve equation (15) the boundary conditions must be known. Dirichlet boundary condition in the form of magnetic scalar potential A_{0z}, is defined on the outer circle of a bearing model as shown in fig. 1.

The AMB is incorporated into the equation of motion (1) by means of the nonlinear electromagnetic force vector, whose elements are computed from equation (15). Two basic methods for calculation of the electromagnetic forces acting between the rotor and stator were used, namely method based on the Maxwell's stress tensor and method based on the principle of the virtual work.

Using the first mentioned method the electromagnetic force $\mathbf{F}_m$ on the body is calculated by the surface integral

$$\mathbf{F}_m = \oint_S \mathbf{T}\,\mathrm{d}S = \oint_S \left[\frac{1}{\mu_0}(\mathbf{Bn})\mathbf{B} - \frac{1}{2\mu_0}\mathbf{B}^2\mathbf{n}\right]\,\mathrm{d}S, \tag{16}$$

where $\mathbf{T}$ is the Maxwell stress tensor in air, $\mathbf{n}$ is the unit outward normal vector of the integration surface S and μ_0 is the permeability of vacuum ($4\pi \cdot 10^{-7}\,\mathrm{Hm}^{-1}$). In a two-dimensional model, the surface integral is reduced to a line integral along the air gap.

In the virtual work method the electromagnetic force is computed as a partial derivative of the coenergy functional with respect to virtual movement

$$\mathbf{F}_m = \left.\frac{\partial W_c}{\partial \mathbf{p}}\right|_{\mathbf{I}_c} = \left[\frac{\partial W_c}{\partial y_p}\;\frac{\partial W_c}{\partial z_p}\right]^{\mathrm{T}}, \quad W_c = \int_{V_e}\left(\int_0^{\mathbf{H}} \mathbf{B}\,\mathrm{d}\mathbf{H}\right)\,\mathrm{d}V_e, \tag{17}$$

where $\mathbf{p}$ is the vector of virtual movement, $\mathbf{I}_c$ is the vector of constant current, y_p and z_p are a horizontal and vertical components of the vector of virtual movement respectively, V_e is the volume element and W_c is the coenergy functional.

The results obtained by finite element calculations of the electromagnetic force are compared with one-dimensional method. This level of approximation is assuming that the size of air gap is uniform and that the magnetic flux crosses the gaps in a straight line. Under these assumptions the components of the electromagnetic force acting on the shaft in horizontal $f_{M,y}$ and vertical $f_{M,z}$ direction are according to [2] given by following relations

$$f_{M,y}(y, i_y) = \frac{1}{4}\cos(\alpha_0)\mu_0 N^2 S_M \left[\left(\frac{I_0 + i_y}{c_0 - y}\right)^2 - \left(\frac{I_0 - i_y}{c_0 + y}\right)^2\right], \tag{18}$$

$$f_{M,z}(z, i_z) = \frac{1}{4}\cos(\alpha_0)\mu_0 N^2 S_M \left[\left(\frac{I_0 + i_z}{c_0 - z}\right)^2 - \left(\frac{I_0 - i_z}{c_0 + z}\right)^2\right], \tag{19}$$

where y, z are the generalized components of displacements in horizontal and vertical direction respectively, c_0 is the size of air gap between the rotor and stator, i_y, i_z are the controlling currents in horizontal and vertical directions respectively, I_0 is the bias current, S_M is the area of the pole of electromagnet in the air part of magnetic circuit, N is the number of winding of the electromagnet coil and α_0 is the angle between the electromagnetic force and vertical axes of the bearing.

4. The finite element analysis of a rotor system supported by radial AMBs

The multiphysics finite element analysis of a shaft and radial AMB has been done with the COMSOL Multiphysics software which allows connection with computational environment of the computer system MATLAB for the modelling of both parts of a rotor system. The presented numerical simulation was obtained using the application of magnetostatic mode of the AC/DC module of the software COMSOL Multiphysics. The magnetostatic module enables to solve Maxwell's equations with certain boundary conditions describing the electromagnetic problem on a macroscopic level.

The automatic procedure for generation of a parametric model of the radial AMB by means of user-defined algorithms in the COMSOL Script [1] was proposed. The shape of stator frame, coils, rotor and smoothness of corners of eighth poles of a radial AMB can be set and modified so that it will automatically generate a number of parameterized configurations. The design of a radial AMB is characterized by several geometry parameters: the shaft diameter, the rotor diameter, the stator diameter, the size of air gap, the thickness of stator frame, the smoothness of corners of stator, rotor and coils, the bearing width, the height and width of pole shoes and the horizontal and vertical movement of a rotor centre. Except the shaft diameter, all user-defined parameters of bearing can be modified manually or automatically. The solid modelling techniques and Boolean operations were used to obtain composite model of a radial AMB (see fig. 2).

