

# Parallel algorithm of trigonometric collocation method in nonlinear dynamics of rotors

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## Abstract

A parallel algorithm of a numeric procedure based on a method of trigonometric collocation is presented for investigating an unbalance response of a rotor supported by journal bearings. After a condensation process the trigonometric collocation method results in a set of nonlinear algebraic equations which is solved by the Newton-Raphson method. The order of the set is proportional to the number of nonlinear bearing coordinates and terms of the finite Fourier series. The algorithm, realized in the MATLAB parallel computing environment (DCT/DCE), uses message passing technique for interacting among processes on nodes of a parallel computer. This technique enables portability of the source code both on parallel computers with distributed and shared memory. Tests, made on a Beowulf cluster and a symmetric multiprocessor, have revealed very good speed-up and scalability of this algorithm.

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*Keywords:* rotor system, periodic response, trigonometric collocation, journal bearing, parallel computation

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## 1. Introduction

A periodic steady-state response of rotors supported by journal bearings can be determined by the approximate trigonometric collocation method. This method expects the periodic response in a form of the finite Fourier series and results in a set of nonlinear algebraic equations, see [3] or [5] for details. Significant advantage of the trigonometric collocation method is its wide possibility of parallelism.

This contribution reports a parallel algorithm which is realized in the MATLAB parallel computing environment – Distributed Computing Toolbox/Engine (DCT/DCE). Proposed algorithm uses message passing technique for interacting among processes on computational nodes of a parallel computer. This technique enables portability of the source code both on parallel computers with distributed and shared memory. Tests have been made on both types of parallel computers, namely on the 8-node Beowulf cluster and on the symmetric multiprocessor with two dual cores CPUs. They have revealed very good speed-up and scalability of this algorithm.

## 2. Steady-state response of rotor on periodic excitation

A model rotor, discretized into finite elements, is assumed to have the following properties: (i) the shaft is linearly flexible and axis symmetric, (ii) discs are axis symmetric rigid bodies, (iii) inertia and gyroscopic effects of rotating parts are taken into account,

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(iv) the stationary part is rigid and fixed, (v) material damping of the shaft is viscous, external damping is proportional to the mass matrix, (vi) the rotor rotates at constant angular velocity, (vii) journal bearings are considered by nonlinear force couplings. Matrices of used finite elements can be found in [6] and [7].

Lateral vibrations of the rotor are described by an equation of motion (after applying boundary conditions)

$$\mathbf{M} \ddot{\mathbf{x}} + (\mathbf{B} + \nu \mathbf{K}_{SH} + \Omega \mathbf{G}) \dot{\mathbf{x}} + (\mathbf{K} + \Omega \mathbf{K}_C) \mathbf{x} = \mathbf{f}_{ST} + \mathbf{f}_E(t) + \mathbf{f}_B(\dot{\mathbf{x}}, \mathbf{x}), \quad (1)$$

where  $\mathbf{M}$ ,  $\mathbf{B}$ ,  $\mathbf{K}_{SH}$ ,  $\mathbf{G}$ ,  $\mathbf{K}$ ,  $\mathbf{K}_C$  are mass, external damping, stiffness of the shaft, gyroscopic, stiffness and circulation matrices,  $\mathbf{f}_{ST}$  is the vector of generalized static forces,  $\mathbf{f}_E(t)$  is the vector of generalized periodic external forces (excitation),  $\mathbf{f}_B(\dot{\mathbf{x}}, \mathbf{x})$  is the nonlinear vector of hydraulic forces,  $\ddot{\mathbf{x}}$ ,  $\dot{\mathbf{x}}$ ,  $\mathbf{x}$  are vectors of generalized accelerations, velocities and displacements of the rotor,  $\nu$  is the coefficient of viscous damping,  $\Omega$  is angular velocity and  $t$  is time.

For simplicity let's denote

$$\mathbf{A}_0 = \mathbf{K} + \Omega \mathbf{K}_C, \quad \mathbf{A}_1 = \mathbf{B} + \nu \mathbf{K}_{SH} + \Omega \mathbf{G}. \quad (2)$$

For other operations it is useful to reorder degrees of freedom of (1), so that the first  $p$  degrees are associated with linear components and the rest  $q$  degrees are associated with nonlinear bearing components. Then (1) with respect to (2) is transformed into

$$\begin{bmatrix} \mathbf{M}_{pp} & \mathbf{M}_{pq} \\ \mathbf{M}_{qp} & \mathbf{M}_{qq} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{x}}_p \\ \ddot{\mathbf{x}}_q \end{bmatrix} + \begin{bmatrix} \mathbf{A}_{1pp} & \mathbf{A}_{1pq} \\ \mathbf{A}_{1qp} & \mathbf{A}_{1qq} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{x}}_p \\ \dot{\mathbf{x}}_q \end{bmatrix} + \begin{bmatrix} \mathbf{A}_{0pp} & \mathbf{A}_{0pq} \\ \mathbf{A}_{0qp} & \mathbf{A}_{0qq} \end{bmatrix} \begin{bmatrix} \mathbf{x}_p \\ \mathbf{x}_q \end{bmatrix} = \begin{bmatrix} \mathbf{f}_{STp} \\ \mathbf{f}_{STq} \end{bmatrix} + \begin{bmatrix} \mathbf{f}_{Ep} \\ \mathbf{f}_{Eq} \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \mathbf{f}_{Bq}(\dot{\mathbf{x}}_q, \mathbf{x}_q) \end{bmatrix}. \quad (3)$$

