Introducing Sweep features in Modeling with Subdivision Surfaces

Chiara E. Catalano
IMATI-CNR - Sez. Genova
Via De Marini, 6
16149, Genova, Italy
chiara @ge.imati.cnr.it

Franca Giannini
IMATI-CNR - Sez. Genova
Via De Marini, 6
16149, Genova, Italy
franca @ge.imati.cnr.it

Bianca Falcidieno
IMATI-CNR - Sez. Genova
Via De Marini, 6
16149, Genova, Italy
bianca@ge.imati.cnr.it

ABSTRACT

In recent times, subdivision surfaces have been considered a powerful representation for shape design. They have been successfully introduced in character animation software packages. In the last few years they have obtained greater attention also from CAD applications due to their potential in overcoming some of the problems intrinsic of spline-based modeling. Anyhow, their major drawbacks are related to the difficulty in constraining the shape of the limit surface and to the limited high level modeling tools to manipulate the shape.

In this paper, we propose a feature-based approach to extend the modeling capabilities of subdivision surfaces and to allow users to deal with this new modeling technique closer to the way they are used to. In particular, features obtainable by means of generalized sweep operations are formalized and treated. This type of feature has been chosen because it covers a large set of shapes commonly appearing in industrial products (e.g. car door internal panel cavities, stiffeners, …).

Keywords

Free-form modeling, feature-based design, subdivision surfaces

1. INTRODUCTION

Product design is a complex activity in which the product shape is the major outcome, resulting from a long and complex loop of evaluations and simulations that normally require several and tedious shape modifications to satisfy the given requirements. All these activities are currently supported by computer tools, which offer the advantage of reusing already defined models and avoiding, or at least reducing, the number of the needed physical prototypes. Each involved activity focuses on specific product aspects and uses particular information, thus needing a proper geometric model. It can be noted that not always a continuous and precise representation is used; on the contrary, in many phases discrete models are adopted mainly aimed at simplifying the process.

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The de-facto standard representation for product design is based on NURBS surfaces, since they guarantee high regularity surfaces, good geometric properties and stable algorithms. Nevertheless, they show limitations concerning the possibility to represent any topology by a unique surface. This causes different kinds of problems when modeling and transferring models to other systems or representations. They are due not only to approximation problems, thus creating gaps or overlapping faces, but also to the user's creation choices, e.g. models created by stylists frequently exhibit a too large number of patches or too long and narrow patches unsuitable for production purposes.

For their nature, subdivision surfaces could overcome this problem defining a discrete surface, which avoids the drawbacks of multi-patch representations. Roughly speaking, a subdivision surface is defined as a sequence of successive meshes that converge to a continuous surface [Zor00]. Subdivision surfaces can be considered in-between continuous surfaces and meshes: on the one hand, they correspond to simple meshes at each refinement step; on the other hand, they converge fast, behaving similarly to a continuous surface: using classical schemes, the limit surface is a C^2

spline almost everywhere. This versatility permits their application even when high-quality geometry is desired, such as it happens in some product development phases.

Greater attention to subdivision surfaces has been recently paid not only from the academic but also from the commercial point of view. In fact, subdivision schemes are incorporated into most of the animation software tools and some CAD providers claimed their intention to include them in their systems [Boi03]. This is possible thanks to both higher hardware capabilities and methods for precise shape evaluation and manipulation. In fact, solutions to many of the typical problems occurring when designing have been devised to some extent also for discrete models (see [Kob00] for a general overview). Among these, the evaluation of coordinates, tangent and curvature at surface points, as well as light lines (e.g. reflection lines, shadow lines), have been treated. In addition, different techniques for mesh parameterization have been developed.

For some problems, solutions providing satisfying results for subdivision surfaces have been found as well. For instance, Stam, J., [Sta98] defined a method for the evaluation of the point coordinates and derivatives on the limit surface for the Catmull-Clark and Loop schemes. Zorin, D., et al [Zor00] illustrate a natural way to describe subdivision surfaces as functions on some parametric domain with values in $\Re^3$ that is convenient to localize points and specific areas. While concerning surface regularity, the commonest schemes guarantee C^2 almost everywhere, but curvature can be unbounded, zero or not continuous; some researchers are working on this point [Loo02].

