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Influence of crucial parameters of the system of an inverted pendulum driven by fibres on its dynamic behaviour

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Abstract

Fibres, cables and wires can play an important role in design of many machines. One of the most interesting applications is replacement of chosen rigid elements of a manipulator or a mechanism by fibres. The main advantage of this design is the achievement of a lower moving inertia, which leads to a higher mechanism speed, and lower production costs. An inverted pendulum attached and driven by two fibres serves as a typical testing system for the investigation of the fibres properties influence on the system dynamic response. The motion of the pendulum of this nonlinear system is investigated using the **alaska** simulation tool. The influence of some parameters of the system of inverted pendulum driven by fibres has been investigated. The evaluation of influence of these crucial parameters of the system of inverted pendulum driven by fibres on its dynamic behaviour is given in this article.

Keywords: inverted pendulum, fibre, kinematic excitation

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1. Introduction

Replacement of the chosen rigid elements of manipulators or mechanisms by fibres or cables [1] is advantageous due to the achievement of a lower moving inertia, which can lead to a higher machine speed, and lower production costs. Drawbacks of using such flexible elements can be associated with the fact that fibres should be only in tension [19, 20] in the course of a motion. The possible fibre modelling approaches should be tested and their suitability verified in order to create efficient mathematical models of cable-based manipulators mainly intended for the control algorithm design. Fibres are modelled using a simple force approach in almost all cases (e.g. [23]). However, in this article a more advanced model based on a point-mass approach is studied in more detail.

An inverted pendulum driven by two fibres attached to a frame (see Fig. 1) is a simplified representation of a typical cable manipulator. Real example of such a manipulator is e.g. HexaSphere redundant parallel spherical mechanism [24] (see Fig. 2), at which the replacement of rigid links by fibres is considered [20]. Functionality of fibres must be verified on this simpler example. The motion of the pendulum of this nonlinear system is investigated using the **alaska** simulation tool and an in-house software created in the MATLAB system. The influence of some parameters of the system of inverted pendulum has been investigated. The influence of

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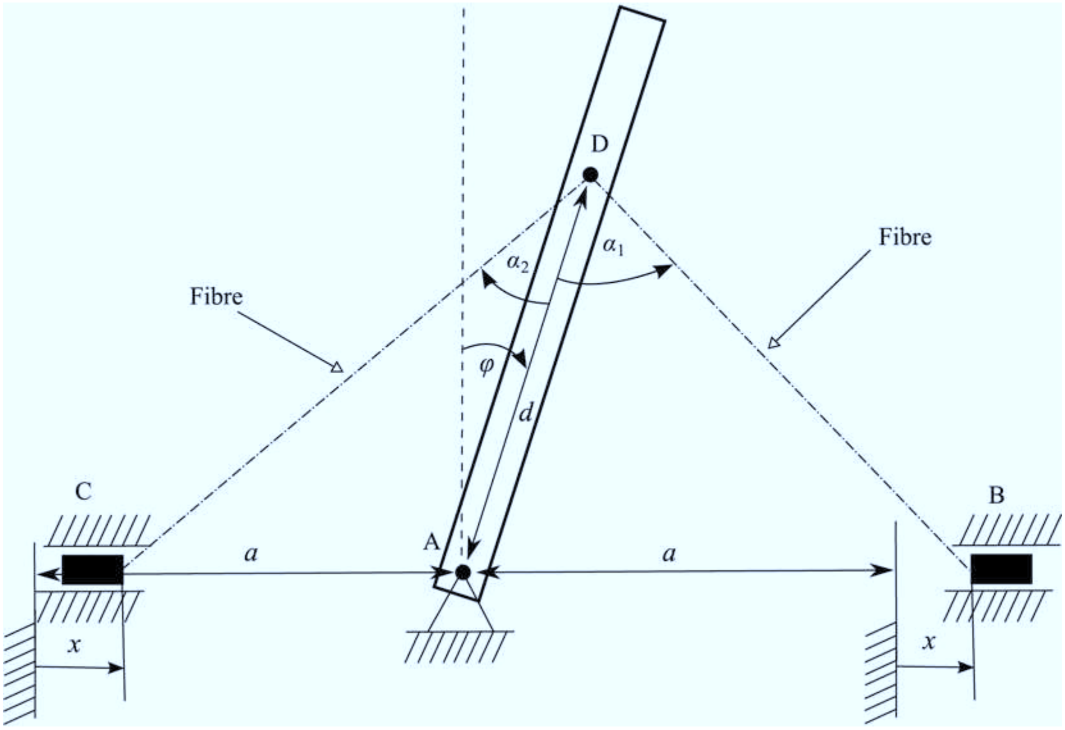