The trigonometric collocation method assumes the periodic response in a form of the finite Fourier series and is expressed by the equation (after reordering degrees of freedom)

$$\begin{bmatrix} \mathbf{x}_p \\ \mathbf{x}_q \end{bmatrix} = \begin{bmatrix} \mathbf{x}_{0p} \\ \mathbf{x}_{0q} \end{bmatrix} + \sum_{j=1}^{L_F} \begin{bmatrix} \mathbf{x}_{jpc} \\ \mathbf{x}_{jqc} \end{bmatrix} \cos\left(j \frac{2\pi}{T} t\right) + \begin{bmatrix} \mathbf{x}_{jps} \\ \mathbf{x}_{jqs} \end{bmatrix} \sin\left(j \frac{2\pi}{T} t\right), \quad (4)$$

where  $L_F$  is the number of considered harmonic terms of the series and  $\mathbf{x}_{0p}$ ,  $\mathbf{x}_{0q}$ ,  $\mathbf{x}_{jpc}$ ,  $\mathbf{x}_{jqc}$ ,  $\mathbf{x}_{jps}$ ,  $\mathbf{x}_{jqs}$  are vectors of Fourier coefficients. The period of the response  $T$  is assumed to be a real multiple of the excitation period.

The trigonometric collocation method requires to specify  $N \geq 2L_F + 1$  collocation points

$$t_k = \frac{T}{N}(k-1), \quad k = 1, 2, \dots, N. \quad (5)$$

A substitution of the assumed solution (4) and its first and second derivatives with respect to time into the reordered equation of motion (3) for all collocation points (5) arrives at the set of nonlinear algebraic equations expressed in the matrix form

$$\begin{bmatrix} \mathbf{S}_{pp} & \mathbf{S}_{pq} \\ \mathbf{S}_{qp} & \mathbf{S}_{qq} \end{bmatrix} \begin{bmatrix} \mathbf{s}_p \\ \mathbf{s}_q \end{bmatrix} = \begin{bmatrix} \mathbf{f}_p \\ \mathbf{f}_q(\mathbf{s}_q) + \mathbf{f}_{qL} \end{bmatrix}. \quad (6)$$

Because only “ $q$  components” in (6) are nonlinear it is suitable to reduce the complexity of the set by eliminating of “ $p$  components” using

$$\mathbf{s}_p = \mathbf{S}_{pp}^{-1}(\mathbf{f}_p - \mathbf{S}_{pq} \mathbf{s}_q). \quad (7)$$

Then (6) is reduced into the set of nonlinear algebraic equations of lower order, which is proportional to the number of bearing coordinates and coefficients of the finite Fourier series

$$\mathbf{R} \mathbf{s}_q + \mathbf{r} - \mathbf{f}_q(\mathbf{s}_q) = \mathbf{0}, \tag{8}$$

where

$$\mathbf{R} = \mathbf{S}_{qq} - \mathbf{S}_{qp} \mathbf{S}_{pp}^{-1} \mathbf{S}_{pq}, \quad \mathbf{r} = \mathbf{S}_{qp} \mathbf{S}_{pp}^{-1} \mathbf{f}_p - \mathbf{f}_{qL}. \tag{9}$$

The nonlinear set (8) is expected to be solved by the Newton-Raphson method. With respect to (9) the Newton-Raphson method is expressed by equations

$$\Delta \mathbf{s}_{q_{i+1}} = \mathbf{s}_{q_i} + \lambda_i \Delta \mathbf{s}_{q_i}, \quad \Delta \mathbf{s}_{q_i} = -\mathbf{J}_i^{-1}(\mathbf{s}_{q_i}) [\mathbf{R} \mathbf{s}_{q_i} + \mathbf{r} - \mathbf{f}_{q_i}(\mathbf{s}_{q_i})], \tag{10}$$

$$\mathbf{J}_i(\mathbf{s}_{q_i}) = \mathbf{R} - \left[ \frac{\partial \mathbf{f}_q(\mathbf{s}_q)}{\partial \mathbf{s}_q} \right]_{\mathbf{s}_q = \mathbf{s}_{q_i}}, \tag{11}$$

where  $\lambda$  is the deceleration parameter and  $i$  is the iteration number.

If the condition (12) is satisfied, the iteration process terminates.  $\varepsilon = 1 \cdot 10^{-5}$  was used.

$$\|\mathbf{R} \mathbf{s}_{q_i} + \mathbf{r} - \mathbf{f}_{q_i}(\mathbf{s}_{q_i})\| \leq \varepsilon \tag{12}$$

### 3. Reynolds equation and calculation of bearing forces

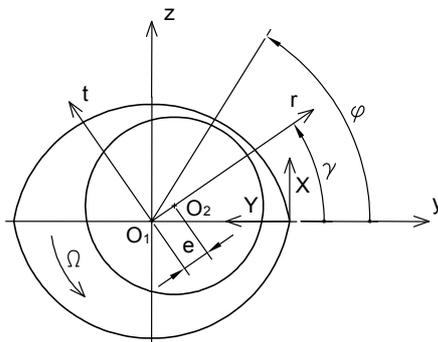


Fig. 1. Journal bearing.