In addition to basic algorithms to evaluate a surface, specific requirements have to be fulfilled to think about subdivision surfaces as a reasonable alternative to NURBS. In particular, the control of the shape is a key issue in product design. Unfortunately, it is also the most critical drawback of the subdivision surfaces. The refinement process for approximation schemes tends to smooth and shrink the final shape. For a better control, an interpolating scheme may be preferred; unfortunately in this case the smoothness of the final surface is not well preserved. A more effective approach to guarantee higher regularity is rather accomplished by using approximating schemes with some constraints. A complete formal taxonomy of the interpolating constraints on subdivision curves and surfaces is provided by Nasri, and Sabin [Nas02a, Nas02b].

The present paper focuses on the insertion of features obtainable through generalized sweep operations in a subdivision surface. Such features produce curve-driven shapes that we have treated as geometric constraints to impose to the underlying surface. In the following, some of the main works aimed at handling constraints are briefly described at the light of our task.

The insertion of sharp edges, as well as the imposition of prescribed tangents or normals at given points are obtained by locally changing some subdivision rules [Hop94] [Bie00].

Other researchers deal with constraining the surface to some specific points, as Qin, H., et al. [Qin98], who introduced dynamic Catmull-Clark subdivision surfaces where a physical-based approach is coupled with subdivision to locally deform an initial surface towards some point constraints. The limits here are typical of the physical-based models: the deformation cannot be controlled both in shape and in size.

The problem of constraining a surface to pass through one or more curves has been also treated by several researchers. In most of them constraints are introduced to build the object model [Nas02c, Mor01] and cannot be directly exploited for the feature-based modeling approach that is intended to be introduced further.

Methods for treating curve constraints applicable both in the creation and in the manipulation phase have been devised, [Lik01a], based on the concept of combined subdivision schemes, which include local samples of the desired curve as subdivision control points [Lev00]. Still based on this approach, trimming operations have been dealt with [Lik01b].

Alternative approaches to curve driven surface modification are followed by Khodakovsky, A., and Schröder, P., [Kho99], and -more accurately- by Biermann, H., et al. [Bie01]. In the latter the constraint line is drawn by the user onto the subdivision surface itself. In both cases only a displacement operation is performed on the points localized on the mesh, so the limit is that only linear constraints are considered.

In the present work, we are interested in the modification phase, where a model, which has to be further enriched with some shape details, already exists and has to be changed only in a limited area submitted to a region constraint, in other words, the control of the shape must be guaranteed in two dimensions. Only the work done by Biermann, H., et al. [Bie02] deals with two-dimensional area constraints. In particular they consider the problem of pasting a given portion of surface on another one. In our case, the portion of surface to be inserted has to be created from the high-level parameters given by the user.

The rest of the paper is organized as follows. Section 2 discusses the proposed feature-based approach and the generalized sweep feature defined. In Section 3 the implemented prototype is described into details and Section 4 contains some results and concluding remarks.

2. FEATURE-BASED APPROACH

Form features group together shape entities having a specific meaning in a given context to treat them as a unique entity and to associate semantic information to geometric data. From the user's point of view, they can be seen as operators allowing for the insertion with few meaningful parameters of shapes having a predictable behavior, normally obtained by repetitive sequences of modeling steps.

In free-form domain it is difficult to define an exhaustive feature classification that is not too wide. Nevertheless, their usefulness in conceptual and detailed free-form design has been recognized and some attempts in bringing the feature concept into the free-form domain have been carried out [Per02, Pol95, Ver01, Cav92, Ber02, Fon00].

While in the mechanical domain few numerical parameters are sufficient to instantiate a specific feature element; on the contrary, in free-form modeling parameters must be higher geometric level entities (e.g. curves) in order to allow for the feature shape specification.

A classification of such features can be based on the spatial dimension of the overall constraint that the final surface must respect, thus named **target**:

- *Point-driven deformation feature* (0D), where the target is one or more points;
- *Curve-driven deformation feature* (1D), where the target is one or more curves;
- *Surface-driven deformation feature* (2D), where the target is an area.

It can be noticed that an overall constraint may be not necessarily described by only one geometric element but few can be used for its specification; for instance, the shape of a region may be univocally defined by a set of curves. These geometric elements correspond to feature parameters, i.e. the entities the user must give as the input to the system.