Fig. 1. Scheme of the inverted pendulum actuated by fibres

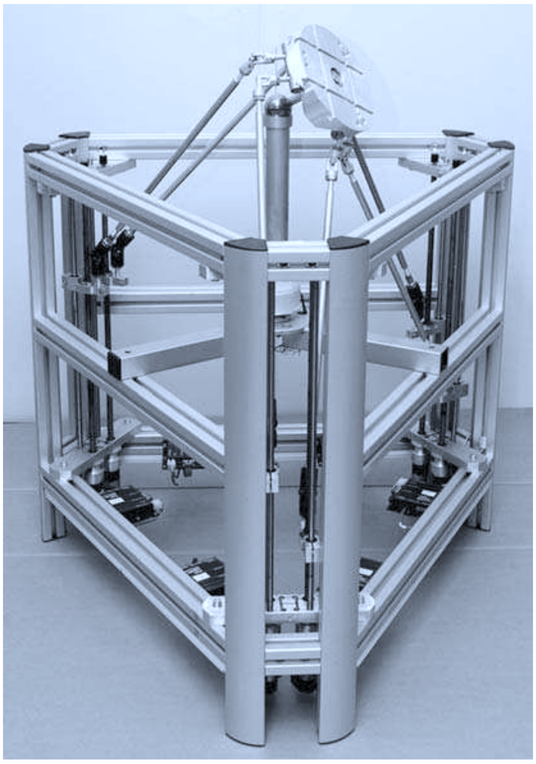


Fig. 2. HexaSphere [24]

the actuated fibres motion on the pendulum motion in the case of simultaneous harmonic excitation of fibres was investigated in [11] or [17], the influence of the phase shift in the case of non-symmetric harmonic excitation of fibres was investigated in [12]. The effect of the fibres preload on the pendulum motion was investigated in [13], the effect of the mass of the fibres on the pendulum motion was investigated in [15] and the influence of the amplitude of the harmonic kinematic excitation of fibres on the pendulum motion was investigated in [14] (all of them in the case of fibres simultaneous harmonic excitation). The evaluation of the influence of these crucial parameters of the system of inverted pendulum on its dynamic behaviour is given in this article.

As it was already mentioned, the point-mass model of the fibres is considered in the model of the system of inverted pendulum. The model of the inverted pendulum system is considered to be two-dimensional. Each fibre is discretized using 10 point masses (e.g. [11]). Each point mass is unconstrained (i.e. number of degrees of freedom is 3) in a two-dimensional model of the system. The adjacent point masses are connected using spring-damper elements. Only axial spring and damping forces are considered in these spring-damper elements. The stiffness and the damping coefficients between the masses are determined in order to keep the global properties of the fibre model based on the force approach. The validation of the point-mass model is given in [16].

2. Possibilities of fibre modelling

The fibre (cable, wire etc.) modelling [5] should be based on considering the fibre flexibility and the suitable approaches can be based on the flexible multibody dynamics (see e.g. [4, 18]). The simplest way how to incorporate fibres in equations of motion of a mechanism is the force representation of a fibre (e.g. [2]; the massless fibre model). It is assumed that the mass of fibres is small to such an extent comparing to the other moving parts that the inertia of fibres is negligible with respect to the other parts. The fibre is represented by the force dependent on the fibre deformation and its stiffness and damping properties. This way of the fibre modelling is probably the most frequently used model in the cable-driven robot dynamics and control.

A more precise approach is based on the representation of the fibre by a point-mass model (e.g. [7]). The fibre can be considered either flexible or rigid. It has the advantage of a lumped point-mass model. The point masses can be connected by forces or constraints.

In order to represent bending behaviour of fibres their discretization using the finite segment method [18] or so called rigid finite elements [22] is possible. Standard multibody codes (SIMPACK, MSC.ADAMS, **alaska** etc.) can be used for this purpose. Other more complex approaches can utilize nonlinear three-dimensional finite elements [3] or can employ the absolute nodal coordinate formulation (ANCF) elements [4, 6, 8, 18].

The approaches to the modelling of the system of inverted pendulum driven by fibres were investigated in [6, 9, 10]. Implementation of the model based on the finite rigid elements into the **alaska** simulation tool proved to be unsuitable [9]. The ANCF elements cannot be implemented in the **alaska** simulation tool, verification on this approach was carried out utilizing the MATLAB system [6, 10].

3. Inverted pendulum

As an example of the investigation of fibres behaviour an inverted pendulum, which is attached and driven by two fibres (see Fig. 1) and affected by a gravitation force, was chosen. When the pendulum is displaced from the equilibrium position, i.e. from the "upper" position, it is returned back to the equilibrium position by the tightened fibre. As it has already been mentioned, this system was selected with respect to the fact that it is a simplification of possible cable-based manipulators. In addition it was supposed that the nonlinear system of the inverted pendulum attached to a frame by two fibres could show an unstable behaviour under specific excitation conditions (e.g. [21]).

For better description of the solved problem a simple massless model is presented. The massless model is shown in Fig. 1. The used point-mass model of the fibres with lumped point masses is geometrically identical [11].

The system kinematics can be described by angle φ of the pendulum with respect to its vertical position (one degree of freedom), angular acceleration $\ddot{\varphi}$ and prescribed kinematic excitation $x(t)$. The equation of motion is of the form

$$\ddot{\varphi} = \frac{1}{I_\mathrm{A}}\left(F_{v1} d \sin\alpha_1 - F_{v2} d \sin\alpha_2 + mg\frac{l}{2}\sin\varphi\right), \tag{1}$$

where I_A is the moment of inertia of pendulum with respect to point A (see Fig. 1), α_1 and α_2 are the angles between the pendulum and the fibres, m is the pendulum mass, F_{v1} and $F_{v2}$are the forces acting on the pendulum from the fibres, g is the gravity acceleration, l is the pendulum length and d is the distance from the axis in point A to the position of the attachment of fibres to the pendulum (point D). Kinematic excitation acts in the points designated B and C (see Fig. 1).