The magnetostatic computation of a radial AMB is described by a few steps. At the beginning, the bearing geometry, the B-H material characteristics that can be nonlinear (see fig. 3), the coils properties and the boundary conditions are defined. Subsequently, an initial discretization by the triangular elements is automatically generated (see fig. 4). The initial mesh is consisting of approximately 130 000 elements and 65 000 degrees of freedom. During the solution an

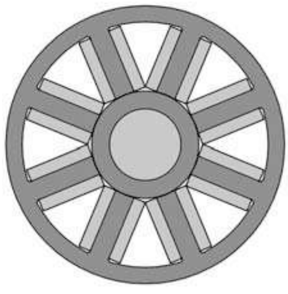

Fig. 2. The model of a radial AMB

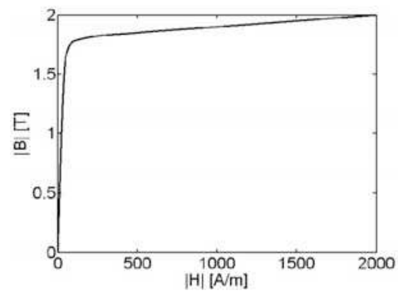

Fig. 3. The norm of the magnetic flux density versus the norm of the magnetic field intensity for the rotor and stator materials

adaptive mesh refinement can be applied based on analysis of errors. In the last step, components of the electromagnetic force acting on the rotor are solved by means of method based on the Maxwell's stress tensor or the method based on the principle of the virtual work.

Furthermore, the algorithm for magnetostatic computation of a radial AMB is incorporated into the computational procedures for the analysis of a lateral vibration of the rotor systems. The motion equation (1) is transformed into the set of nonlinear algebraical equations (2) for solving the problem of a equilibrium position. To its solution Newton-Raphson method can be used.

5. The results of numerical simulation of the magnetic field analysis and the computation of the equilibrium position

The investigated radial AMB is designed for the supported rotor system with two discs fixed on the cantilever shaft described in detail in [2]. The heteropolar radial AMB containing eight pole shoes, the stator frame, the rotor and coils (see fig. 2 and fig. 4) is proposed. The considered radial AMB has the following primary dimensions and parameters: the shaft diameter – 76.6 mm, the rotor diameter – 105 mm, the stator diameter – 273 mm, the size of air gap – 0.5 mm, the thickness of stator frame – 15.9 mm, the bearing width – 200 mm, the number of coil turns – 64, the bias current – 4 A and the current range $0 \div 8$ A.

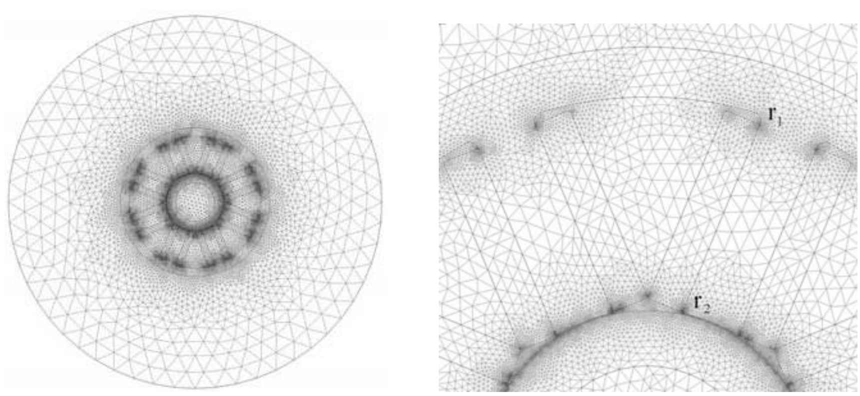

Fig. 4. The generated mesh of a radial AMB (left) and the detail of generated mesh in the surrounding electromagnet (right)

The shaft is represented in the computational model by a beam-like body that is discretized into twenty four finite elements of equal length (95 mm). More detailed data are given in [2]. The rotor system supported in designed AMBs is controlled by means of current PD controllers. The current PD feedback controllers of both bearings are assumed to be identical and their parameters are given in [2].

