

## Crack propagation in the vicinity of the interface in layered materials

L. Šestáková<sup>a,b,\*</sup>, L. Náhlík<sup>a,b</sup>, P. Hutař<sup>a</sup>, Z. Knésl<sup>a</sup>

<sup>a</sup>*Institute of Physics of Materials, Academy of Sciences of the Czech Republic, Žitkova 22, 616 62 Brno, Czech Republic*

<sup>b</sup>*Institute of Solid Mechanics, Mechatronics and Biomechanics, Brno University of Technology, Technická 2896/2, 616 69 Brno, Czech Republic*

Received 5 September 2008; received in revised form 6 March 2009

---

### Abstract

The paper deals with the problem of crack propagation in the vicinity of the interface in layered materials. Layered materials are often used in practice, primarily because of their better mechanical properties in comparison with the properties of individual materials components. The configuration of a crack with its tip at the bi-material interface can be created during crack propagation in the structure. It is important to decide if the crack propagates into the second material in this case. The step change of material properties at the bi-material interface means that classical linear elastic fracture mechanics are not appropriate. A generalized approach has to be applied. In this paper, two criteria are applied for the determination of the critical value of an applied load. Knowledge of these values is important for the estimation of the service life time of such structures. The results obtained can be used especially for multi-layer polymer composites designs. On the basis of the procedures presented, suitable materials combinations can be suggested for new composite structures.

© 2009 University of West Bohemia. All rights reserved.

*Keywords:* crack, bi-material interface, stability criteria, layered materials

---

### 1. Introduction

A mathematical description of crack propagation in materials is important for the estimation of the service life time of structures damaged in this way. Cracks are mostly initiated in defective locations, i.e. in locations with a high stress concentration. The production of absolutely homogeneous materials without flaws is difficult, economically inconvenient and sometimes even impossible. Therefore technologists are trying to find other ways of avoiding failures and disasters caused by crack propagation and the consequent fracture of structures. It has been discovered that the lamination of individual materials can improve the fracture mechanics parameters of these components, e.g. [1, 2]. On the other hand, the existence of interfaces between individual material layers can play a negative role and can decrease the utility of laminated material. To estimate the influence of interfaces on the fracture behaviour of laminated structures, the interaction between cracks and the interfaces needs to be studied and the threshold stress values for crack propagating through interfaces estimated.

An analytical expression of the stress distribution around a crack terminating at a bi-material interface can be found e.g. in [6, 7]. Attempts at a generalization of fracture criteria of classical linear elastic fracture mechanics, which would be suitable for general singular stress concentrators, have also previously been published, e.g. [12, 16]. The general form of fracture criteria

---

\*Corresponding author. Tel.: +420 532 290 362, e-mail: sestakova@ipm.cz.

used for the determination of an influence of material interface on crack propagation is published in the paper [9].

The paper presented deals with practical applications of the published criteria and their common comparison of real engineering structures. The objective of this paper is to extend and complete the previous analysis, see [17], and to consider configurations with a crack tip touching the bi-material interface.

A three-layer polyethylene (PE) pipe that is used for gas and water distribution is studied. The system is formed by two protective layers (inner and outer) made of extremely durable PE material (XSC 50) and a middle jointing part of PE 100 material (see fig. 1). The second geometry studied here is a composite plate under tensile load made of the same materials, (see fig. 2). The real material combination (PE 100/XSC 50) of polyethylene composites is considered and two model material combinations are supplemented for determination of the effect of material on crack propagation.

