UNIVERSITY OF WEST BOHEMIA

Applied and Computational Mechanics 1 (2007) 631 - 640

Applied and Computational Mechanics

Use of elastic stresses for a multiaxial fatigue prediction

M. Růžička [a], J. Papuga [a,*], Z. Hrubý [a], K. Doubrava [a]

[a] *Faculty of Mechanical Engineering, CTU in Prague, Technická 4, 166 07 Prague, Czech Republic*

Received 7 September 2007; received in revised form 10 October 2007

Abstract

A new computational method derived from Papuga PCr multiaxial criterion is presented in the paper. While the PCr criterion is suitable for a comparison of a local multiaxial loading with a fatigue limit, the PCF criterion derived is focused on computation within a finite life. Its use is intended for a variable amplitude multiaxial loading, where the Palmgren-Miner damage cumulation law is applied. The PCF method is based on local elastic stresses and their action within the standard S-N curves of smooth specimens. No arrangement concerning the stress gradient effect was applied, since the experiments serving for comparison were carried on smooth and unnotched specimens. The experimental set covers different load paths applied to specimens manufactured of CSN 41 1523 structural steel. Computational results are promising for cases with load paths formed from single unclosed lines, but for the cases with load paths related to closed constructs it provides too conservative solution. A need for a further term counting for the multiaxial hardening is discussed.

Keywords: multiaxial fatigue calculation, high-cycle fatigue, low-cycle fatigue, load path effect

1. Introduction

Any multiaxial fatigue calculation in the region of a finite life is a complicated matter. The local load level at some evaluated places exceeds the elastic region. Induced plasticity corresponds to a form of energy accumulation, release of which at last is accompanied by a formation of micro-cracks. The effect of plasticity on a damage process is therefore very substantial. Nevertheless, the necessity to involve the plasticity into the FE-solution presents a very unfavourable obstacle for any engineering application. First, the use of elastic-plastic model in the FE-calculation slowdowns its processing. Second, use of such a solution for cases with applied random or very long load history is beyond our imagination. Since material changes its elastic-plastic characteristics during the loading even for very simple load histories, a further question, which of the characteristics is the relevant one for a fatigue computation, is upon the place.

There are therefore some reasons why to try to stick on methods based on the use of purely elastic S-N curves. These curves are well documented for many materials. The use of elastic load data can speed up the FE-computation and also any further work with its results. According to our up-to-date knowledge, the only acknowledged solution processing elastic stresses is the multiaxial method implemented in FemFat[†]. Because of we are not equipped with this tool in our department, we were not able to prepare an adequate evaluation of FemFat's calculation over our experimental data.

The Papuga PC method was defined in [3] and later revised to Papuga PCr method presented in [4]. Its very good results can be observed in the comparison available in the FatLim

[*] Corresponding author. Tel.: +420 737 977 741, e-mail: papuga@pragtic.com.
[†] Commercial fatigue FE-postprocessing tool (http://www.femfat.com)

database, as is mentioned in the text further. The PCr method compares the evaluated local loads with the fatigue limit. Its results are thus suitable only for declaration, whether the critical place will withstand the number of cycles at fatigue limit or even higher. The present results of the method in this domain of computation lead us to an attempt to shift its validity or applicability into a finite life domain.

2. New Method

2.1. Papuga PCr Method

The method was developed empirically during testing of different combinations of load parameters in the PhD thesis of Papuga [3]. It focuses on computation of a fatigue limit under complex multiaxial loading by a definition of an equivalent fatigue limit, which is compared with a fatigue limit in fully reversed axial loading f_{-1}:

$$\sqrt{a_c \cdot C_a^2 + b_c \cdot \left(N_a + \frac{t_{-1}}{f_0} \cdot N_m \right)} \leq f_{-1} . \qquad (1)$$

As follows, the impact of changes in the C_a shear stress amplitude on some examined plane is more pronounced than it is by N normal stress on the same plane in the linear form. The form of the inequality is chosen in order to limit the allowable state of loading. Its violation marks the cases, where the loading applied is higher than the expected fatigue limit. The criterion is dependent on two resting material parameters t_{-1} and f_0, which correspond to fatigue limit in fully reversed torsion and fatigue limit in repeated axial loading. The parameters a_c and b_c are set by maximizations of the formula on the left hand side of (1) over all possible planes. The maximization is solved for load cases of reversed axial loading and reversed torsion successively. The detailed solution can be found in [4], here only final formulas are shown:

$$\begin{aligned} &\kappa < \sqrt{\frac{4}{3}} \cong 1.155 \ : \ a_c = \frac{\kappa^2}{2} + \frac{\sqrt{\kappa^4 - \kappa^2}}{2}, \quad b_c = f_{-1}, \\ &\kappa \geq \sqrt{\frac{4}{3}} \cong 1.155 \ : \ a_c = \left(\frac{4\kappa^2}{4 + \kappa^2} \right)^2, \ b_c = \frac{8 f_{-1} \kappa^2 \left(4 - \kappa^2 \right)}{\left(4 + \kappa^2 \right)^2}. \end{aligned} \qquad (2)$$