The forces acting on the pendulum from the fibre are

$$\begin{aligned} F_{v1} &= \left[k_v(l_{v1} - l_{v0}) + b_v\frac{\mathrm{d}l_{v1}}{\mathrm{dt}}\right] \cdot H(l_{v1} - l_{v0}), \\ F_{v2} &= \left[k_v(l_{v2} - l_{v0}) + b_v\frac{\mathrm{d}l_{v2}}{\mathrm{dt}}\right] \cdot H(l_{v2} - l_{v0}), \end{aligned} \tag{2}$$

where k_v is the fibre stiffness, b_v is the fibre damping coefficient, l_{v0} is the original length of the fibres and $H(\cdot)$ is the Heaviside function. It is supposed that forces act in the fibres only when the fibres are in tension.

Actual lengths l_{v1} and l_{v2} of the fibres should be calculated in each time

$$\begin{aligned} l_{v1} &= \sqrt{(d\cos\varphi)^2 + (a + x(t) - d\sin\varphi)^2}, \\ l_{v2} &= \sqrt{(d\cos\varphi)^2 + (a - x(t) + d\sin\varphi)^2}. \end{aligned} \tag{3}$$

Kinematic excitation is given by function

$$x(t) = x_0 \sin(2\pi f t + \psi), \tag{4}$$

where x_0 is the chosen amplitude of motion, f is the excitation frequency, ψ is the phase shift (in case of symmetric excitation $\psi = 0$) and t is time.

The chosen model parameters (see Fig. 1) are: $l = 1$ m, $a = 1.2$ m, $d = 0.75$ m, $I_\mathrm{A} = 3.288\ \mathrm{kg \cdot m^2}$, $m = 9.864$ kg, $k_v = 8.264 \cdot 10^3$ N/m (stiffness), $b_v = 5 \cdot 10^{-4} \cdot k_v\ \mathrm{N \cdot s/m}$ (damping coefficient). Additional parameters of the point-mass models are [11–13, 17] fibre cross-section area $A_v = \pi \cdot 0.000\,001\ \mathrm{m^2}$ and fibre density $\rho_v = 4\,000\ \mathrm{kg \cdot m^{-3}}$ (these parameters represent the wattled steel wire — see Fig. 3). In this case mass of one fibre is 17.783 grams.

The natural frequency of the linearized system of the inverted pendulum in equilibrium position is 5.04 Hz. Thus the extreme values of pendulum angle without the parameters change appears at excitation frequency 5 Hz [11, 17].

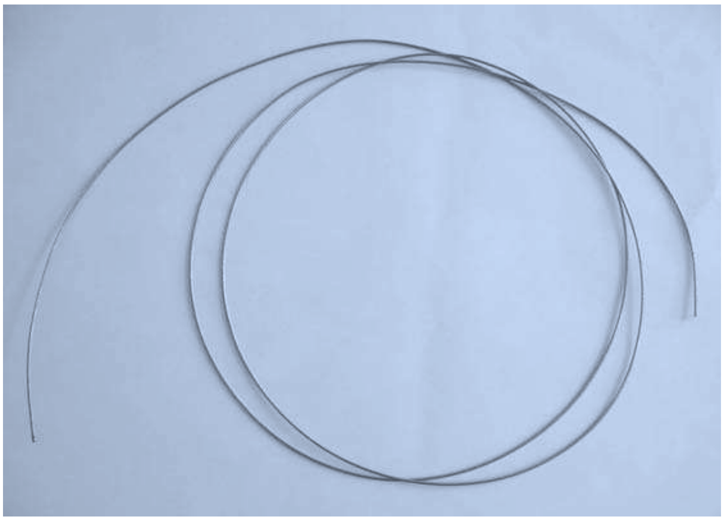


Fig. 3. Wattled steel wire

4. Simulation results

As it was already stated, the fibres preload [13], the mass of the fibres [15], the amplitude of the harmonic kinematic excitation of fibres [14] (all in the case of fibres simultaneous harmonic excitation) and the phase shift the case of non-symmetric harmonic excitation of fibres were evaluated as the crucial parameters of the system of the inverted pendulum driven by fibres [12].

Presented results are obtained using the **alaska** simulation tool. Generated nonlinear equations of motion are solved by means of numerical time integration. The simulations results presented in this article were obtained utilizing the Livermore Solver for Ordinary Differential Equation (LSODE) for stiff systems, maximum relative error that **alaska** allows at each integration step was chosen $0.000\,1$ and maximum absolute error that **alaska** allows at each integration step was chosen $0.000\,1$, too. Time step of this integration routine is variable.

Kinematic excitation amplitude was chosen $x_0 = 0.02$ m (excepting the investigation of the influence of the amplitude of the harmonic kinematic excitation of fibres on the pendulum motion [14], where it is changed in the range from $x_0 = 0.02$ m to $x_0 = 0.2$ m). Excitation frequency f was considered in the range from 0.1 Hz to 200 Hz. Some of results are given in the frequency range from 0.1 Hz to 10 Hz because the upper limit of excitation frequencies 200 Hz is too high for the practical use in manipulators.