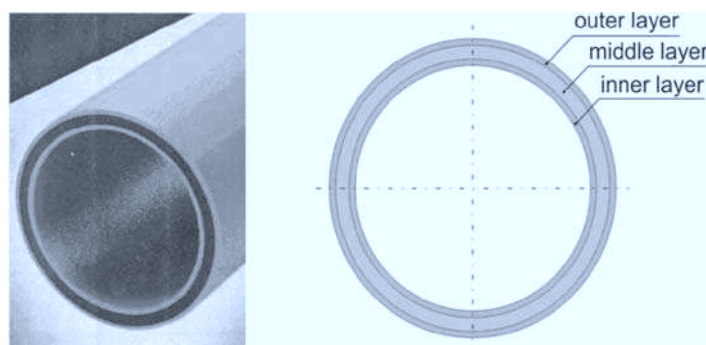


Fig. 1. Three-layer pipe studied

## 2. Solution methodology

### 2.1. Stress distribution around a crack tip

In the frame of linear elastic fracture mechanics (LEFM) the perpendicular crack touching the interface between two dissimilar elastic materials represents a kind of general singular stress concentrator. The singular stress distribution around the tip of a concentrator of this type can be expressed in general form as (e.g. [4, 6, 7]):

$$\sigma_{ij} = \frac{H_I}{(2\pi r)^p} f_{ij}(p, \alpha, \beta), \quad (1)$$

where  $f_{ij}(p, \alpha, \beta)$  is a known function of composite parameters  $\alpha$  and  $\beta$  [3, 6] and  $p$  is the stress singularity exponent ( $0 < p < 1$  and  $p \neq 1/2$ ). Composite parameters  $\alpha$  and  $\beta$  express the elastic mismatch of the materials.  $H_I$  is the generalized stress intensity factor that describes stress amplitude in the vicinity of the crack tip.

The stress singularity exponent  $p = 1 - \lambda$  can be calculated from a characteristic equation [6, 8]:

$$\lambda^2(-4\alpha^2 + 4\alpha\beta) + 2\alpha^2 - 2\alpha\beta + 2\alpha - \beta + 1 + (-2\alpha^2 + 2\alpha\beta - 2\alpha + 2\beta) \cos(\lambda\pi) = 0. \quad (2)$$

The generalized stress intensity factor  $H_I$  is proportional to the applied load and has to be estimated numerically. Using direct method, its value is determined by comparison of analytical expression of stress distribution (1) with values of stress component (the crack opening stress component is usually used) ahead of crack tip obtained from the numerical solution.

## 2.2. Stability criteria

Two stability criteria (one based on average stress ahead of the crack tip and the other based on the strain energy density factor) have been used here for the estimation of the critical value of an applied load for crack propagation through the interface. Two different geometries are considered: a three-layer polyethylene pipe with inner crack and cracked composite plate. Both of the criteria used are based on the phenomenological assumption that the mechanism of the crack propagation from the bi-material interface into the material “m” is the same as in the case of the crack propagation in the homogeneous material “m” only.

The former criterion is based on the average stress ahead of the crack tip [9]. The critical value of generalized stress intensity factor  $H_{IC}$  for a crack perpendicular to and terminating at the bi-material interface can be estimated from the value of the fracture toughness of the material to which the crack will propagate as:

$$H_{IC} = K_{IC} \frac{2d^{p-\frac{1}{2}}}{2-p+g_R}, \quad (3)$$

where  $K_{IC}$  is the fracture toughness of the second material, parameter  $d$  relates to the microstructural characteristic and its choice depends on the crack propagation mechanism and  $g_R$  is a known function of the material properties:

$$g_R(\lambda) = \lambda - \cos \lambda\pi - \frac{\beta[\alpha + 2\lambda - (1 + 2\alpha - 4\alpha\lambda^2) \cos \lambda\pi + (1 + \alpha) \cos 2\lambda\pi]}{1 + 2\alpha + 2\alpha^2 - 2(\alpha + \alpha^2) \cos \lambda\pi - 4\alpha^2\lambda^2}. \quad (4)$$

The latter criterion is based on generalization of the strain energy density factor concept [14, 15]. Sih introduced the strain energy density factor  $S$  for a crack in homogeneous material as:

$$S = wr = a_{11}K_I^2 + 2a_{12}K_I K_{II} + a_{22}K_{II}^2, \quad (5)$$

where  $w$  is strain energy density,  $a_{11}$ ,  $a_{12}$  and  $a_{22}$  are known functions of polar coordinate  $\theta$ ,  $K_I$  and  $K_{II}$  are stress intensity factors for mode I and II of loading and  $r$  is radial distance from the crack tip.