The numerical simulations show that the shape of pole shoes, their height and width, the size of air gap and the coil parameters significantly change the electromagnetic force value produced by the electromagnet. The magnetic flux density is cumulated on all sharp edges in the bearing, because in their proximity the material saturation is bigger than on the other parts of the magnetic circuit. Therefore, all sharp edges in the magnetic circuit must be appropriately rounded. The maximum magnetic flux density concentration is located in the proximity of fillet of the pole shoe on side of the stator (r_1) and rotor (r_2) part (see right fig. 4). The numerical results show, that a minimal concentration of the magnetic flux density in the bearing appears, when the value of fillet of pole shoes at stator is $r_1 = 3$ mm and at rotor is $r_2 = 0.2$ mm.

In the next step, an analysis is focused on the influence of shaft movement on magnetic field properties in the AMB. Fig. 5 and fig. 6 show the field distribution of magnetic potential for four different locations of the rotor centre with respect to the bearing centre (whose detailed specifications are mentioned below). The magnetic potential is changed due to the rotor displacement and the current density in coils remains the same. The magnetic flux strongly influences the rotor and stator material and depends on the size of the air gap between the rotor and the pole shoe. Thus, when the air gap increases the electromagnetic force decreases and conversely.

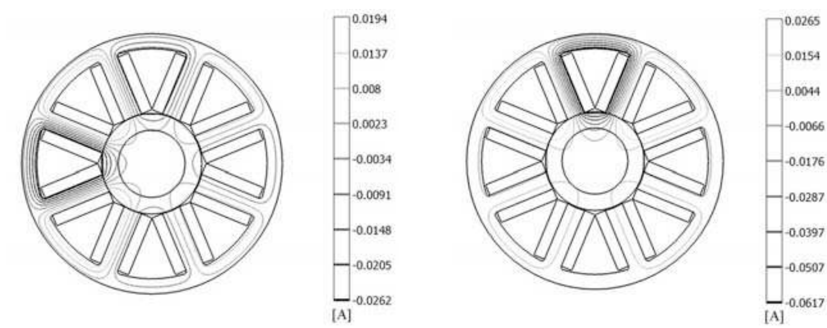

Fig. 5. The magnetic potential in a radial AMB for the rotor moved 0.35 mm to the left side (left) and the rotor moved 0.49 mm upward (right) with respect to the bearing centre

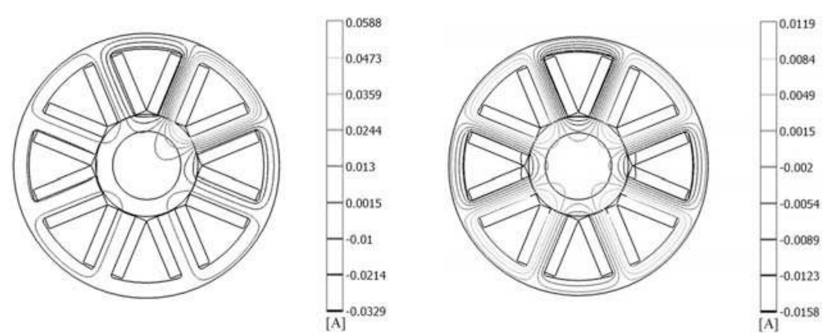

Fig. 6. The magnetic potential in a radial AMB of the rotor moved 0.35 mm upward and to the right side (left) and the rotor located at the centre position (right) with respect to the bearing centre

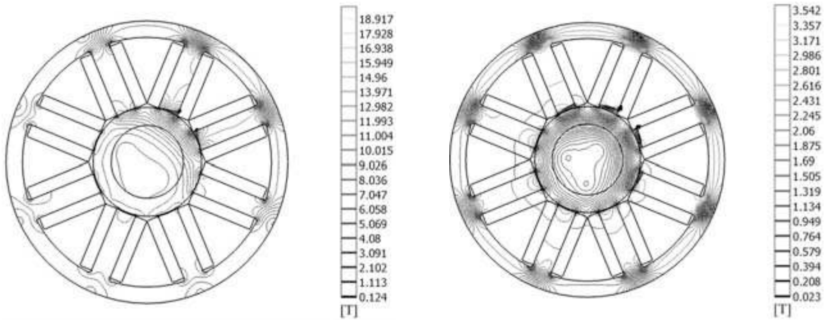

Fig. 7. The Maxwell surface stress tensor and the magnetic flux density in a AMB of the rotor moved 0.35 mm upward and to the right side (left) with respect to the bearing centre and same physical quantities in the case that the material of stator and the outer part of a rotor has a nonlinear B-H curve (right)

The rotor and stator material is composed of thin laminated plates for the purpose of reducing eddy current effects. The distribution of magnetic flux density for linear and nonlinear B-H magnetic characteristic of a rotor and stator material is presented in fig. 7. The computed distribution of magnetic flux density and its maximal value and the Maxwell's stress tensor are considerably dependent on the rotor and stator material characteristics (see fig. 7).