According to the classic lubrication theory, the pressure distribution into the journal bearing is described by the Reynolds equation. Assumptions of the Reynolds equation can be found e.g. in [4]: (i) the lubricant is massless, incompressible and adheres to bearing surfaces, (ii) the lubricant is Newtonian and its viscosity is constant, (iii) the flow is laminar, (iv) the pressure is constant in the radial direction, (v) the flow velocity in the radial direction is neglected, (vi) velocity gradients in the radial direction are large in relation to those in the tangential and axial directions, (vii) the oil-film thickness is small compared to the journal radius,

(viii) the curvature of the oil-film is negligible and (ix) bearing surfaces are stiff and smooth. Then the Reynolds equation can be written in the form

$$\frac{1}{R^2} \frac{\partial}{\partial \varphi} \left( h^3 \frac{\partial p}{\partial \varphi} \right) + \frac{\partial}{\partial Z} \left( h^3 \frac{\partial p}{\partial Z} \right) = 6\eta\Omega \frac{\partial h}{\partial \varphi} + 12\eta \frac{\partial h}{\partial t}, \tag{13}$$

where  $\varphi, Z$  are circumferential and axial coordinates (fig. 1),  $R$  is the radius of journal,  $\eta$  is dynamical viscosity of the lubricant. According to fig. 1, the oil-film thickness is in form (14), where  $h_0$  is the oil-film thickness at the centric position,  $e$  is the eccentricity of the journal centre and  $\gamma$  is the position angle of the line of centres.

$$h(\varphi, t) = h_0(\varphi) - e(t) \cdot \cos[\varphi - \gamma(t)] \tag{14}$$

Boundary conditions and the axial symmetry condition of (13) can be given by

$$p(0, Z) = p(2\pi, Z), \quad p(\varphi, Z = \pm L/2) = p_a, \tag{15}$$

$$\left[ \frac{\partial p(\varphi, Z)}{\partial Z} \right]_{Z=0}^{\varphi} = 0, \tag{16}$$

where  $L$  is the bearing length and  $p_a$  is atmospheric pressure.

The Reynolds equation (13) is valid only for pressures which are higher than the cavitation pressure. Realized calculations considered this phenomenon approximately by the Gumbel's boundary condition, i.e. pressures calculated from the Reynolds equation (here numerically by using finite difference method), which are lower than cavitation pressure, are replaced by the magnitude in the cavitation. More details about this phenomenon can be found in [1].

Components of the nonlinear coupling vector  $\mathbf{f}_B$  in directions of  $y$  and  $z$  axes (fig. 1) are obtained numerically by integration of the pressure function over the journal surface

$$F_{B_y} = -R \int_0^{2\pi} \int_{-L/2}^{L/2} p \cos \varphi dZ d\varphi, \quad F_{B_z} = -R \int_0^{2\pi} \int_{-L/2}^{L/2} p \sin \varphi dZ d\varphi. \tag{17}$$

#### 4. Description of parallel algorithm

Before the parallel algorithm was written some tests were made on a serial algorithm of the trigonometric collocation method. They have revealed that almost all execution time is needed for the numerical evaluation of the Jacobi matrix (11).

With respect to (13) and (14) the pressure distribution into the journal bearing and thus components (17) depends only on the displacement and the velocity of the node at the place of bearing. Therefore elements of (11) and  $\mathbf{f}_q(\mathbf{s}_q)$  can be calculated in a parallel manner. The evaluation of hydraulic forces components (17) are set as an elemental parallel task.

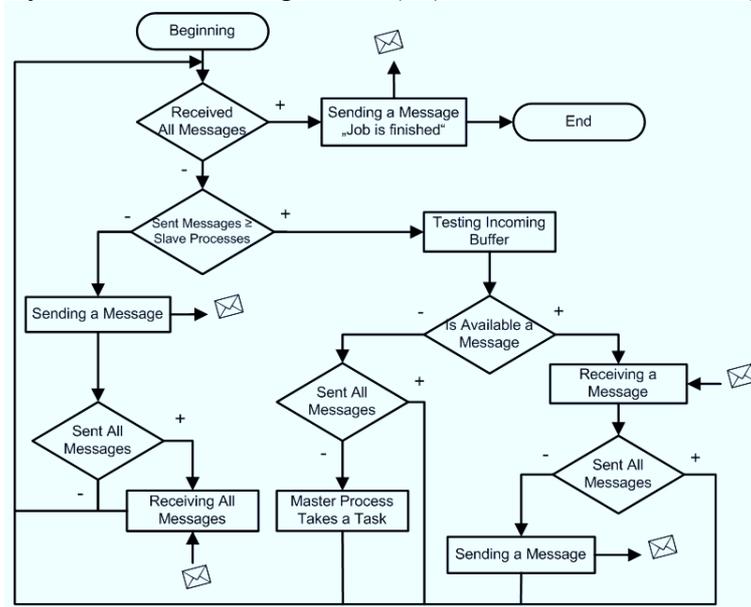


Fig. 2. Block diagram of the master process execution.

As it was reported earlier, the parallel algorithm uses the message passing technique. The algorithm establishes a master process on one computation node of a parallel computer. The master process manages the Newton-Raphson algorithm and also sends and receives messages from slave processes. These are established on the rest of computational nodes. Slave processes only receive a message from the master process, execute a task and send

results back to the master process. If it is necessary to calculate e.g. (11), the master process proceeds according to the block diagram in fig. 2.

