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Tool Center Position Determination of Deformable Sliding Star by Redundant Measurement

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Abstract

This paper deals with the procedures for the determination of TCP (tool center position) of machine tool of PKM (parallel kinematical machines) structure in the case that PKM must be considered as compliant mechanism. Two different approaches are described and investigated. The first approach is based on the model of compliant mechanism of PKM. The second approach is based on the redundant measurement that can replace the stiffness knowledge and force balance computation by pure geometric consideration. These procedures are described on PKM Sliding Star.

Keywords: parallel kinematic machine, compliant mechanism, TCP position, calibration, redundant measurement

1. Introduction

Machine tools, robots and measuring machines must be positioned with high accuracy despite that they cannot be in principle manufactured with necessary accuracy. Especially for parallel kinematical machines (structures) (PKM) it holds that despite the very accurate manufacture it is not possible to use the design dimensions for the nonlinear kinematical transformation in the control system. This is solved by the calibration process where based on measurement of some variables during the motion of PKM its real dimensions compared to the nominal (designed) dimensions are determined. In case of PKM it is not possible to determine the real dimensions by direct measurements, therefore these dimensions must be computed from some indirect measurements. This is done using external measuring device (e.g. [1, 2]) or using redundant measurements of relative motions in kinematical joints of PKM (e.g. [3, 4, 5]). These calibration procedures suppose PKM as perfect rigid mechanism. Recently it was reported that the process of calibration is influenced by the stiffness of PKM and PKM was considered as compliant mechanism [6]. Nevertheless the nonlinear kinematical transformation in the control system has used subsequently the equations for the rigid mechanism.

However, the fundamental purpose of calibration is to enable the control system to determine the TCP position accurately during the operation of machine tool. Therefore if the mechanism of PKM must be treated as compliant mechanism, then the determination of TCP position should be also determined using the concept of compliant mechanism. The possible procedures for determination of TCP are described in details for the example of PKM Sliding Star.

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2. TCP determination of deformable PKM by two approaches

Two approaches for the determination of TCP position of deformable PKM can be distinguished. The first one is based on the model of compliant mechanism of PKM. The second one replaces this complex model by redundant measurement.

The first approach uses relatively complex model of compliant mechanism and is based on the belief that simulation can truly predict the reality. The model includes geometrical dimensions, their deformations and stiffnesses and forces both internal and external. It uses usually minimum input measurements. Its advantage is complete physical description of PKM deformation. Its disadvantage is the uncertainty of forces, especially friction forces, and the uncertainty of stiffnesses. It requires accurate model calibration.

The second approach uses just the geometrical model where the flexibility is described by the variability of mechanism dimensions. The model includes only geometrical quantities. This approach is based on extensive redundant measurement. Its advantage is the simplicity of the model, removal of uncertain quantities of stiffnesses and forces. Its disadvantage is large number of sensors.

3. Models of Sliding Star

The procedures for the determination of TCP are described for PKM Sliding Star. Sliding Star is a functional model of machine tool with parallel kinematical structure (Fig. 1). It is redundantly actuated mechanism, i.e. it has more drives than the degrees of freedom. Sliding Star has 3 degrees of freedom and 4 drives (moving screws).