The term κ stands for a fatigue limit ratio:

$$\kappa = f_{-1} / t_{-1} . \qquad (3)$$

The term accompanying the N_m mean normal stress component on the examined plane is a fatigue ratio, which was found empirically. Testing of the proposed criterion seems to be much thorough than the most of similar criteria could experience. The Internet database called FatLim‡ was prepared, where any visitor can confront this method with other 17 criteria of a similar specialization. The current number of experiments collected in FatLim achieved total of 437 items. The Papuga PCr method still remains the most attractive one there. The database has some limitations – it covers only experiments on smooth unnotched specimens, where the loading on any load channel was either harmonic or static. No other complex load paths were tested up to now.

‡ FatLim database is a non-commercial database of fatigue experiments and computational results accessible from http://www.pratic.com/experiments.php

2.2. PCF Method – Idea, Premises and Expectations

The promising features of the PCr method led us to a conclusion to try to extend its applicability to a finite life. The fatigue limit itself is in fact a point on the S-N curve, determination of which is relatively complicated and financially demanding. Many of researchers working in this category thus decide to set its position on the abscissa to some specific value within the range from 1e6 to 1e7 cycles. Because of tests like this are also covered in the FatLim database, the PCr method already works with the S-N. Our premise is, that the knowledge of the relevant S-N curves at load modes, with which the PCr method operates, could allow us determination of a number of cycles, which the examined place survives without a breakage. The PCr method originally presented in (1) as inequality transforms for our intention to an equivalent axial fully reversed loading. The S-N curve is expected to be known for this case:

$$\begin{aligned}
&\sqrt{a_C(N)\cdot C_a^2 + b_C(N)\cdot\left(N_a + \frac{t_{-1}(N)}{f_0(N)}\cdot N_m\right)} = f_{-1}(N);\\
&\kappa(N) < \sqrt{\frac{4}{3}} \cong 1.155: \quad a_c(N) = \frac{\kappa^2(N) + \sqrt{\kappa^4(N) - \kappa^2(N)}}{2}, \qquad b_c(N) = f_{-1}(N);\\
&\kappa(N) \geq \sqrt{\frac{4}{3}} \cong 1.155: \quad a_c(N) = \left(\frac{4\kappa^2(N)}{4+\kappa^2(N)}\right)^2, \qquad b_c(N) = \frac{8 f_{-1}(N)\kappa^2(N)\left[4-\kappa^2(N)\right]}{\left[4+\kappa^2(N)\right]^2};\\
&\kappa(N) = \frac{f_{-1}(N)}{t_{-1}(N)}.
\end{aligned} \tag{4}$$

All the *N* number of cycles dependent terms are fatigue limits utilizing the following well-known formula in our solution:

$$\sigma^w \cdot N = A, \tag{5}$$

where the actual loading presented in the form of stress σ relates to an exponent *w* of the S-N curve and the coefficient *A*, which is constant for each individual S-N curve.

The use of elastic stresses negates the necessity to take in the plasticity effects. We are far away from this statement, but we expect that at least areas of lifetimes higher than 1e5 cycles could be covered with a sufficient closeness to experimental values. Important point in this expectation plays the fact, that the specimens used in our experimental program were un-notched.

Our next question was, whether the complex load histories presented in the sec. 3 Experiments will affect the quality of the prediction by the newly defined method substantially.

2.3. PCF Method – Realization

The method in the final form is implemented to PragTic fatigue freeware[§], where also the rest of methods covered in the FatLim database are programmed. There are three fatigue limits in (1), thus we expect a use of three S-N curves. The four parameters joint in (5) leave three known material variables necessary to compute the unknown one. The usual solution depends on the knowledge of fatigue limit, the exponent *w* and number of cycles corresponding to the fatigue limit. These values enable to calculate the value of constant *A* for each of the three load modes. If we introduce the following substitution:

$$f = f_{-1}(N), \quad p = f_0(N), \quad t = t_{-1}(N) \tag{5}$$

[§] PragTic fatigue solver enabling also post-processing of the FE-data is distributed as a freeware downloadable from the website: http://www.pragtic.com in its current version PragTic v.0.2.

to avoid the confusion of fatigue limits and endurance limits, the three fatigue curves can be described:

$$f_f^{w_f} \cdot N = A_f, \quad f_p^{w_p} \cdot N = A_p, \quad f_t^{w_t} \cdot N = A_t. \tag{6}$$