Numerical results of the total electromagnetic force of the first magnetic bearing (MB1) and the second magnetic bearing (MB2) computed by method based on the Maxwell's stress tensor and method based on the principle of the virtual work are shown in right fig. 8. The difference between results obtained by using both methods is negligible in the studied case. The percentage errors of both numerical computations of electromagnetic forces have been calculated and its maximum value is less than 1 %. If not stated otherwise, electromagnetic forces were calculated by means of the Maxwell's stress tensor method.

The finite element computations of the total electromagnetic forces with their analytical formula are also compared (see left fig. 8). Due to the non-uniformity of magnetic flux distribution, the value of electromagnetic force obtained from the analytical formula is bigger than value computed by the finite element method. If a magnetic saturation of the shaft material and cross coupling effects of electromagnetic forces originating from close pole shoes are negligible, than the percentage errors of computed electromagnetic forces are less than 5 %.

A study of the influence of the nonlinear B-H magnetic characteristics of a rotor and stator material on the equilibrium position of a rotor system is carried out. In fig. 9 and fig. 10, the results for equilibrium position at 10 000 rpm with the electromagnetic forces computed by means of the analytical nonlinear formula (o), the magnetostatic finite element method with a linear (+) and a nonlinear (x) material characteristic are shown. Calculations based on the finite element method with a linear and nonlinear B-H curve give considerably different equilibrium positions.

6. Conclusion

A multiphysics finite element simulation of the rotor system supported by means of radial AMBs is presented in this paper. In the COMSOL Multiphysics software the electromagnetic analysis of a radial AMB was carried out and subsequently the rotor dynamic finite element analysis in connection with the computational environment of the computer system MATLAB is performed.

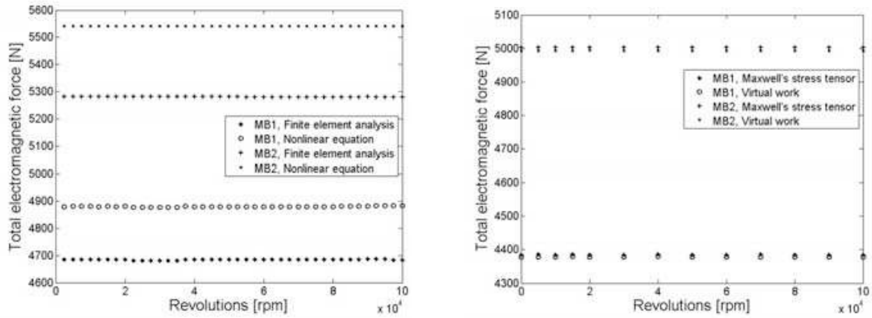

Fig. 8. The dependence of total force in the equilibrium position on revolutions (left) and the dependence of total force in the equilibrium position on revolutions for two methods of solution (right)

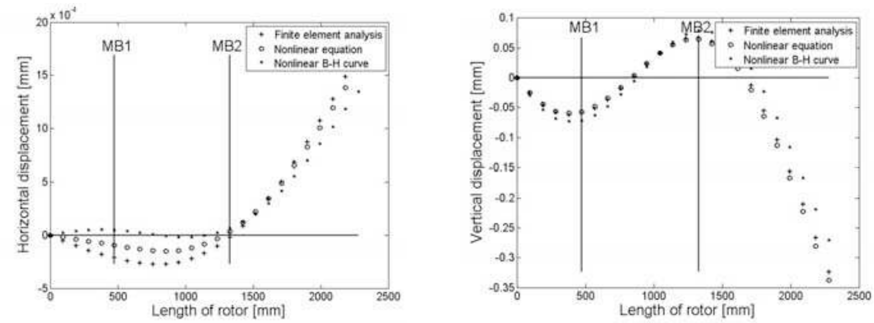

Fig. 9. The equilibrium position of a rotor system in the horizontal (left) and vertical (right) direction