The fracture criterion assumes that the direction of the crack propagation coincides with that of the minimum strain energy density factor  $S$  around the crack tip and the crack extension starts in this direction when  $S$  reaches critical value  $S_c$ .  $S_c$  is a material constant and is usually correlated with fracture toughness value corresponding to the normal mode of loading:

$$S_c = \frac{(1 + \nu)(1 - 2\nu) K_{IC}^2}{2\pi E} \text{ for plane strain condition and} \quad (6)$$

$$S_c = \frac{(1 - \nu) K_{IC}^2}{2\pi E} \text{ for plane stress condition,} \quad (7)$$

where  $E$  is Young’s modulus and  $\nu$  Poisson’s ratio of the material.

The relation (5) can be analogically written for a general singular stress concentrator. If the crack is perpendicular to a bi-material interface and mode I load prevails, the critical value  $\Sigma_c$  can be estimated from relation

$$\Sigma_c = A_{11} H_{IC}^2, \quad (8)$$

where

$$A_{11} = \frac{(1 + \nu)(1 - p)^2}{2\pi E} \cdot (4(1 - 2\nu) + (g_R - p)^2) \quad (9)$$

and  $\nu$ ,  $E$  are material properties of the material behind the interface (where the crack propagates). Note that contrary to a crack in homogeneous material the strain energy density factor for a general singular stress concentrator depends on variable  $r$ , i.e.  $\Sigma = \Sigma(r)$  and the criterion (8) was applied at the distance  $r = r_c$ . Several approaches exist, e.g. [12, 16], for determination of the value of  $r_c$ . In the approach here, the ultimate strength  $\sigma_C$  is used for the estimation of the length of the critical ligament:

$$r_c = \frac{1}{2\pi} \frac{K_{IC}^2}{\sigma_C^2}. \quad (10)$$

The equality between parameter  $d$  (see equation (3)) and  $r_c$  is assumed for further calculations.

The relation for  $H_{IC}$  value can be derived using equation (8) and considering a constant value of critical strain energy density [13]:

$$H_{IC} = \sqrt{\frac{S_c}{A_{11}}} \cdot r_c^{p-\frac{1}{2}}. \quad (11)$$

In equation (11),  $S_c$  value is determined from equation (6) or (7) for the material that the crack will propagate to and  $r_c$  is calculated from (10).

Both mentioned criteria make it possible to estimate the critical value of the generalized stress intensity factor  $H_{IC}$ . Then, from the condition  $H_I(\sigma_{crit}) = H_{IC}$  follows the relation for the critical value of the applied load:

$$\sigma_{crit} = \frac{H_{IC}}{H_I(\sigma_{appl} = 1 \text{ MPa})}, \quad (12)$$

where  $\sigma_{appl}$  is the value of the remote applied load.

### 3. Numerical examples

Two different geometries were considered for numerical calculations. To estimate the influence of material properties on critical stress, a three-layer plate with the crack perpendicular to and terminating at interface between two layers and subjected to remote tensile applied stress (see fig. 2) was modelled first.