Regarding nature of the Newton-Raphson method (iterations execute sequentially), it is assured, during calling functions for the parallel evaluation of the Jacobi matrix (11) and the nonlinear vector  $\mathbf{f}_q(\mathbf{s}_q)$ , that slave processes are idling. That is why the master process in fig. 2 sends messages to all slave processes at first. This avoids the master process to redundantly test the incoming buffer. In the other cases the master process periodically tests the incoming buffer. If there is a message from any slave process, the master process receives it and immediately sends to the slave process another one (of course if it is necessary to execute any messages yet). If there is no message, the master process starts execute one parallel task and then again tests the incoming buffer. This feature enables increasing of the algorithm efficiency. Also programming complexity is lower because there is no need to handle how many messages are to be executed by the certain slave process. Therefore load balancing of CPUs is fully dynamical, i.e. before calculation it is not known computational load of particular CPUs. On the other hand certain fluctuation of execution times must be expected due to dynamical load balancing.

### 5. Scalability analysis, obtained results

The algorithm, realized in the MATLAB parallel environment (DCT/DCE), was tested on two different parallel computers. The first was 8-node Beowulf cluster THEA. Each node dispose of CPU AMD Athlon 1.4 GHz and 1.5 GB RAM. The second one was the symmetric multiprocessor equipped with two dual-cores CPUs AMD Opteron 2.4 GHz, 4 GB of shared RAM. Installed operating system was LINUX in Debian and Ubuntu distribution. There are schemes of both parallel computers in fig. 3.

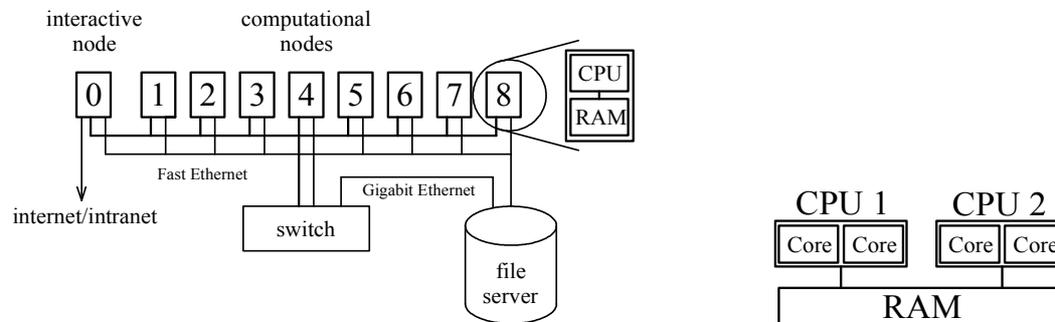


Fig. 3. Schemes of used parallel computers.

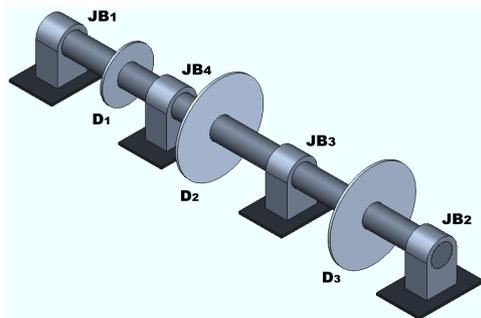


Fig. 4. Model of the rotor.

The investigated rotor in fig. 4 consists of the shaft discretized into 26 identical shaft elements and three rigid discuses ( $D_1, D_2, D_3$ ). During tests the rotor was successively supported by two ( $JB_1, JB_2$ ), three ( $JB_1, JB_2, JB_3$ ) and four ( $JB_1, JB_2, JB_3, JB_4$ ) identical two-lobe journal bearings of the finite length. This was done in order to test a reaction of the algorithm to increasing size of the problem.

The rotor was excited by an unbalance of discuses and the steady-state response was approximated by an absolute and the first four harmonic terms of the Fourier series. It was assumed that the period of the response is equal to the period of the excitation.

MATLAB’s functions “tic” and “toc” were utilized for the measurement of execution times. Each measurement was repeated for 10 times for all combinations of CPUs and cores. Obtained execution times showed out quite high fluctuation – up to 20% (probably due to dynamic load balancing). Only the best results were taken into account of graphics processing. For reaching the convergence criterion (12) between 7 and 8 Newton-Raphson’s iterations (depends on the rotor configuration) were needed if the initial guess was zero vector.

The parallel algorithm was evaluated in terms of achieved speed-up and efficiency (fig. 5 and fig. 6). Another important indicator is a scalability of the algorithm, i.e. its ability to increase speed-up proportionally to the number of used CPUs. According to [2] the speed-up is defined by the rate of execution times of the serial and parallel algorithm and its efficiency is obtained by the rate of the speed-up and number of CPUs. It is evident that the parallel algorithm has reached very good speed-up and efficiency which is always higher then 80%. Slightly better absolute execution times are obtained by using the symmetric multiprocessor. Absolute times are mentioned in tab. 1 in case of using all CPUs of the parallel computer. Finally execution times per one Newton-Raphson’s iteration are in fig. 7.