The load modes of each of the S-N curve above should be apparent from the use of letters *f*, *p* and *t* designating the alternate push-pull, repeated pull and alternate torsion consecutively. The use of the three equations gathered in (6) within (4) calls for some simplification in terms:

$$F = 1/w_f, \quad P = 1/w_p, \quad T = 1/w_t. \tag{7}$$

Then the current fatigue limit ratio $\kappa(N)$ can be re-formulated to:

$$\kappa(N) = \left(\frac{A_f}{N}\right)^F \cdot \left(\frac{N}{A_t}\right)^T = \frac{A_f^F}{A_t^T} \cdot N^{T-F} \tag{8}$$

and the basic formula to:

$$\sqrt{a_C(N) \cdot C_a^2 + b_C(N) \cdot \left(N_a + \frac{A_f^T}{A_t^P} \cdot N^{P-T} \cdot N_m\right)} = \left(\frac{A_f}{N}\right)^F. \tag{9}$$

The final formula (9) is not explicit. The number of cycles N cannot be derived from it directly. The implicit scheme demands the use of Newton-Raphson iterative formula.

A typical S-N curve has a characteristic point at a fatigue limit – its slope behind it either decreases significantly or the curve becomes horizontal line. A full description involving such a change in the slope is not suitable for our experimental programme. The transition to the nearly horizontal slope would produce great differences in the final lifetimes for slight changes of the load applied. We do not want to set the final lifetime of some specific engineering component. We would like to compare the behaviour of the criterion in the limited lifetime. S-N curves used here are thus expected to continue with the same slope even below the fatigue limit.

The mean stress effect is introduced via the term $t_{-1}/f_0 \cdot N_m$. There is a convention applied in PragTic that whenever the mean normal stress N_m is lower than zero (i.e. it is the compressive mode), the N_m term is set equal to zero. Thus it is not activated for the most of the load paths defined.

3. Experiments

The experimental program using tubular specimens manufactured of CSN 41 1523 steel was conducted at CDM, IT AS CR[**]. The shape and dimensions of the specimens are shown in fig. 1. The load paths applied to the axial and torsion load channels are depicted in fig. 2. The load frequency was originally set to 2 Hz, but it had to be decreased to 1 Hz by some load paths due to an excessive generation of a heat. Numbers of cycles for different load set-ups are available in tab. 1. The numbers of cycles relate to the final breakage, when the specimen cannot anymore transfer the load applied. Experiments include tests in the low-cycle and also in high-cycle fatigue areas. More details on the experimental programme can be found in [2].

Material parameters found during testing are provided in tab. 2. The description of the S-N curves is used in this report only. All the values corresponding to alternate axial or alternate torsion loading were obtained from experiments. The S-N curve for the load case of repeated tension was not set experimentally, but was prepared as an artificial construct. The relevant fatigue limit was computed with the use of the Smith, Watson & Topper rule:

[**] Centre for Diagnosis of Materials, Institute of Thermomechanics, Academy of Sciences of the Czech Republic

$$\sigma_{a,eq} = \sqrt{\sigma_a \cdot (\sigma_{\acute{a}} + \sigma_m)}\,. \tag{9}$$

If we want to set the fatigue limit in repeated loading, its application produces:

$$f_{-1} = \sqrt{\frac{f_0}{2} \cdot f_0}\,, \quad \text{i.e.:} \quad f_0 = \sqrt{2} \cdot f_{-1}\,. \tag{9}$$

The numbers of cycles for both fatigue limits stay the same. The second required point of the S-N curve is the tensile strength related to 1 cycle. These two points fully define parameters of the S-N curve as presented in tab. 2

The number of other methods for evaluation of the mean stress effect is high (a more detailed overview can be found in [3]), but we had no special clue, which one to use. The parameter used here is very simple. Moreover, as have been already noted in [5], the original PCr method is sensitive to deviations in this material parameter only slightly.