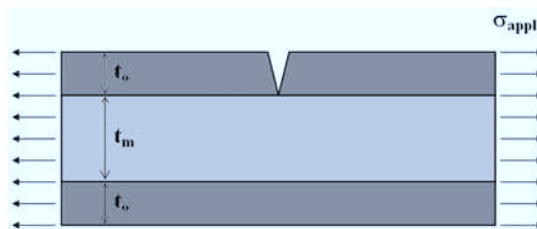


Fig. 2. Scheme of the loaded three-layer plate studied

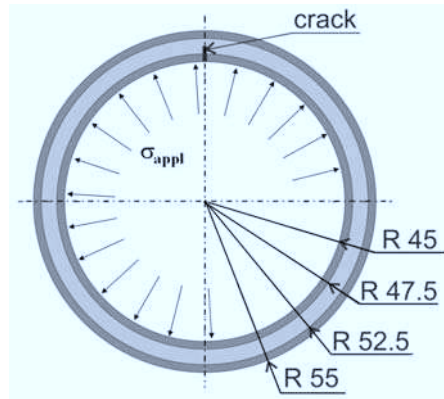


Fig. 3. Basic geometry of the three-layer pipe studied

Table 1. Young's moduli of individual layers

$E_i, E_o$ [MPa]	240	600	828
$E_m$ [MPa]	1 200	1 200	1 213

The second geometry modeled was that of a three-layer polyethylene pipe, see fig. 3. The thicknesses of outer layers  $t_o$  and middle layer  $t_m$  were chosen as the same for both the plate and the pipe, see figs. 2 and 3, and correspond to the real geometry of the three-layer polyethylene pipe produced. With regard to the real function of the pipe (water and gas distribution), a numerical model of the pipe was subjected to inner pressure. Due to the symmetry of the problem, only one half of the model was modelled.

Plane strain conditions were considered for numerical calculations. It was assumed further that the material interfaces were of welded type (ideal adhesion). Both of the materials used are homogenous, isotropic and linear elastic. The mechanical properties of the layer materials are characterised by values of the Young's moduli  $E_i$  (inner),  $E_o$  (outer) and  $E_m$  (middle) in the case of pipe and  $E_o$  (outer) and  $E_m$  (middle) in the case of the layered plate. The values of material parameters used are summarized in tab. 1. In the first and the second column in tab. 1 Young's moduli of model material are written and in the third column values corresponding to the three-layer produced pipe at room temperature are presented. These values were determined from standard tensile tests, see [10] for details.

The entire numerical simulations were performed by finite element method (FEM) in software ANSYS 10.0.

#### 4. Results and discussion

The solution procedure was the same in the case of the three-layer plate and in that of the three-layer pipe. In the case of a cracked three-layer pipe, two configurations were considered, see fig. 4. Note that the case of a crack terminating at the inner interface is more important from the practical point of view than the second one.

Firstly, the stress singularity exponent for each configuration was established as the solution of the characteristic equation (2). Then the generalized stress intensity factor was estimated

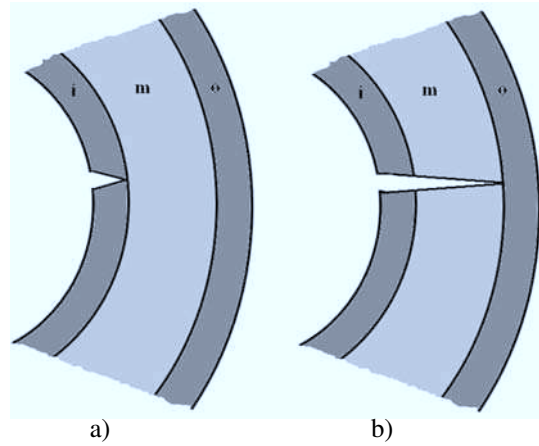


Fig. 4. Two modelled configurations of cracked pipe: a) crack terminating at inner interface, b) crack terminating at outer interface

from the numerical solution by the help of the direct method [5, 11]. The results obtained are summarized in tab. 2 for the three-layer plate and in tables 3 and 4 for the pipe. The case of a crack in homogeneous material is also introduced in these tables for comparison.

Table 2. Estimations of generalized stress intensity factors for the first interface of three-layer plate

$E_o - E_m - E_o$ [MPa]	$E_o/E_m$ [-]	$p$ [-]	$H_I$ [MPa · m <sup>p</sup> ]
240 – 1 200 – 240	0.20	0.380 68	16.27
600 – 1 200 – 600	0.50	0.438 74	15.29
828 – 1 213 – 828	0.68	0.464 54	14.53
homogeneous plate, 1 213	1.00	0.500 00	13.27

Table 3. Estimations of generalized stress intensity factor for the inner interface of three-layer pipe

$E_i - E_m - E_o$ [MPa]	$E_i/E_m$ [-]	$p$ [-]	$H_I$ [MPa · m <sup>p</sup> ]
240 – 1 200 – 240	0.20	0.380 68	75.75
600 – 1 200 – 600	0.50	0.438 74	63.47
828 – 1 213 – 828	0.68	0.464 54	57.71
homogeneous pipe, 1 213	1.00	0.500 00	49.88