Time histories and extreme values of pendulum angle, of the forces in the fibres and of the positions of the point masses are the monitored quantities. At excitation amplitude $x_0 = 0.02$ m maximum value of pendulum angle at quasi-static loading is $\varphi = 1.52°$; minimum value of pendulum angle at quasi-static loading is logically $\varphi = -1.52°$. Selected results of the numerical simulations are presented in Figs. 4 to 12. Simulation time is 10 seconds. It was verified that after this period the character of the system response to the kinematic excitation, apart from exceptions, does not change (e.g. [11]).

Besides the excitation frequency [11, 17] of moving fibres the pendulum motion influences all the investigated parameters (i.e. the phase shift in the case of non-symmetric harmonic excitation of fibres, the fibres preload, the mass of the fibres and the amplitude of the harmonic kinematic excitation of fibres).

Based on the obtained results it is evident that the pendulum motion is mostly influenced (besides the excitation frequency of the moving fibres) by the fibres preload [13] and by the

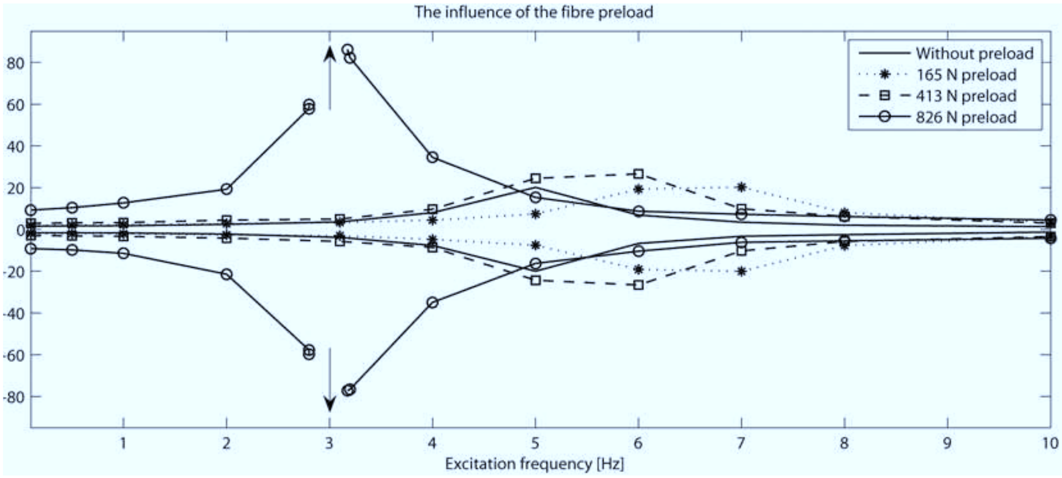


Fig. 4. Extreme values of time histories of pendulum angle φ in dependence on the excitation frequency, investigation of the influence of the fibre preload

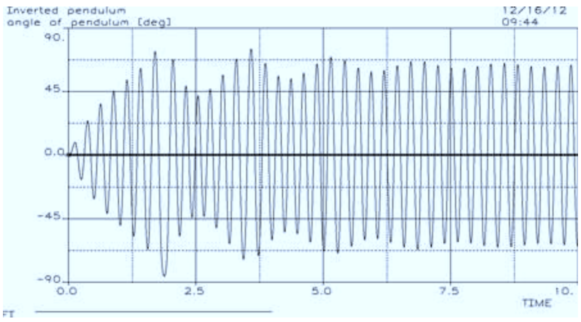
amplitude of the harmonic kinematic excitation of fibres [14]. At the change of these parameters an unstable behaviour of the studied system was detected.

The next mentioned extreme values of the pendulum angle refer to the dependence on excitation frequency f (not to the time histories of the pendulum angle at definite excitation frequency f).

When investigating the effect of the fibres preload on the pendulum motion results at preload 165 N in fibres (i.e. at shortening by the fibres by 2 % of the free length), at preload 661 N in fibres (at shortening of the fibres by 8 % of the free length), at preload $1\,033$ N in fibres (at shortening of the fibres by 12.5 % of the free length) and at preload $1\,099$ N in fibres (at shortening of the fibres by 13.3 % of the free length) are commented in this article.

Owing to fibres preload the excitation frequency at which the pendulum angle reaches extreme values at low preload first increases (at preload 165 N in fibres up to 7 Hz) and then decreases (down to 3 Hz) at growing preload but the extreme values of pendulum angle increase — see Fig. 4. The pendulum vibration without fibres preload is more stable — in the time histories of the pendulum angle vibration damping occurs; in some cases the pendulum does not vibrate at all [11, 17]. For the first time at preload 661 N in fibres, at excitation frequency 3.83 Hz, the extreme value of pendulum angle is already greater than 90 degrees and the pendulum "oscillates" between both semiplanes defined by B and C points (see Fig. 5). From the investigated cases, e.g. at preload $1\,033$ N in fibres, the extreme values of pendulum angle are greater than 90 degrees at excitation frequencies in the range from 0.1 Hz to 2.6 Hz. At fibres preload $1\,099$ N in fibres (and higher) the extreme values of pendulum angle are greater than 90 degrees independently of excitation frequency in the whole investigated range (i.e. from 0.1 Hz to 200 Hz) [13].