Here, the attention has been drawn to surface-driven features; in particular, to those that can be obtained by generalized sweep operations. For simplicity, in the following they are generally indicated as : **Sl-Features** (*Sweep-like Features*).

Sweep-like Features

Fontana, M., and some colleagues [Fon00] proposed a taxonomy of free-form features, aiming at enriching the modeling functionalities. The authors identified two categories in the different phases of computer-assisted styling activity: *structural features* and *detail features.* The latter correspond to local modifications of the surface adding aesthetic and functional details; two examples are shown in Fig. 1.

Considering the detail features, we focused on the subclasses of features which produce a deformation obtained by propagating a profile *s* (***section***) along a specific curve *d* (***directrix)***.

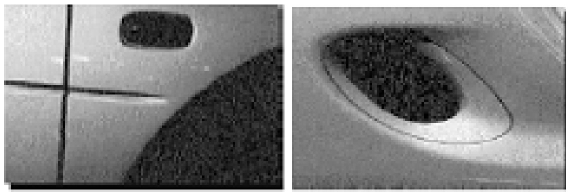

Figure1. Two detail features, a cross gap and a hole, respectively

In particular, we considered the classes of features shown in Figure 2.

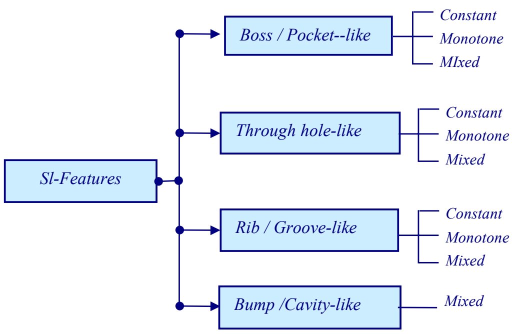

Figure 2. Sweep-like Features taxonomy

These classes are characterized by the properties of the two driving curves (whether open or closed) and their position with respect to the surface to which the feature has to be applied. Both can be either closed or not and can lay either on the surface or not -but not at the same time. The distinction between *boss* and *pocket*, *rib* and *groove*, *bump* and *cavity* is due to the direction of deformation with respect to the object to which it is applied: towards the exterior or the interior, respectively.

If the section is a closed curve laying on the surface and the directrix is an open curve in the 3D space, a Sl-Feature belongs to the *boss/pocket-like* or the *through hole-like* class. If the section is open, the directrix must lay on the surface and the feature is a *rib/groove-like* or a *bump/cavity-like*. Since sections can vary in size along the directrix, additional sub-categories have been specified introducing the concept of *scaling function sf* associating a scale factor to each point of the directrix.

Then, the Sl-Feature can be seen as a couple (*C*, *sf*), where *C* indicates the class of the feature (*Pocket, Boss, Through pocket, Rib, Groove, Bump, Cavity*) and *sf* the associated scaling function.

Let be

$$d: [0,1]=I \rightarrow \Re^3$$

$$t \rightarrow d(t)$$

the directrix; we define a *scaling function*

$$sf: I \rightarrow \Re$$

The following subclasses have been identified in accordance with the definition of *sf*, dependent on the curve length evaluated on the directrix.

Constant Sl-Feature. The section is unaltered along the directrix. In this case, *sf* is a constant function:

$$sf_c: t \rightarrow k_{const}.$$

Monotone Sl-Feature. The section size decreases or increases monotonically. If L is the length of the directrix curve and $l(t)$ the length in the interval $[0,t]$, we define

$$sf_m(t) = \frac{L - l(t)}{L} k_0 + \frac{l(t)}{L} k_1$$

where $k_0=sf(0)$, $k_1=sf(1)$. It can be noticed that the Sl-Feature is increasing if $k_0<k_1$, decreasing if $k_0>k_1$.

Here, for the sake of simplicity, we indicated a uniform scaling function, but, on user's request, they can be different in the width and height direction. It is even possible to apply it only in one direction as in the example of the "hat" in Figure 3, where the height is scaled but the width remains constant.