Table 4. Estimations of generalized stress intensity factor for the outer interface of three-layer pipe

$E_i - E_m - E_o$ [MPa]	$E_m/E_o$ [-]	$p$ [-]	$H_I$ [MPa · m <sup>p</sup> ]
240 – 1 200 – 240	5.00	0.679 27	110.05
600 – 1 200 – 600	2.00	0.573 29	211.87
828 – 1 213 – 828	1.46	0.539 14	260.51
homogeneous pipe, 1 213	1.00	0.500 00	316.75

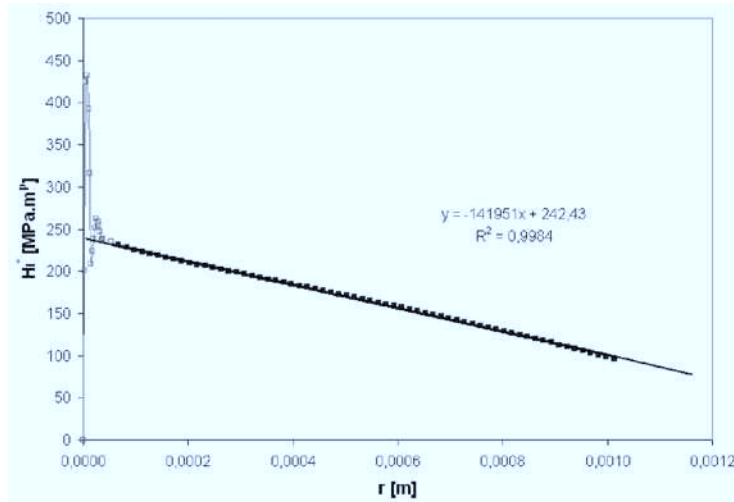


Fig. 5. Extrapolation of  $H_I^*$  values into the crack tip  $r = 0$

The principle of the direct method lies in the extrapolation of the linear part of dependence  $H_I^* = H_I^*(r)$  into the crack tip ( $r = 0$ ), see fig. 5 for clarity's sake. This method requires a very fine mesh and some experience of how to estimate a suitable distance of the linear part of dependence  $H_I - r$  for extrapolation. Values of  $H_I^*$  in the nearest vicinity of the crack tip should not be considered in extrapolation because of substantial numerical errors.

When the values of generalized stress intensity factors for the applied load chosen ( $\sigma_{appl} = 100$  MPa) are known, it is possible to calculate the critical value of the applied load by substitution equations (3) or (11) in equation (12). The obtained value of the critical applied load will cause further crack propagation through the interface into the second material, see tabs. 5 to 7 for resultant critical stresses for crack propagation through the material interface.

In tables 5 to 7,  $\nu_m$  and  $\nu_o$  represent values of Poisson's ratios of middle and outer layers, respectively.  $K_{IC,m}$  is the fracture toughness of the middle layer material and  $K_{IC,o}$  is the fracture toughness of the outer layer material. Note that all values of  $\sigma_{crit}$  were determined for parameter  $d = r_c = 1$  mm, see equation (10). As mentioned in chapter 2, these two quantities are related to the mechanism of crack propagation.

Table 5. Values of critical load determined for the first interface of three-layer plate by:  $\sigma_{crit}^1$  – criterion based on average stress ahead of the crack tip,  $\sigma_{crit}^2$  – criterion based on the generalized strain energy density factor

$E_o - E_m - E_o$ [MPa]	$\nu_m$ [-]	$g_R$ [-]	$K_{IC,m}$ [MPa · m <sup>1/2</sup> ]	$H_I$ [MPa · m <sup>p</sup> ]	$\sigma_{crit}^1$ [MPa]	$\sigma_{crit}^2$ [MPa]
240 – 1 200 – 240	0.35	0.444 31	3.0	16.27	40.76	33.89
600 – 1 200 – 600	0.35	0.456 24	3.0	15.29	29.70	26.68
828 – 1 213 – 828	0.35	0.470 45	3.0	14.53	26.31	24.64
homogeneous plate, 1 213	0.35	0.500 00	3.0	13.27	22.61	22.61