The pendulum motion is influenced by the amplitude of the harmonic kinematic excitation of fibres (see Figs. 6 to 8). As it has already been stated the extreme value of pendulum angle (without the parameters change) at the investigated excitation amplitude $x_0 = 0.02$ m appears at excitation frequency 5 Hz [11–13, 17]. Owing to the amplitude of the harmonic kinematic excitation of fibres the excitation frequency at which the pendulum angle reaches the extreme value decreases (for example at excitation amplitude $x_0 = 0.06$ m down to 3.7 Hz) and on the contrary the (absolute) extreme value of the pendulum angle increases — see Fig. 6. For the first time at excitation amplitude $x_0 = 0.055$ m of fibres (at excitation frequencies in the range



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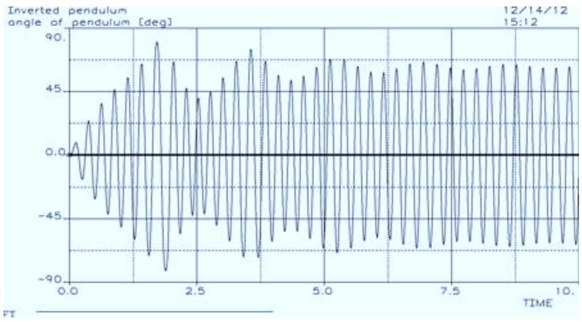

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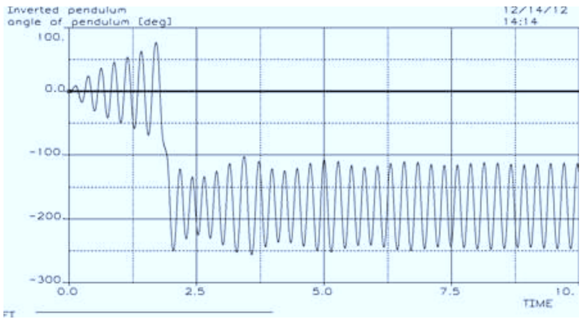


c)

Fig. 5. Time history of pendulum angle φ, at preload 661 N in fibres, a) excitation frequency $f = 3.82$ Hz, b) excitation frequency $f = 3.84$ Hz, c) excitation frequency $f = 3.83$ Hz, investigation of the influence of the fibre preload

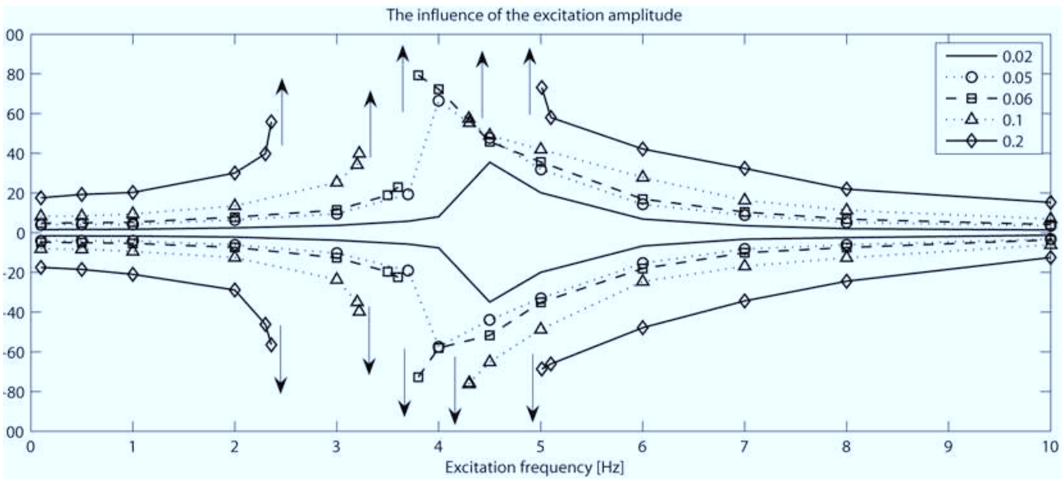


Fig. 6. Extreme values of time histories of pendulum angle φ in dependence on the excitation frequency, investigation of the influence of the excitation amplitude

from 3.76 Hz to 3.78 Hz) the extreme value of the angle is already greater than 90 degrees and the pendulum "oscillates" between both semiplanes defined by B and C points (see Fig. 8). At higher excitation amplitudes the extreme values of pendulum angle are greater than 90 degrees in some range of excitation frequencies, which extends at increasing amplitudes (e.g. at excitation amplitude $x_0 = 0.2$ m the extreme values of pendulum angle are greater than 90 degrees at excitation frequencies in the range from 2.4 Hz to 5 Hz) — see Fig. 6. Besides extreme values of pendulum angle at lower frequencies in the courses of extreme values of pendulum angle in dependence on the excitation frequency local extremes appear at higher frequencies (e.g. at