Mixed Sl-Feature. It is so-called if the sweep combines constant and monotone parts. This means

$$sf_i(t) : I_i \rightarrow \Re \text{ s.t. } sf_i(t) \in \{ sf_c(t), sf_m(t)\},$$

$$I = \bigcup_{i=1,\dots,n} I_i, \quad I_i \cap I_j = \varnothing, \quad \forall i \neq j$$

For this class, the user has to specify the starting points of the different portions, i.e. I_i, with the associated characteristics, i.e. the corresponding sf_i.

A very common case of mixed Sl-Features is given by the juxtaposition of two monotone parts joined at the common minimum or maximum, respectively. Here:

$$sf(t) = \begin{cases} \dfrac{l(\bar{t}) - l(t)}{l(\bar{t})} k_0 + \dfrac{l(t)}{l(\bar{t})} \bar{k}, & \text{if } 0 \le t \le \bar{t} \\ \dfrac{L - l(t)}{L - l(\bar{t})} \bar{k} + \dfrac{l(t) - l(\bar{t})}{L - l(\bar{t})} k_1, & \text{if } \bar{t} \le t \le 1 \end{cases}$$

where $k_0=sf(0)$, $k_1=sf(1)$ and $\bar{k} = sf(\bar{t})$, with $\bar{t} \in (0,1)$, is the relative maximum or minimum of *sf*. For simplicity, in the following we will refer to this particular case when talking of mixed Sl-Features.

Ribs and grooves, as well as pockets and holes, may contemplate the three different possibilities, while bumps and cavities can be only mixed since, by definition, $sf(0)=sf(1)=0$.

In Figure 3, examples of a cavity (mouth), a constant groove-like (on the neck), monotone (pieces composing the skirt) and variable rib-like (hat) are shown.

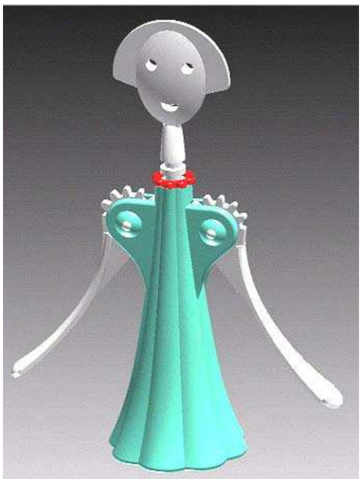

Figure 3. Examples of the different Sl-Features (by courtesy of *Alessi s.p.a.*)

For predefined shapes of sections, it is possible to give dimensional information (e.g. height, width) to immediately create families or patterns of features on the surface.

In the following Section, the algorithm for the implementation of the rib/groove-like and bump/cavity-like classes, is described.

3. RIB/GROOVE-LIKE AND BUMP/CAVITY-LIKE SL-FEATURE CREATION

The proposed algorithm to insert a Sl-Feature on a subdivision surface can be split into three main parts.

The first one handles the parameters of the feature in order to create the feature surface. The first operation done is a consistency test on the input data according to the feature type. Then, the section and the directrix are manipulated for the successive phase.

The second step builds the feature itself from the section and the directrix data as a separate discrete surface.

The third stage modifies locally the initial surface. An influence area around the directrix is defined such that its boundary corresponds to the one of the feature surface. Finally, the initial surface and the feature surface are glued together along the common boundary.

If the subdivision surface to be modified is refined at a level n, the feature insertion is performed at a level k, with $k<n$; by default $k=n-1$. In fact, the feature to create is a detail feature, inserted in a second step of the modeling phase: the product model is generally a rather refined surface, i.e. the number of mesh points is fairly high, and then heavy for manipulation. Therefore, it has been decided to operate directly on a coarser level of refinement, inserting a coarse version of the feature that will eventually be refined together with the entire surface.

In the algorithm, the Catmull-Clark scheme is adopted. It is an extension of cubic B-Splines: the initial tessellation is quadrangular almost everywhere, similarly to a NURBS control polyhedron, and it converges to a bicubic at the limit. In this way, the new geometry can be understood and manipulated by designers in an easier way.

In the following subsections, the three parts will be described and some pictures of a simple case (a regular portion of a possible mesh) are used to exemplify the algorithm more clearly.

Parameters' Check and Elaboration

According to the specific Sl-Feature type, a different number of parameters needs to be considered for generating the desired shape.