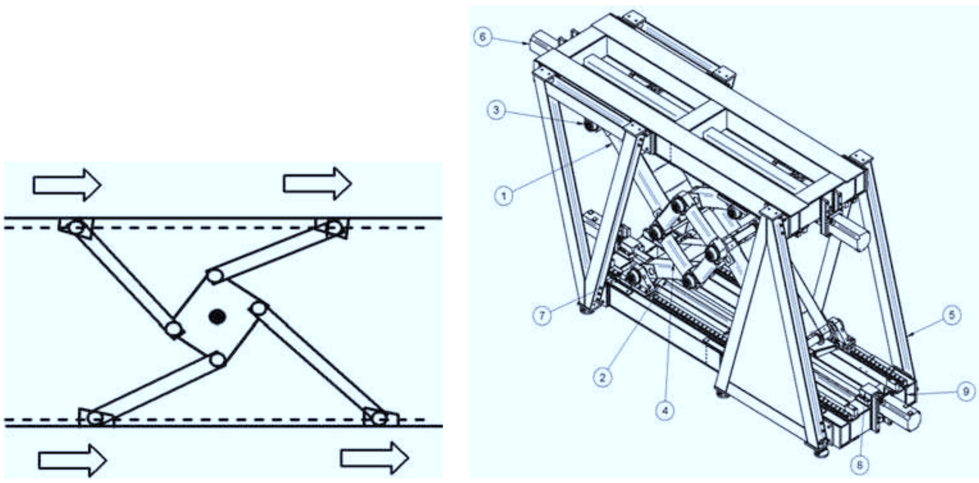

Fig. 1. Sliding Star

The simplified simulation model of Sliding Star as compliant mechanism is in Fig. 2. The flexibility of important construction elements is considered. It is introduced the stiffness of arms of the platform of the lengths $L1$ and $L2$, the stiffness of machine frame, the stiffness k_2 of moving screws and the stiffness k_1 of machine mounting to the ground. The basic dimensions were taken from the machine design $a = 1.201$ m, $b = 0.483$ m, $c = 1.354$ m, $d = 0.966$ m, $e = 0.07$ m, $h = 0.4$ m, $L1 = 0.85$ m, $L2 = 0.6$ m, $k1 = 1e15$ N/m, $k2 = 4e8$ N/m, the sections of machine frame are derived from the module 0.19 m.

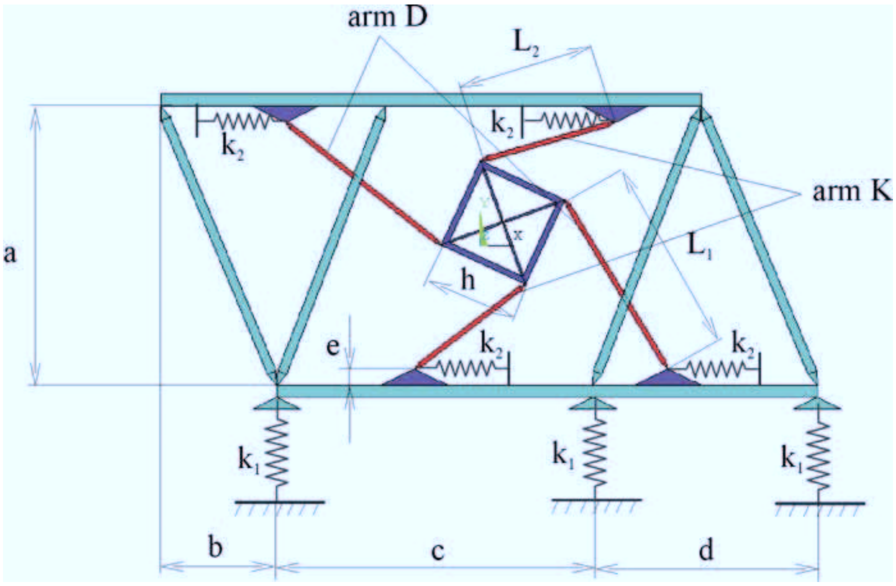

Fig. 2. Flexible model of Sliding Star

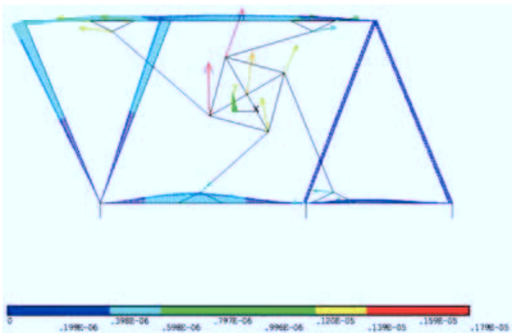

Fig. 3. Character of the deformation

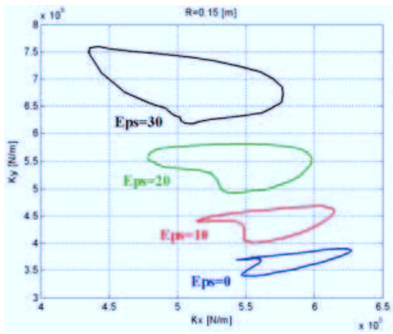

Fig. 4. Course of stiffness on a circle

The character of deformation of the structure of Sliding Star is in Fig. 3. If the ideal node connection of arms and the platform is considered then according to the mass and bending stiffness of the arms their deformation can be supposed only in the axial direction. During the TCP motion on a circle with the radius $R = 150$ mm the resulting stiffnesses were analysed and the influence of platform rotation ε (see Fig. 8) was investigated (Fig. 4).