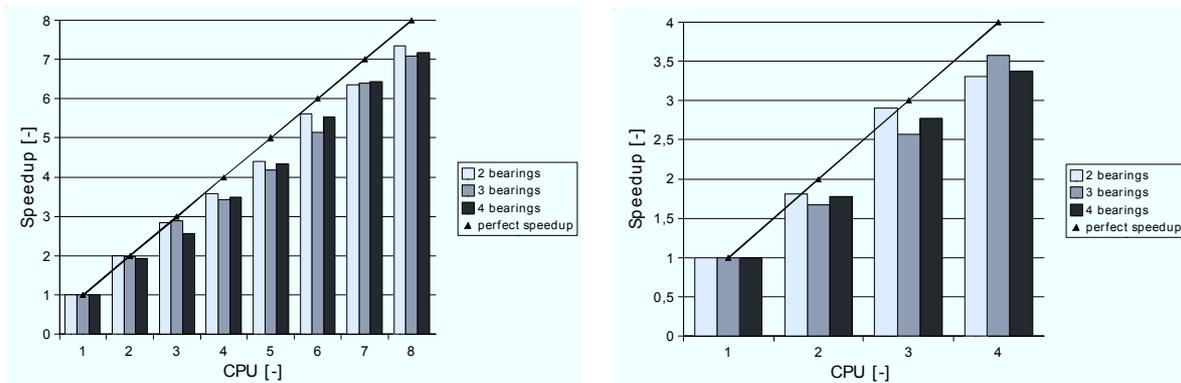


Fig. 5. Speed-up of the parallel algorithm.

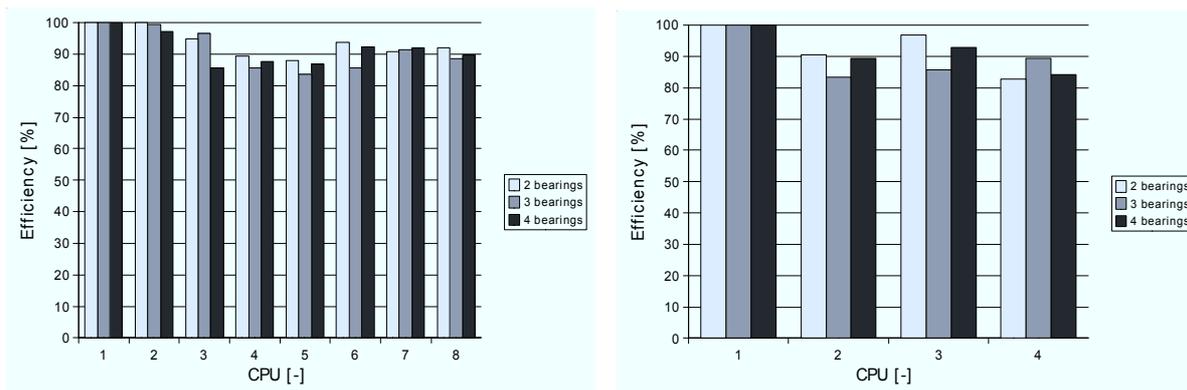


Fig. 6. Efficiency of the parallel algorithm.

Number of journal bearings	Execution times [min]	
	Beowulf cluster	Symmetric multiprocessor
2 bearings	8.8	7.7
3 bearings	19.3	15.4
4 bearings	29.2	25.6

Tab. 1. Execution times (all CPUs used).

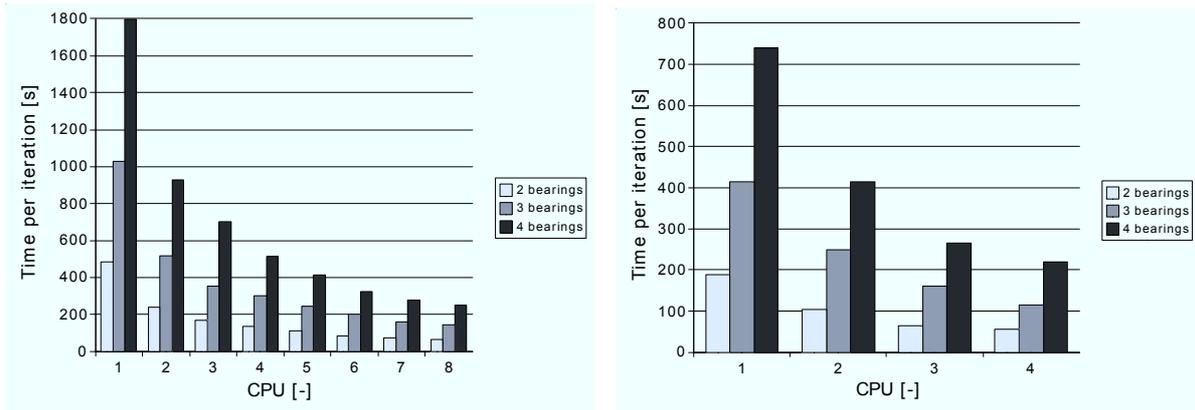


Fig. 7. Execution times per one Newton-Raphson's iteration.

By reason of the lucidity only results for the rotor supported by two journal bearings are presented bellow. Orbits of the shaft centre at places of bearings are illustrated in fig. 8. Shapes of these orbits resemble circle in case of JB<sub>1</sub> and ellipse (JB<sub>2</sub>).