In all the cases, the defining parameters s (open curve) and a directrix d must be specified. We give the users the possibility to initially scale s of a factor α in order to allow them to instantiate already defined curves, e.g. corresponding to shape archetypes or to create patterns, without having to treat separately the curvilinear parameter to provide. Note that α is set equal to 1 by default; then, the input values of the scaling function sf previously introduced have to be chosen starting from $s'=\alpha s$.

For *constant ribs/grooves* no additional parameters are needed and no consistency check is necessary: $sf(t)=1, \forall t\in[0,1]$.

In case of *monotone ribs/grooves*, the given section s is placed at the first endpoint of d and an additional factor specifying the ratio of the final section of the sweep surface is needed, i.e.:

$sf(0)=k_0=1$;

$sf(1)=k_1$.

For *mixed ribs/grooves*, s is the maximum/ minimum section; two scaling factors are required for the endpoints together with the position $Q=Q(\bar{t})$ of s on the directrix, i.e.

$sf(0)=k_0$;

$sf(1)=k_1$;

$sf(\bar{t})=\bar{k}=1$.

For *bumps/cavities* only the position of s, $Q=Q(\bar{t})$, on the directrix is required since, by definition, $sf(0)=sf(1)=0$, and $sf(\bar{t})=\bar{k}=1$.

Depending on the specific type some checks on the values of the provided parameters have to be performed in order to avoid inconsistent situations. Examples of consistency checks are:

- *Monotone rib/groove*: $d(0)\neq d(1)$ and $sf(0)\neq sf(1)$.
- *Mixed rib/groove*: if $d(0)\neq d(1)$, then, $sf(0)<1$ iff $sf(1)<1$ and $\bar{t}\in(0,1)$; if $d(0)=d(1)$, then $sf(0)=sf(1)$.
- *Bump/Cavity*: $d(0)\neq d(1)$.

Geometrically speaking, the constraint lines can be given arbitrarily, both as polygons and as B-splines. For the sake of simplicity, the section is assumed to be planar, but this choice covers most of the practical needs during the modeling phase.

Since the feature surface to create is a tessellation, a discretization of the two curves is necessary. The control points of the section are retrieved (a polyline can be seen as a B-spline of degree 1). Using the Catmull-Clark scheme, the refinements of the curve can be maintained consistent with the ones of the surface.

In the implementation, the directrix has to be discretized such that it belongs to the edges and the vertices of the mesh. At present it is directly built as a polyline fitting some edges of the mesh, supposing the error ε between a proper sampling of the curve and the vertices of the initial surface is small enough, i.e. $|d(t_i)-v_i|<\varepsilon \ \forall i=0,\ldots,n-1$, where v_i are vertices of the initial surface.

Feature Surface Creation

The feature surface is naturally created as the tensor product between the polygonal section and the directrix. The tessellation obtained is the base control polyhedron of a new subdivision surface.

First of all, the discretized section $\tilde{s}$ is duplicated n times as the number of vertices of the directrix d. A local coordinate system Lc is associated to $\tilde{s}$ in order to place the various sections consistently with the underlying mesh. The local coordinate system chosen for $\tilde{s}$ is given by

$$Lc(\tilde{s})=\{S_m-S_0, (S_m-S_0)\wedge \underline{\tilde{N}}, \underline{\tilde{N}}\},$$

where S_m, S_0 are the endpoints of the section and $\underline{\tilde{N}}$ is the normal at the plane containing the curve (see Fig. 4(a)). Each copy s_i is positioned such that $Lc(\tilde{s})$ coincides with the local reference system

$$Lc(s_i)=\{\underline{T_i}, \underline{N_i}, \underline{T_i}\wedge\underline{N_i}\}, \ \forall i$$

where $\underline{T_i}$ is an appropriate tangent in v_i and $\underline{N_i}$ the normal to the mesh in v_i, as shown in Fig. 4(b). Clearly, in case of grooves and cavities, we consider $-\underline{N_i}$. In this way, the feature surface will adapt to the behavior of the initial surface determined by the normal direction. Analogously, it takes into account the behavior of the directrix through the choice of a proper tangent: $\underline{T_i}=\underline{b_i}$^$\underline{N_i}$, being $\underline{b_i}$ the direction of the bisector of the angle θ between the edges of d incident to v_i.