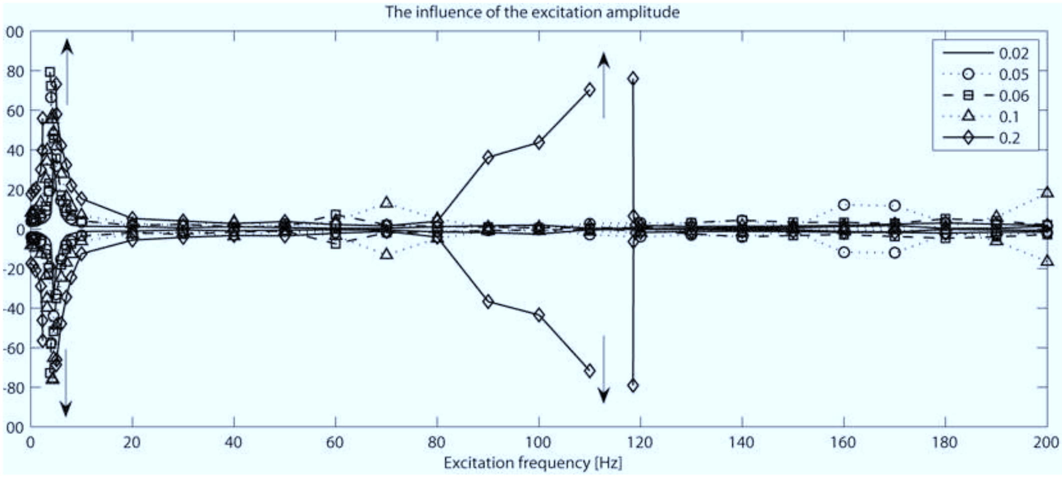
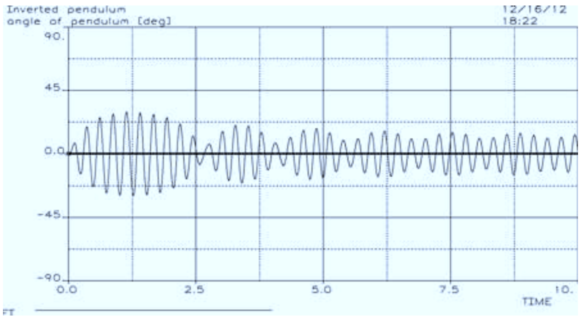


Fig. 7. Extreme values of time histories of pendulum angle φ in dependence on the excitation frequency (in the whole investigated frequency range), investigation of the influence of the excitation amplitude



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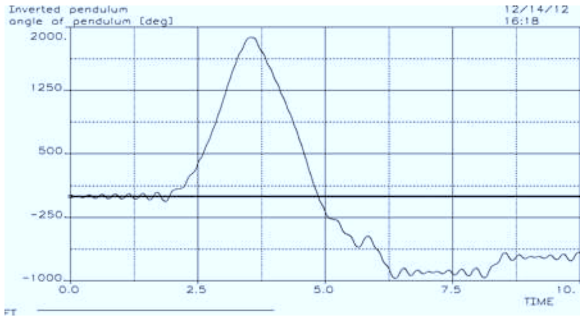

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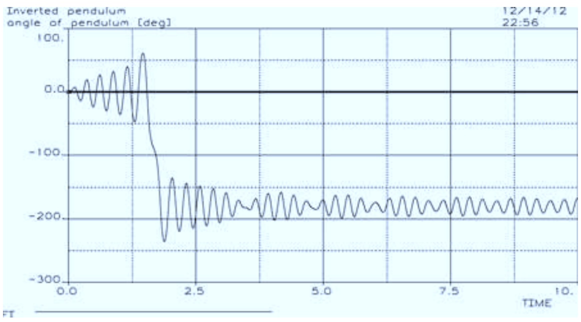

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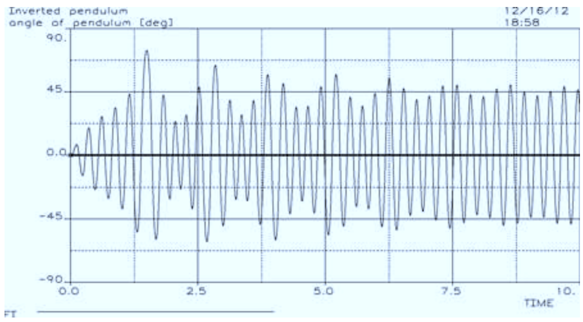

d)

Fig. 8. Time history of pendulum angle φ, amplitude of excitation $x_0 = 0.055$, a) excitation frequency $f = 3.75$ Hz, b) excitation frequency $f = 3.76$ Hz, c) excitation frequency $f = 3.78$ Hz, d) excitation frequency $f = 3.79$ Hz, investigation of the influence of the excitation amplitude

excitation amplitude $x_0 = 0.02$ m at 40 Hz; at excitation amplitude $x_0 = 0.2$ m in the range from 111 Hz to 118 Hz the extreme value of pendulum angle is even greater than 90 degrees — see Fig. 7) [14].

Influences of the change in the mass of the fibres [15] and of the change in the phase shift in the case of non-symmetric harmonic excitation of fibres [12] do not affect the pendulum motion as significantly as the fibres preload and as the amplitude of the harmonic kinematic excitation of fibres. These parameters do not cause the unstable behaviour of the system of the inverted pendulum.