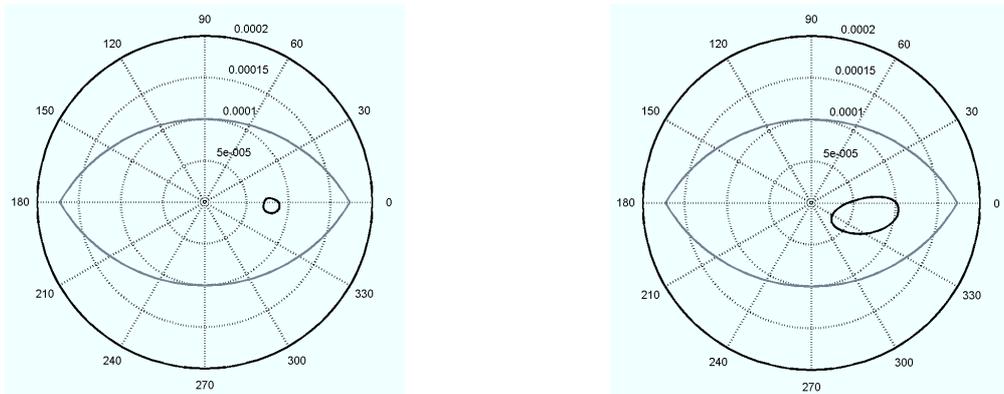


Fig. 8. Orbits of the shaft centre at places of journal bearings (left JB<sub>1</sub>, right JB<sub>2</sub>).

Next four figures record horizontal and vertical components of the displacement, velocity, acceleration and hydraulic forces at places of journal bearings over one period of the response. Discrete Fourier transformation in fig. 13 reveals influence of particular terms of the Fourier series. It is not surprise that the greatest influence has the first term; vice versa influence of the fourth term is almost insignificant. Figures are depicted in the horizontal direction and again at places of bearings. In the vertical direction the situation is analogical.

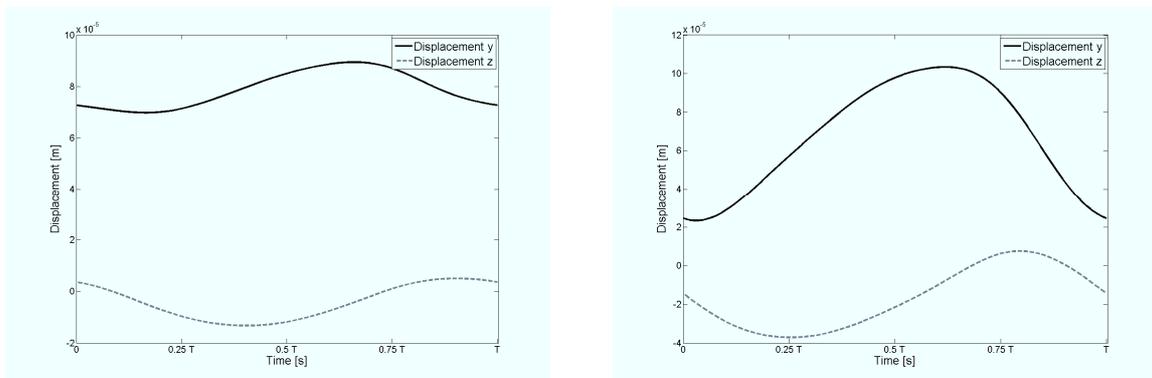


Fig. 9. Displacements in horizontal and vertical directions (left JB<sub>1</sub>, right JB<sub>2</sub>).

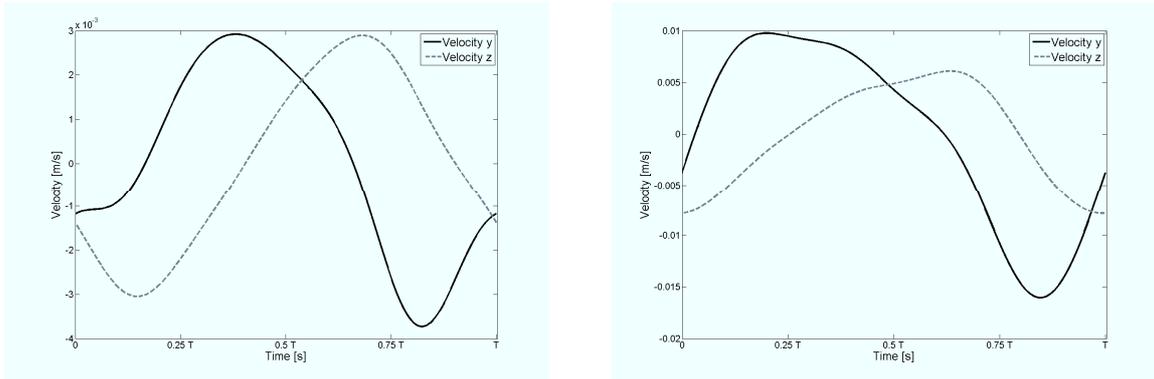


Fig. 10. Velocities in horizontal and vertical directions (left JB<sub>1</sub>, right JB<sub>2</sub>).