The detailed analysis of stiffness of arms in axial and bending directions was carried out (Fig. 5–6). The computed axial stiffness for arm D was $kx = 1.389e9$ N/m the stiffness in the bending direction was $ky = 1.422e7$ N/m. The axial stiffness for arm K was $kx = 1.572e9$ N/m the stiffness in the bending direction was $ky = 2.537e7$ N/m.

Similarly the machine frame deformation due to the loading in x and y directions was investigated (Fig. 7). The results demonstrate very large values of stiffness in general and specifically hundred times lower bending than axial stiffness. These values demonstrate the potential of PKM as truss structure if the joints are designed as large planar ones.

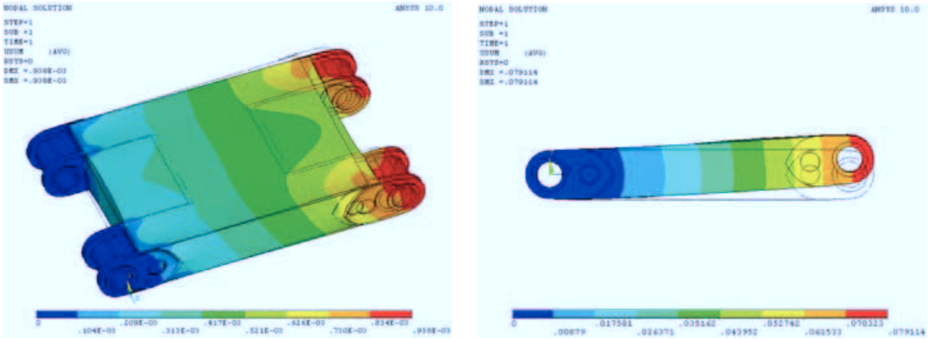

Fig. 5. Arm D, deformation [m] in axial and bending direction

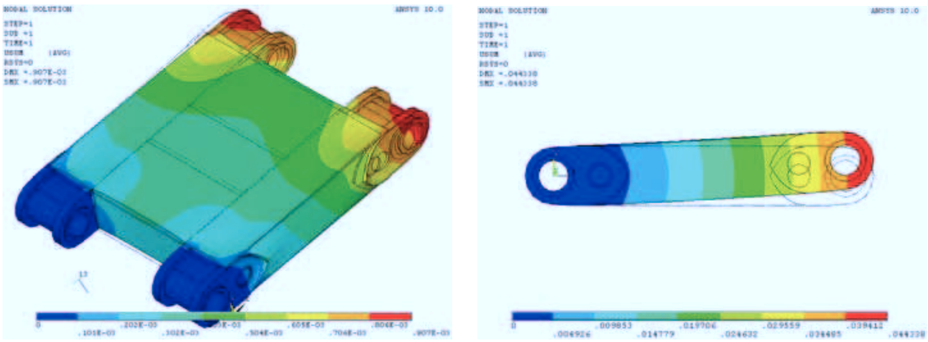

Fig. 6. Arm K, deformation [m] in axial and bending direction

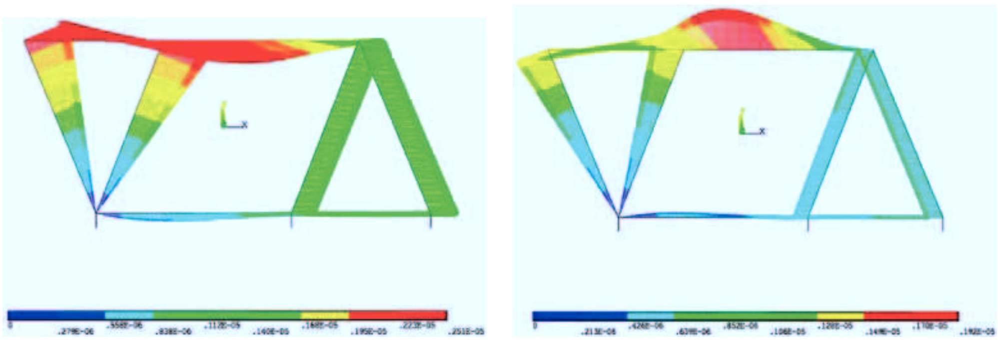

Fig. 7. Machine frame deformation for loading in the x and y directions of global axes