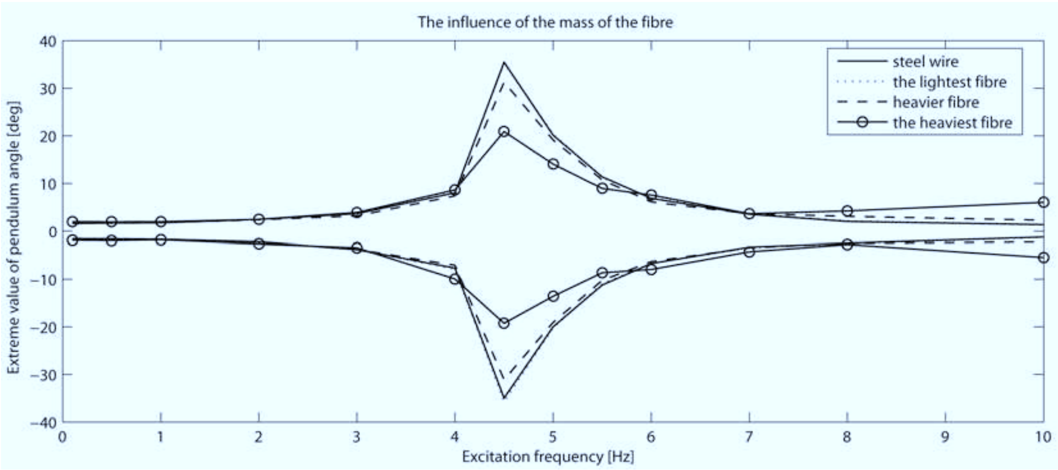

Fig. 9. Extreme values of time histories of pendulum angle φ in dependence on the excitation frequency, investigation of the influence of the mass of the fibres

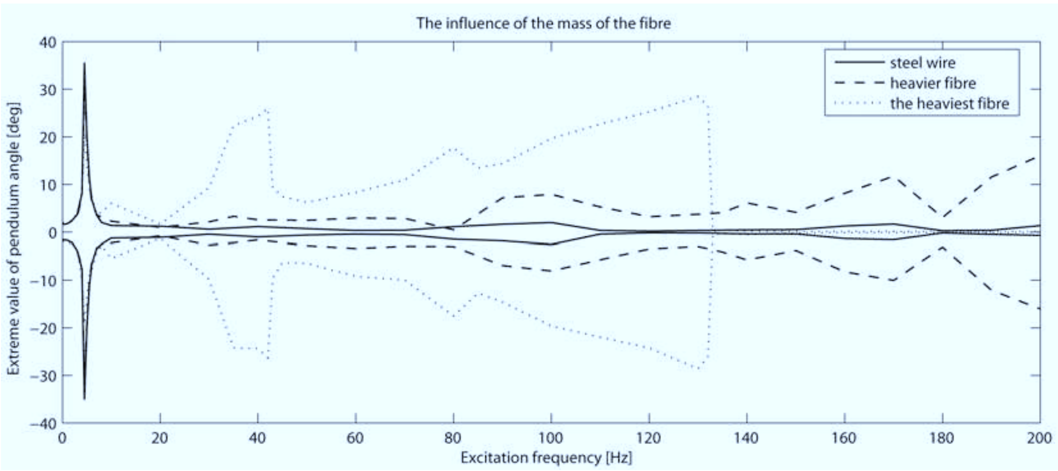

Fig. 10. Extreme values of time histories of pendulum angle φ in dependence on the excitation frequency (in the whole investigated frequency range), investigation of the influence of the mass of the fibres

At investigating the effect of the mass of the fibres on the pendulum motion results which were obtained using the "the lightest" fibres (carbon fibres; mass of one fibre is 3.846 grams), using (already mentioned) wattled steel wire parameters, using "heavier" fibres (virtual fibres of the mass ten times larger than the mass of the wattled steel wire; mass of one fibre is 177.83 grams) and using "the heaviest" fibres (virtual fibres of the mass of one fibre $1\,269$ grams) are presented in this article.

The results which were obtained using "the lightest" fibres are (almost) the same as the results which were obtained using the massless models [15]. The global extreme values of pendulum angle (with exception of using "the heaviest" fibres, where only the local extreme values are concerned) appear at excitation frequency 5 Hz irrespective of the mass of the fibres (see Fig. 9) in the monitored interval of the excitation frequencies (i.e. up to 200 Hz). The lower the mass of the fibres the higher the extreme values of the pendulum angle (see Figs. 10 and 11) at this frequency. When using "heavier" fibres pendulum angle increases at higher excitation frequencies (above 170 Hz) (see Fig. 10). When using "the heaviest" fibres pendulum angle achieves considerable local extreme value at excitation frequency 40 Hz and high extreme values of pendulum angle appear at the excitation frequency up to 130 Hz [15] — see Fig. 10.