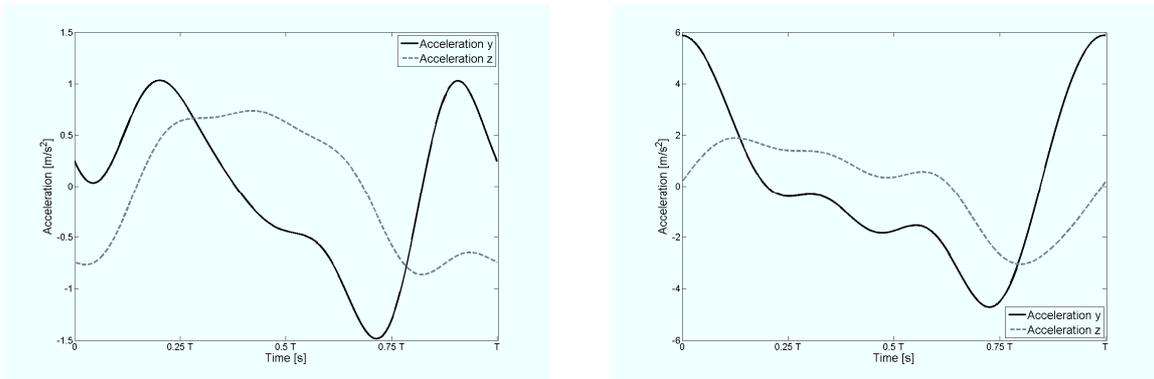


Fig. 11. Accelerations in horizontal and vertical directions (left JB<sub>1</sub>, right JB<sub>2</sub>).

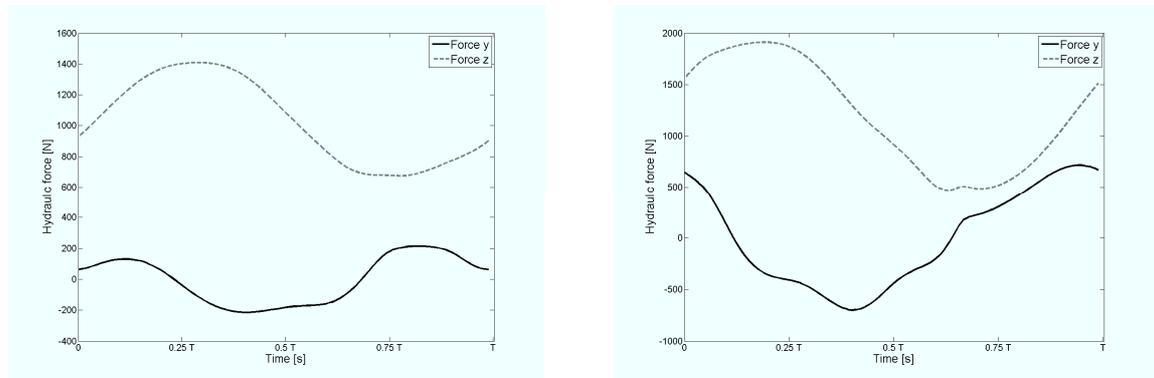


Fig. 12. Hydraulic forces in horizontal and vertical directions (left JB<sub>1</sub>, right JB<sub>2</sub>).

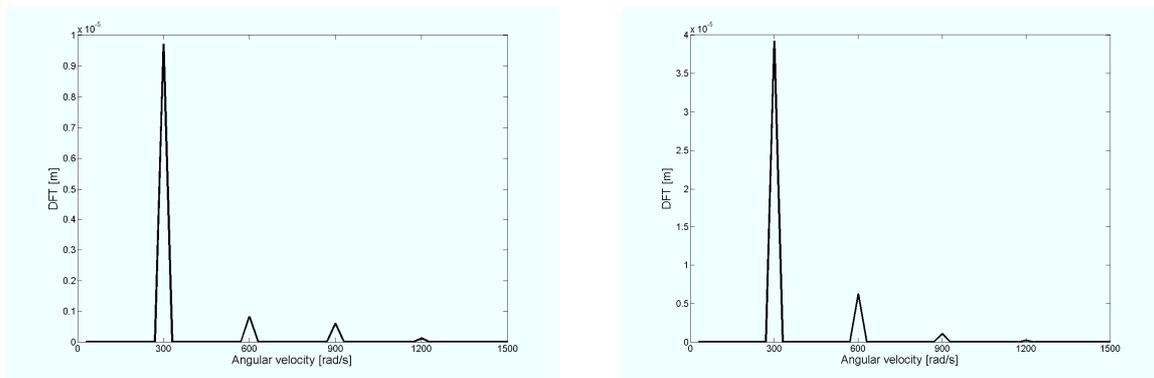


Fig. 13. Discrete Fourier transformation in horizontal direction (left JB<sub>1</sub>, right JB<sub>2</sub>).

The accuracy of the solution was assessed by the MAC. Its running over one period is depicted in fig. 14 left. In collocation points MAC takes the value nearly one. The minimum takes the value 0.977. In the same figure on right hand side it can be seen the steady-state response of the rotor's shaft.

Results obtained by the trigonometric collocation method are compared to the steady-state response calculated by the Runge-Kutta method (ODE45). Figures 15 and 16 contain comparison in terms of hydraulic forces because they are a function of the displacement and velocity. Apparently the error of the trigonometric collocation method is low, so the approximation by the absolute and the first four harmonic terms of the Fourier series seems to be sufficient.