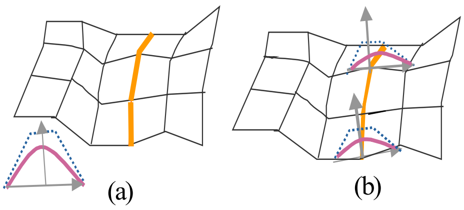

Figure 4. (a) Assignment of a local frame to $\tilde{s}$ and (b) duplication of $\tilde{s}$ along *d*

After positioning each s_i, the values of the scaling function in t_i are evaluated, depending on the feature type and the given scaling values: the choice of *sf* as defined in the previous Section guarantees a smooth size variation to avoid undesired artifacts.

Since the directrix may be arbitrary, it does not necessarily follow a specific direction: corners may be present and can be detected by the angle θ. In order to avoid shrinking effects, each section is resized. Considering how the local coordinate system is defined for $\tilde{s}$, it can be noticed that the shrinking happens along the local *x*-axis (i.e. the vector $\underline{T_i}$). The relationship to exploit in order to preserve the feature shape is therefore the following:

$$\forall S \in CV_{\tilde{s}}, \quad x'_S \rightarrow \beta x'_S, \quad \text{with} \quad \beta = 1/\cos\frac{\theta}{2},$$

where $CV_{\tilde{s}}$ is the set of the points of $\tilde{s}$ and the prime indicates that the calculations are performed in the local frames. If $\theta=\pi$, the section does not need to be resized since the two edges are collinear.

Another aspect we have considered after duplicating the sections is the correspondence with the mesh. In fact, the section endpoints do not generally lie exactly on the mesh, but they can totally or partially be over or below (see Fig. 5). It has been chosen to calculate the intersection points between each section and the surface itself and then to move the section endpoints to the obtained intersections (if the section does not intersect the mesh, the extension of the end segments will do it). In our opinion, such a choice better preserves the design intent. In fact, we are namely considering details features which intrinsically have a limited size with respect to the entire surface: modifying the geometry of the sections in this way corresponds to give a priority to the underlying surface shape and, at the same time, does not alter the specific section too much. Besides, if we had chosen to rotate the sections such that their endpoints were closer to the initial surface, torsions would have appeared on the feature surface.

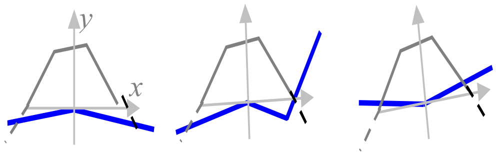

Figure 5. Relative positions (profile view) of the copies of $\tilde{s}$ with respect to the mesh (darker line)

When all the sections are adapted, the tensor product between the two generating curves of the feature surface can be finally computed.

Feature Insertion

The third step performs a remeshing of a localized area around the directrix. The intersections found in the previous step constitute the boundary of the feature to insert; then, they are added as new vertices of the surface and the topology around the segmented directrix is properly modified.

The new vertices are inserted in the initial mesh, while the ones of the trajectory removed. To illustrate a generic step of the local remeshing, let P be an internal vertex of the directrix, P_s the successive point in d, P_1 and P_2 the points inserted at the previous step, P_{1s} and P_{2s} the intersection points of the section relative to P to insert as the successors of P_1 and P_2, respectively (see Figure 6(a)).

First, the vertex P of the edges incident to P between the edge P_j-P and P-P_s, is changed with P_{js}, $\forall j$=1,2. Then, the edges P_j-P_{js} are created, together with the temporary edges P_{js}-P_s (since they will be eliminated at the next step), while P-P_s, edge of d, is deleted.

In Figure 6(b), the new faces Ft_j, adjacent to the temporary edge P_{js}-P_s, are tagged as temporary, while all the other modified faces are tagged as definitive.

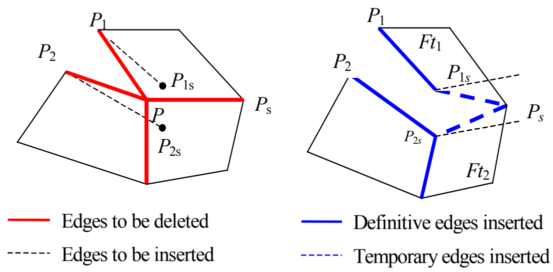

Figure 6. Upgrading the original surface along a point P on *d* (a) according to the $\tilde{s}$ extremes (b)

The topology updated in this way preserves the structure of the mesh. In fact, the strategy has been thought to be able to apply the Catmull-Clark scheme in the regular case for the most of the vertices of the considered area: quads or triangles are kept quadrangular or triangular, respectively, and concave faces cannot be created in the general case. In critical situations, if a concave face appears it is split in two triangles. However, triangular faces have to be created in correspondence with the initial and final sections to join the new quads with the old ones and maintain the surface manifold, as shown in Figure 7.