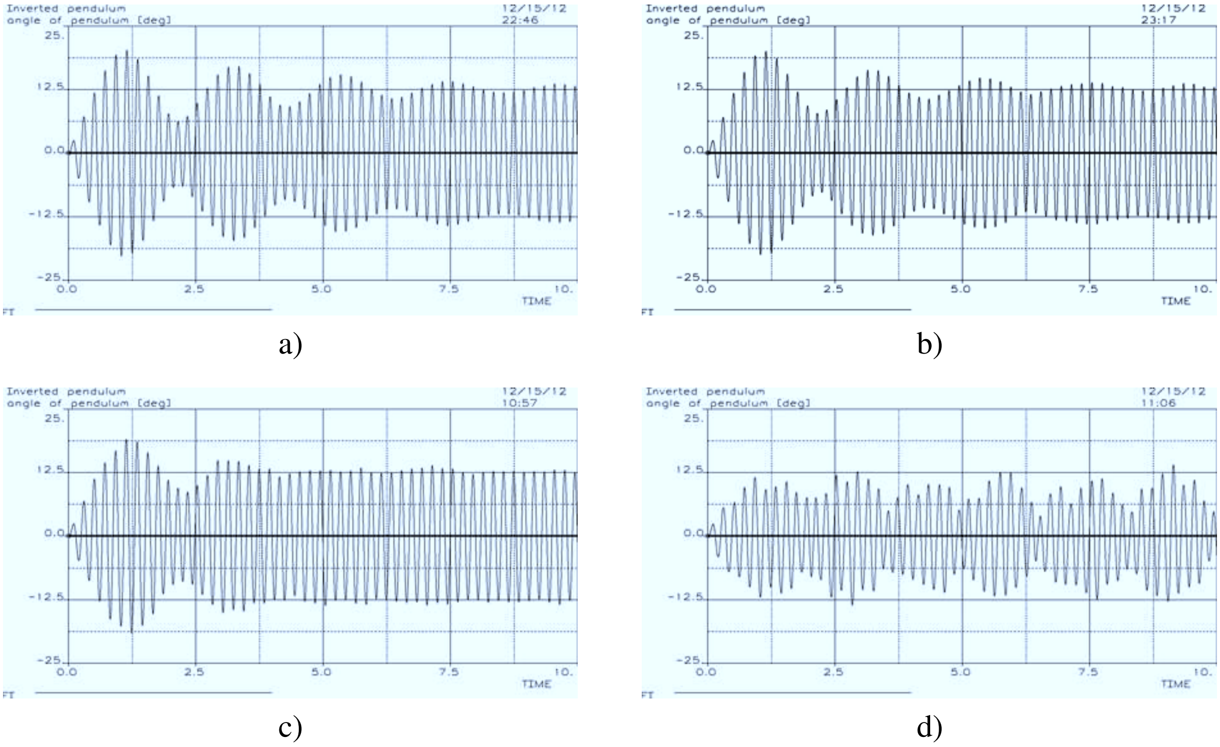

Fig. 11. Time history of pendulum angle φ, excitation frequency $f = 5$ Hz, a) carbon fibres, b) wattled steel wires, c) "heavier" fibres, d) "the heaviest" fibres, investigation of the influence of the mass of the fibres

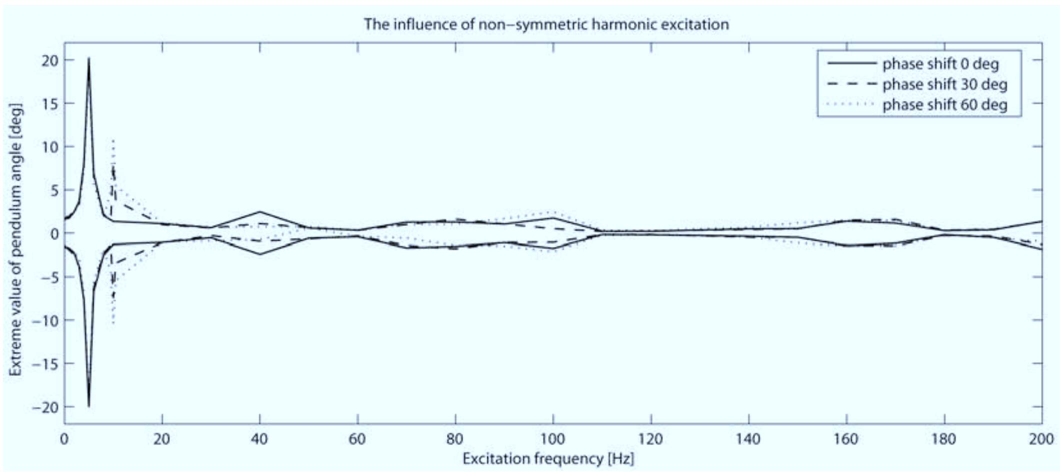

Fig. 12. Extreme values of time histories of pendulum angle φ in dependence on the excitation frequency, investigation of the influence of the phase shift in the case of non-symmetric harmonic excitation of fibres

The phase shift of in the case of non-symmetric harmonic excitation of the fibres does not manifest itself too much in the change of the character of pendulum vibration at the definite excitation frequency but especially at excitation frequency 10 Hz in the local extreme values of pendulum angle. The local extreme value of pendulum angle at this excitation frequency increases with the increasing phase shift — see Fig. 12.

5. Conclusion

The approach to the fibre modelling based on the lumped point-mass representations was utilized for the investigation of the influence of the crucial parameters of the system of the inverted pendulum driven by two fibres attached to a frame on its motion. Harmonic excitation of the fibres was considered.

Based on the obtained results it is evident that the pendulum motion is mostly influenced (besides the excitation frequency of the moving fibres) by the fibres preload [13] and by the amplitude of the harmonic kinematic excitation of fibres [14]. At the change of these parameters an unstable behaviour of the studied system was detected. Changes in other investigated parameters of this system — i.e. the change in the mass of the fibres [15] and the change of the phase shift in the case of non-symmetric harmonic excitation [12] — do not cause the unstable behaviour of the pendulum.

Experimental verification of the fibre dynamics within the manipulator systems and research aimed at measuring the material properties of selected fibres are considered important steps in further research.

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