Table 6. Values of critical load determined for the inner interface of three-layer pipe by:  $\sigma_{crit}^1$  – criterion based on average stress ahead of the crack tip,  $\sigma_{crit}^2$  – criterion based on the generalized strain energy density factor

$E_i - E_m - E_o$ [MPa]	$\nu_m$ [-]	$g_R$ [-]	$K_{IC,m}$ [MPa · m <sup>1/2</sup> ]	$H_I$ [MPa · m <sup>p</sup> ]	$\sigma_{crit}^1$ [MPa]	$\sigma_{crit}^2$ [MPa]
240 – 1 200 – 240	0.35	0.470 45	3.0	75.75	8.75	7.28
600 – 1 200 – 600	0.35	0.476 90	3.0	63.47	7.15	6.43
828 – 1 213 – 828	0.35	0.481 66	3.0	57.71	6.62	6.20
homogeneous pipe, 1 213	0.35	0.500 00	3.0	49.88	6.01	6.01

Table 7. Values of critical load determined for the outer interface of three-layer pipe by:  $\sigma_{crit}^1$  – criterion based on average stress ahead of the crack tip,  $\sigma_{crit}^2$  – criterion based on the generalized strain energy density factor

$E_i - E_m - E_o$ [MPa]	$\nu_o$ [-]	$g_R$ [-]	$K_{IC,o}$ [MPa · m <sup>1/2</sup> ]	$H_I$ [MPa · m <sup>p</sup> ]	$\sigma_{crit}^1$ [MPa]	$\sigma_{crit}^2$ [MPa]
240 – 1 200 – 240	0.35	0.809 66	3.5	110.05	0.86	1.43
600 – 1 200 – 600	0.35	0.597 72	3.5	211.87	0.98	1.17
828 – 1 213 – 828	0.35	0.546 26	3.5	260.51	1.02	1.11
homogeneous pipe, 1 213	0.35	0.500 00	3.0	316.75	0.95	0.95

By comparison of the two last columns in tables 5 and 6, i.e. values of critical applied load, it can be concluded that both criteria give similar results. All of the  $\sigma_{crit}$  values introduced in tab. 7 are very small and close to 1 MPa stress, but it is not probable that this configuration (an inner crack terminating at the outer interface) will occur in real cases.

The results of the critical stresses obtained make it possible to assess the influence of more compliant protective layers on the fracture behaviour of the structures studied. It can be concluded that the three-layer structures (plate or pipe) considered in this paper have better fracture properties than the structure made only of homogeneous material with properties of the middle layer. The increase of critical value of an applied load in the case of three-layer plate subjected to tensile applied stress for  $E_o/E_m = 0.2$  is nearly 50 % compared to the homogeneous case. The increase of critical stress in the three-layer pipe is due to different geometry and loading conditions less than in the case of the plate. The results obtained have an important practical effect, i.e. the material with worse mechanical properties (Young’s modulus) can be used for the inner surface of pipes (plates) without decrease of their fracture resistance or can be used for the design of new layered structures with better fracture properties, which will be safer during their service life.

## 5. Conclusion

A three-layer polyethylene pipe used for gas and water distribution is studied in this paper. Specifically, the configuration of a pipe with a perpendicular inner crack terminating at either of both bi-material interfaces is analyzed. A similar problem is studied in the case of a three-layer



plate subjected to a remote tensile applied load. Calculations of the values of generalized stress intensity factors that are essential for the estimation of the value of the critical load  $\sigma_{crit}$  were performed by means of the finite element method.

Two fracture criteria were chosen for the assessment of the stability of a crack of mentioned type, with its tip at the bi-material interface (the first criterion was based on the average stress ahead of the crack tip and the second criterion was based on the generalized strain energy density factor). Based on criteria used, the effect of bi-material interface on the crack propagation can be determined and the value of critical stress  $\sigma_{crit}$  for crack propagation through material interface can be estimated. The values of parameters  $d$  and  $r_c$ , which are involved in the criteria used, were chosen in compliance with the assumed mechanism of failure.