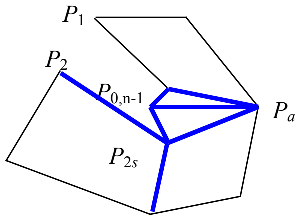

Figure 7. Remeshing at the end points of *d*, $P_{0,n-1}$.

The final operation to perform is gluing the modified surface and the feature hull. It can be done with no approximation, since there is a one-to-one correspondence between the boundary of the feature surface and the created hole in the surface itself.

4. RESULTS AND CONCLUSIONS

In this paper we presented a feature-based approach for modeling with subdivision surfaces. It is aimed at providing designers with the possibility of adding details by means of few meaningful parameters. The focus has been on the definition of features having a shape obtainable by means of sweep operations.

The algorithm proposed in this paper has been developed in *Maya* by *Alias|wavefront* [1] a commercial modeler which supports subdivision surfaces together with more common geometric representations.

In Fig. 8 examples of the insertion of constant Sl-Features in a bottle are shown. At first, a rib-like is created to model the enlargement around the neck; then, a decorative pattern of grooves is added. In both cases the section is a cubic spline laying in the 3D space; the directrix is a curve on the surface, closed and open respectively. The Figures 8(a) shows the features inserted at a coarse level, whereas in Figs. 8 (b) and (c) two successive refinement steps are applied.

[1] URL: http://www.aliaswavefront.com

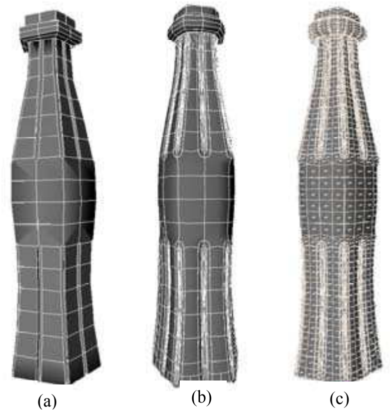

Figure 8. The insertion of a rib-like (neck) and of a pattern of groove-like features at a coarse level (a) and after two steps of refinement (b,c)

To show the feasibility of the presented method on a real model, a mixed rib-like feature has been inserted in the lateral side of a *Ferrari Modena* by Pininfarina (Figure 9). The character line, i.e. the directrix, is an open curve and it is highlighted on the initial surface together with the starting vertices of each portion in Fig. 9(top). The feature is composed by two monotone parts joined with a constant one. The user selects the directrix edges, the curve corresponding to the section and the starting vertices as geometric parameters, and couples them with the scaling factors of each portion. In this example, the rib has been chosen to vanish at the extremes. Fig. 9(bottom) shows the final shape with the feature inserted.

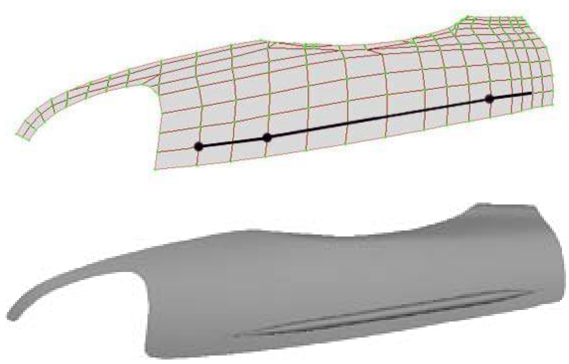

Figure 9. Selected directrix and starting vertices on the initial mesh (top) and final refined surface with a mixed rib-like inserted (bottom)

The future work will concentrate on the improvement of the algorithm efficiency. Moreover, the discretization of the directrix in case is given by the user as a B-spline will be considered: if the points of the original surface are not sufficient for an acceptable sampling, a local refinement and a replacement of the surface control points are planned.

5. ACKNOWLEDGMENTS

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