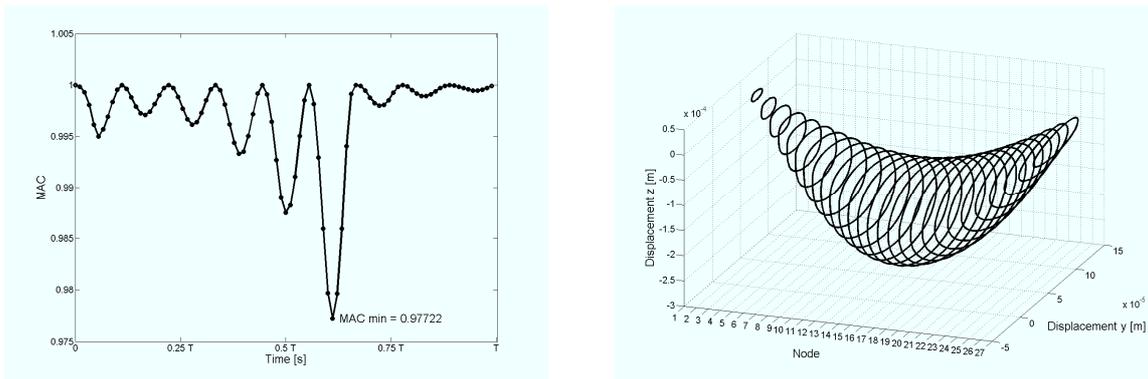


Fig. 14. MAC and steady-state response of the shaft.

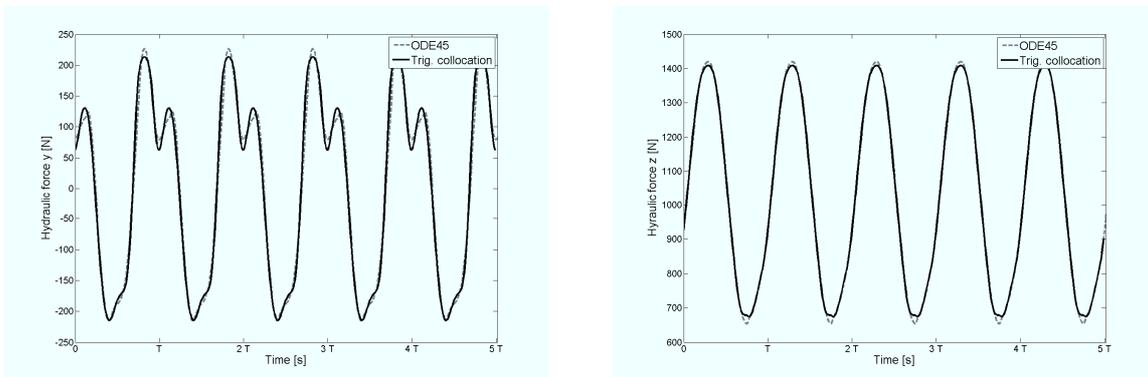


Fig. 15. Comparison of hydraulic forces  $JB_1$  (left horizontal, right vertical direction).

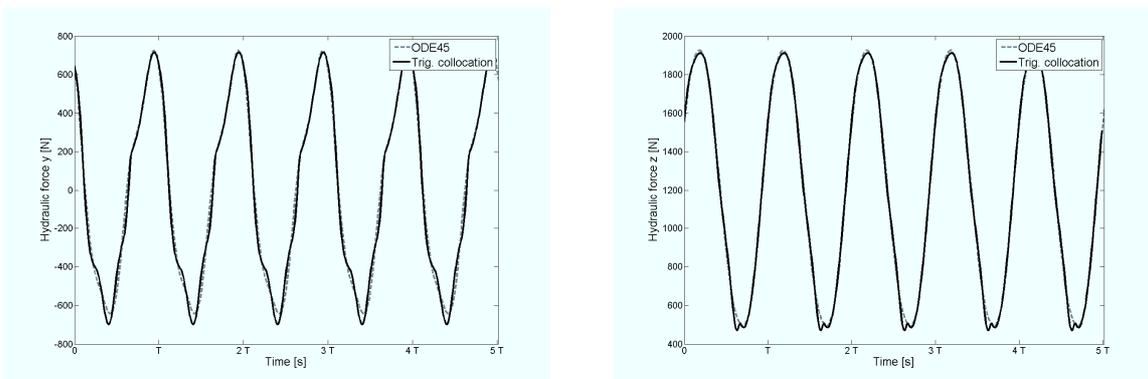


Fig. 16. Comparison of hydraulic forces  $JB_2$  (left horizontal, right vertical direction).

## 6. Conclusion

This contribution is dealing with the parallel algorithm of the trigonometric collocation method in the dynamics of the rotors supported by journal bearings. The algorithm was realized in the MATLAB parallel computing environment (DCT/DCE) and uses message passing technique for interacting among processes. This algorithm was successfully tested on two different types of parallel computers – the Beowulf cluster THEA and the symmetric multiprocessor (two dual-cores CPUs).

Performed tests have revealed very good speed-up, scalability and also efficiency of this algorithm. Values of obtained speed-up on THEA (8 processing units) were 7.4, 7.1 and 7.2, on the symmetric multiprocessor (4 processing units) were 3.3, 3.6 and 3.4 for the rotor supported by two, three and four journal bearings respectively. These results correspond to absolute times 8.8, 19.3 and 29.2 minutes on THEA and 7.7, 15.4 and 25.6 minutes on the multiprocessor. The efficiency of the algorithm was not lower than 83% on both parallel computers.

Advantage of the approximate trigonometric collocation method is its wide possibility of parallelism. Therefore results are obtained much faster compared to the equation of motion integration methods.

Future work is going to be dedicated to implementation of more appropriate boundary conditions for the Reynolds equation, namely the Reynolds boundary condition. Also the universal cavitation algorithm proposed by Elrod will be tested. This algorithm preserves mass conservation within the entire flow domain (directly incorporates so called *JFO* model).

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