4. Method of compliant model

The computational model of compliant mechanism of Sliding Star consists of force (or deformation) model in Fig. 8 and of geometrical model in Fig. 9. The usually considered measurement scheme is in Fig. 10a, i.e. the measurement of x positions of sliders. It consists just from the sensors of drives of moving screws. As the variables it is necessary to introduce the lengths of arms $L1, L2, L3, L4$, the cartesian coordinates of points A, B, C, D x_A, x_B, x_C, x_D, y_A, y_B, y_C, y_D, the arm angles φ_1, φ_2, φ_3, φ_4 and the platform turning ε, the stiffnesses of all elements, the acting forces of drives, cutting and the reaction forces.

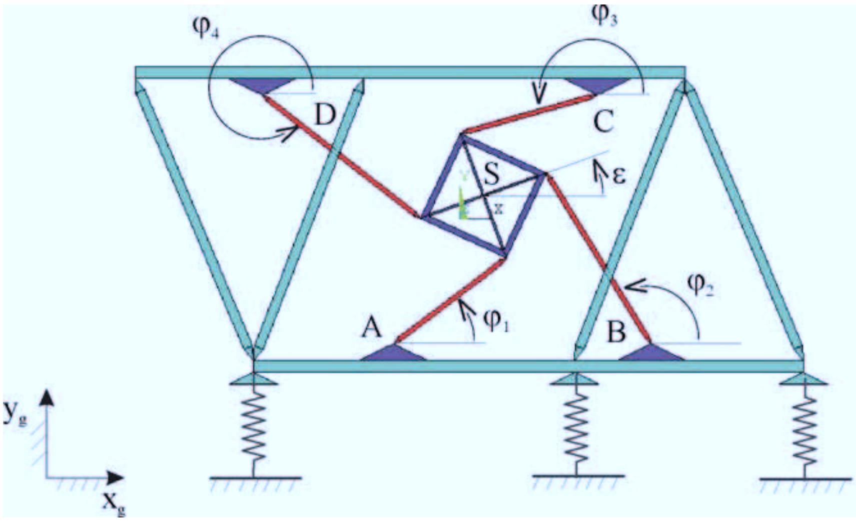

Fig. 8. Force model

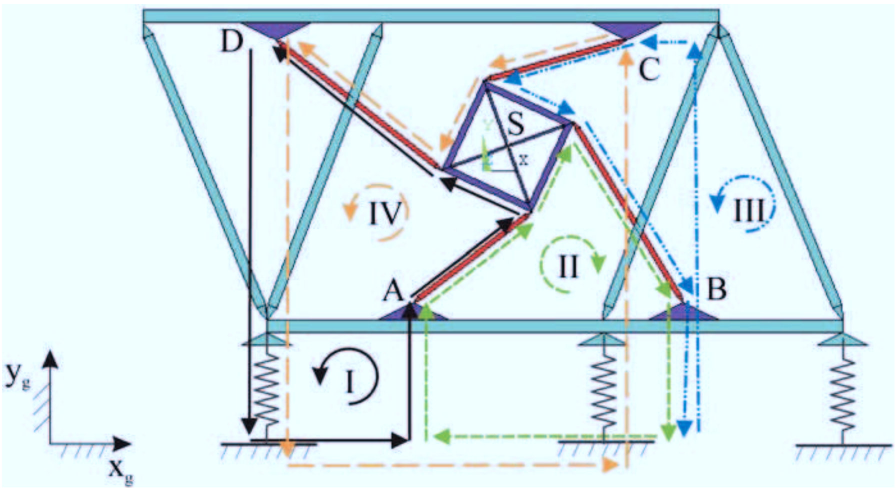

Fig. 9. Geometrical model

For simplicity the platform described by the dimension h is supposed to be completely rigid. From the geometrical model the equations of closure of kinematical loops are assembled, for example for the loop AEFB

$$\begin{aligned} x_A + (L_1 + \Delta L_1)\cos(\varphi_1) + h\cos\left(\varepsilon + \frac{\pi}{4}\right) &= x_B + (L_2 + \Delta L_2)\cos(\varphi_2),\\ y_A + (L_1 + \Delta L_1)\sin(\varphi_1) + h\sin\left(\varepsilon + \frac{\pi}{4}\right) &= y_B + (L_2 + \Delta L_2)\sin(\varphi_2). \end{aligned} \tag{1}$$