Although each of the criteria is based on different physical principles, the results of  $\sigma_{crit}$  established by the help of both procedures are in quite good agreement and give qualitatively similar critical values for the applied load causing the crack propagation through the interface into the second material layer.

The results obtained are particularly relevant to multi-layer polymer composites design. On the basis of the procedures presented here, a suitable combination of materials can be suggested for composite structures.

### Acknowledgements

The work has been supported by the grants No. 106/09/0279 of the Czech Science Foundation and grant KJB200410803 of the Grant Agency of Academy of Sciences of the Czech Republic.

### References

- [1] R. Bermejo, et al., Residual stresses, strength and toughness of laminates with different layer thickness ratios, *Acta Materialia* 54 (2006) 4745–4757.
- [2] W. J. Clegg, K. Kendall, N. M. Alford, T. W. Buton, J. D. Birchall, A simple way to make tough ceramics, *Nature* 347 (1990) 455–457.
- [3] J. Dundurs, Effect of elastic constants on stress in a composite under plane deformation, *Journal of Composite Materials* 1 (1967) 310–322.
- [4] D. N. Fenner, Stress singularities in composite materials with an arbitrarily oriented crack meeting an interface, *International Journal of Fracture* 12 (1976) 705–721.
- [5] P. D. Hilton, G. C. Sih, Applications of the finite element method to the calculations of stress intensity factors, in: Sih, G. C., *Mechanics of fracture – Methods of analysis and solutions of crack problems*, Noordhoff International Publishing, Leyden, 1973, pp. 429–477.
- [6] K. Y. Lin, J. W. Mar, Finite element analysis of stress intensity factors for cracks at bi-material interface, *International Journal of Fracture* 12 (1976) 521–531.
- [7] S. A. Meguid, M. Tan, Z. H. Zhu, Analysis of cracks perpendicular to biomaterial interfaces using a novel finite element, *International Journal of Fracture* 73 (1995) 1–23.
- [8] L. Náhlík, Propagation of fatigue cracks in vicinity of an interface between two elastic materials, Ph.D. thesis, Brno University of Technology, Brno, 2002 (in Czech).
- [9] L. Náhlík, Z. Knésl, J. Klusák, Crack initiation criteria for singular stress concentrations — part III: An application to a crack touching a bimaterial interface, *Engineering Mechanics* 15 (2) (2008) 99–114.
- [10] E. Nezbedová, Z. Knésl, B. Vlach, R. Lach, W. Grellman, Fracture behaviour of three-layer PE-HD pipe, *Proceedings of the 11<sup>th</sup> Problemseminar: Deformation and fracture behaviour of plastic materials*, Merseburg, 2007.

- [11] D. R. J. Owen, A. J. Fawkes, *Engineering fracture mechanics: Numerical methods and applications*, Pineridge Press Ltd, Swansea, 1983.
- [12] A. Seweryn, A. Lukaszewics, Verification of brittle fracture criteria for elements with V-shaped notches, *Engineering Fracture Mechanics* 69 (2002) 1 487–1 510.
- [13] G. C. Sih, A field model interpretation of crack initiation and growth behavior in ferroelectric ceramics: change of poling direction and boundary conditions, *Theoretical and Applied Fracture Mechanics* 38 (2002) 1–14.
- [14] G. C. Sih, *A special theory of crack propagation*, Mechanics of fracture, Noordhoff International Publishing, Leyden, 1977.
- [15] G. C. Sih, *Mechanics of fracture: Methods of analysis and solutions of crack problems*, Noordhoff International Publishing, Leyden, 1973.
- [16] G. C. Sih, J. W. Ho, Sharp notch fracture strength characterized by critical energy density, *Theoretical and Applied Fracture Mechanics* 16 (1991) 179–214.
- [17] L. Šestáková, Z. Majer, Numerical analysis of non-homogeneous specimens used for measurement of polyethylene fracture mechanics parameters, *Proceedings of seminar Brittle fracture 2007*, Brno, Institute of Physics of Materials AS CR, 2007, pp. 145–156 (in Czech).