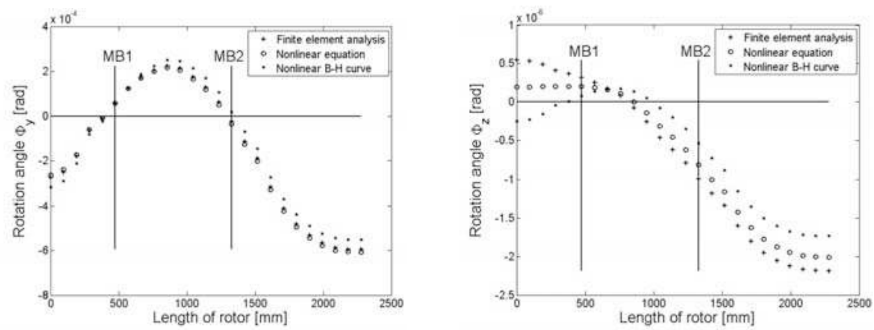

Fig. 10. The rotation angle about y-axis (left) and z-axis (right) of a rotor system in the equilibrium position

Unlike [3] and [5] the shaft of rotor system is discretized by a flexible beam-like body. The flexibility of the shaft and its viscous material damping is taken into account in the computational model of a rotor system. This paper presents improved mathematical model of a rotor system supported by radial active magnetic bearings modelled by means of finite element method, which makes possible to study relations between mechanical properties of a rotor (flexibility, damping) and electromagnetic forces (influence of geometrical dimensions, nonlinear material characteristics) with higher precision. The mathematical models of rotor systems supported by radial active magnetic bearings are usually built with either discrete model of a rotor system or bearing. Due to presented design methodology it is possible to create and solve finite element model of both active magnetic bearings and rotor system. Improved mechanical and electromagnetic properties obtained by its analysis enable better design of both parts.

The suggested procedures were used to perform two dimensional and the stationary magnetostatic finite element analysis. The method based on the Maxwell's stress tensor and method based on the principle of the virtual work was implemented into the algorithm for computation of electromagnetic forces. The comparison of forces computed by means of these methods was carried out and negligible differences between them have been found in this study. In proposed computation procedures the geometric dimensions and physical parameters influencing the distribution of magnetic field (e.g. the course of field lines, the distribution of magnetic flux density, the magnetic field intensity, etc.) can be simply determined. When shaft centre lies near centre of stator bearing the difference between electromagnetic forces solved by analytical nonlinear formulas and by means of the finite element method is small, because the effect of a cross coupling between mutually perpendicular electromagnets is not present.

Finally, the numerical results obtained from the solution of the equilibrium position of a rotor system show that incorporation of the nonlinear magnetization characteristic has considerable influence on bearing electromagnetic forces. Differences in results between electromagnetic forces solved with a linear and nonlinear material characteristic are caused by a saturation of material in the proximity of transition from stator to pole shoes of electromagnets and their edges.

The introduced simulation allows to achieve more precise computation of the electromagnetic forces with respect to the geometry of bearing, the nonlinear magnetization characteristic of a rotor and stator material and the cross coupling effects between each pole shoe. This analysis plays an important role in the design procedures of a radial AMB and helps with the verification of analytical assumptions. The future work will be focused on assumptions made in this analysis (e.g. the stationary magnetic field, two dimensional modelling and the hysteresis effect of the material parts).

Acknowledgements

This research work has been supported by the postdoctoral grant of the Grant Agency of the Czech Republic (No. 101/07/P368) and by the research project of the Czech Ministry of Education (No. MSM6198910027). Their help is gratefully acknowledged.

References

[1] COMSOL 3.3, Documentation Set, COMSOL AB, Sweden, 2006.

[2] P. Ferfecki, Computational Modelling of a Rotor System Supported by Radial Active Magnetic Bearings, Ph.D. thesis, VŠB-Technical University of Ostrava, Ostrava, 2005, in Czech.

[3] B. Polajžer, Design and Analysis of an Active Magnetic Bearing Experimental System, Ph.D. thesis, University of Maribor, Faculty of Electrical Engineering and Computer Science, Maribor, 2003.

[4] B. Sedlák, I. Štoll, Electricity and Magnetism, 2nd corrected and extended edition, Praha, Academia, 2002, in Czech.

[5] A. Tenhunen, Electromagnetic Forces Acting between the Stator and Eccentric Cage Rotor, Doctoral thesis, Helsinki University of Technology, Laboratory of Electromagnetics, Report 69, Helsinki, 2003.