It is possible to assemble 6 such equations. From the force model the equations for the solution of truss are assembled, for example for the node A

$$\begin{aligned} x &: F_1 + k_1 \Delta L_1 \cos(\varphi_1) = 0, \\ y &: k_{yB} y_B + k_1 \Delta L_1 \sin(\varphi_1) = 0. \end{aligned} \tag{2}$$

It is possible to assemble 16 such equations. These 22 equations are solved for the 22 unknowns ΔL_1, ΔL_2, ΔL_3, ΔL_4, ε, φ_1, φ_2, φ_3, φ_4 reactions R_{yA}, R_{yB}, R_{yC}, R_{yD}, in the sliders, the axial forces S_1, S_2, S_3, S_4, S_5, in the rigid truss of the platform, the loading forces from the cutting acting on the platform F_x, F_y, M_z, and possible also the redundant position x_D.

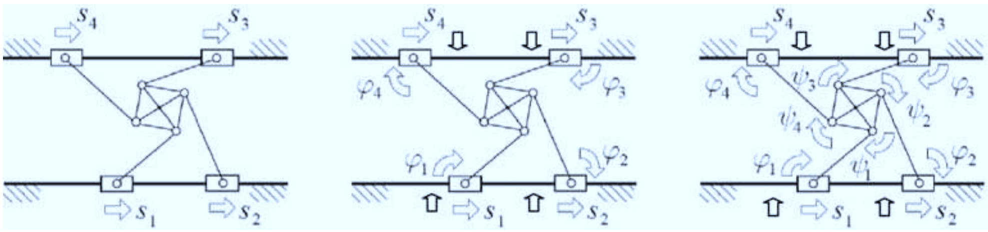

Fig. 10. Different measurement schemes for Sliding Star

4.1. Redundant geometry-stiffness calibration

If the redundant measurements are available (e.g. the schemes Fig. 10b, c) then the equations (1)–(2) can be used for the simultaneous calibration of geometry and stiffness of the compliant mechanism of PKM.

The assembled equations consist of system of redundant equations. The assembled equations are finally collected for n calibration positions into summary system similarly to the pure geometrical calibration

$$\mathbf{F}(\mathbf{d}, \mathbf{S}, \mathbf{F}_m) = \mathbf{0}, \tag{3}$$

where for the calibration position $i = 1, \ldots, n$ the constraint $\mathbf{f}_i = \mathbf{f}(\mathbf{d}, \mathbf{s}_i, \mathbf{f}_{mi}) = \mathbf{0}$ holds and $\mathbf{F} = [\mathbf{f}_1, \mathbf{f}_2, \ldots, \mathbf{f}_n]^T$, $\mathbf{S} = [\mathbf{s}_1, \mathbf{s}_2, \ldots, \mathbf{s}_n]^T$, $\mathbf{F}_m = [\mathbf{f}_{m1}, \mathbf{f}_{m2}, \ldots, \mathbf{f}_{mn}]^T$. The vector $\mathbf{s}_i$ represents the geometrical measurements and $\mathbf{f}_{mi}$ the force measurements, $\mathbf{d}$ is vector of calibration parameters. The basic solution of the mixed geometry-stiffness calibration problem can be realized based on the Newton methods for overconstrained system [1]. The quality of the calibration can be measured by the calibrability measure $C = \mathrm{cond}\,(\mathbf{J}_d^T \mathbf{J}_d)$ [5, 2], where $\mathbf{J}_d$ is the Jacobi matrix of the system (3) with respect to the calibration parameters $\mathbf{d}$. The calibrability of the system is the important criterion for the accurate calibration similarly as for the pure geometrical calibration. Besides the influences applicable for the geometrical calibration (optimisation of mechanism dimensions, sensor placement optimisation, calibration positions optimisation) also the manner of the simplified flexibility parameterization can affects the final calibrability.

5. Method of redundant measurement

The computational model of TCP determination by redundant measurement consists just of the geometrical model in Fig. 9. The measurement scheme must be more redundant and it is in Fig. 10b. It is measured the displacement of all drives (sliders) x_A, x_B, x_C, x_D, the rotation of arms φ_1, φ_2, φ_3, φ_4 and the vertical deformation of the frame at the position of sliders y_A, y_B,

y_C, y_D. In this case only the equations from the geometrical model are used, however modifies, for example the equations (1) are

$$x_A + LL_1\cos(\varphi_1) + h\cos\left(\varepsilon + \frac{\pi}{4}\right) = x_B + LL_2\cos(\varphi_2),$$
$$y_A + LL_1\sin(\varphi_1) + h\sin\left(\varepsilon + \frac{\pi}{4}\right) = y_B + LL_2\sin(\varphi_2), \tag{4}$$

where LL_1 describes the unknown deformed length of the arm $L1$. It is possible to assemble 6 such equations and these 6 equations are solved for 5 unknowns LL_1, LL_2, LL_3, LL_4, ε where two of the measurements x_D, y_D, φ_4 are redundant. The redundancy is however only helpful for the solution of the equations. The results of application of this approach for the TCP determination of Sliding Star for the loading force $F = 1\,000$ [N] are in Fig. 11 that describes the deformation of TCP during the motion on the circular path of $R = 150$ mm for the Young modulus $E = 2.1e5$ MPa of the frame. In Fig. 12 are presented sensitivity TCP on the stiffness parameters of the frame.

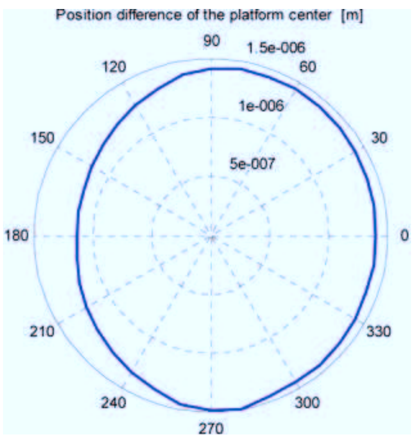

Fig. 11. The compliant deformation of TCP during circular path

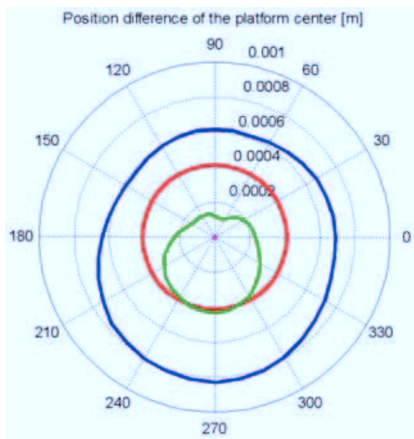

Fig. 12. The sensitivity TCP on the stiffness parameters of the frame

6. Conclusion

The determination of tool center position for PKM being considered as compliant mechanism is an important task for control of PKM with improved path tracking. Several approaches are possible. The method of redundant measurement seems more robust and promising for industrial usage.

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References

[1] G. Stengele, Cross Hueller Specht Xperimental, a machining center with new hybrid kinematics, In: R. Neugebauer (ed.), Development methods and application experience of parallel kinematics, IWU FhG, Chemnitz 2002, pp. 609–627.

[2] A. Nahvi, J. M. Hollerbach, The noise amplification index for optimal pose selection in robot calibration. In: Proc. of IEEE Int. Conf. on Robotics and Automation, 1996, pp. 647–654.

[3] H. Zhuang, Self-calibration of parallel mechanisms with a case study on Stewart platforms, IEEE Trans. On Robotics and Automation, 13 (1997), 3, pp. 387–397.

[4] M. Valášek, et al., Redundant Measurement and Calibration of Parallel Kinematical Structures. In Proceedings of the 15th International Symposium on Measurement and Control in Robotics (ISMCR-2005) [CD-ROM], Brussels, 2005.

[5] M. Valášek, Z. Šika, V. Hamrle, From Dexterity to Calibratability of Parallel Kinematical Strutures, In: 12th IFToMM World Congress [CD-ROM], Besancon, 2007.

[6] G. Ecorchard, R. Neugebauer, P. Maurine, Self-Calibration of a Redundantly Actuated Paralel Kinematic Machine Tool, In: The 5th Chemnitz Parallel Kinematics Seminar, Chemnitz, 2006, pp. 477–496.

[7] T. Vampola, M. Valášek, Z. Šika, S. Gráf, Reconstruction of the Elastic Model of 3D Structures from Redundant Measurements. In: Engineering Mechanics 2007. Praha, Ústav termomechaniky AV ČR, 2007, vol. 1, p. 307–308.