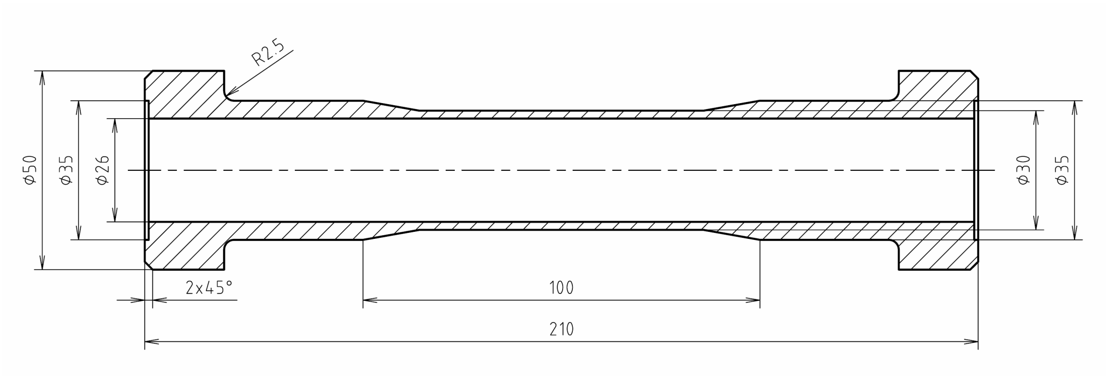

Fig. 1. The smooth unnotched specimens used in the experimental programme, results of which are used for the validation of the PCF method.

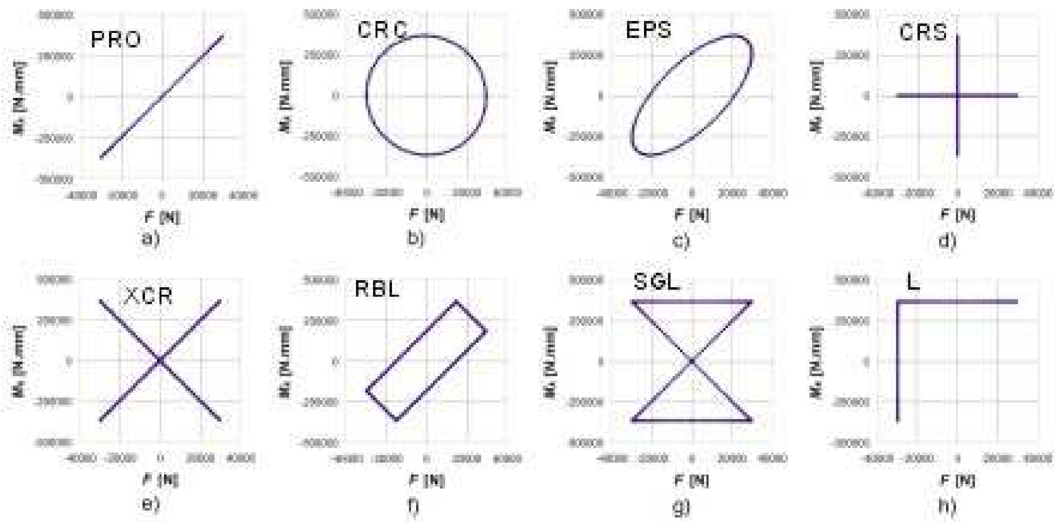

Fig. 2. Load paths used in the experimental programme relate the axial and torsion load channels.

Mark		Phase shift	Real nominal loads on individual load channels				Number of cycles	
			Axial		Torsion		Experiment	Prediction
		δ [deg]	σ_a [MPa]	σ_m [MPa]	τ_a [MPa]	τ_m [MPa]	N_e [-]	N_p [-]
CRC	CRC150	90	150.60	-1.10	147.95	1.95	508 255	33 622
	CRC160	90	157.40	1.50	158.85	-1.45	162 396	10 450
	CRC180	90	177.25	1.05	176.45	0.85	25 063	1 518
CRS	CRS160		160.30	-0.60	157.75	-0.35	742 305	532 406
	CRS170		169.95	-0.05	166.00	1.10	572 163	246 636
	CRS180		178.45	-0.55	175.10	-2.60	56 152	110 551
EPS	EPS140	45	138.70	-1.70	132.65	-3.05	270 846	193 755
	EPS150	45	149.20	-1.40	148.35	-1.25	115 232	32 680
	EPS180	45	183.25	-7.25	175.40	-7.30	7 830	1 364
L	L150		153.75	-1.45	146.95	8.65	98 794	197 401
	L160		169.95	-2.25	156.35	15.45	62 522	60 543
	L180		184.75	-1.65	181.20	15.90	16 804	5 412
PRO	PRO140		139.00	-2.00	139.00	-3.30	262 324	100 490
	PRO150		150.65	-4.55	149.20	-7.70	51 711	27 865
	PRO160		157.20	-2.00	152.20	-5.70	44 148	17 476
	PRO180		175.35	-1.95	171.35	-7.05	10 642	2 313
RBL	RBL140		139.75	-0.05	139.50	0.60	1 582 327	103 556
	RBL150		148.35	-0.55	148.95	-1.05	294 125	32 808
	RBL180		181.45	0.65	175.35	2.15	24 597	1 435
SGL	SGL130		128.75	-0.85	128.90	1.00	200 715	11 853
	SGL140		139.25	-1.15	139.45	0.45	63 989	2 527
	SGL150		150.65	-1.15	149.55	-0.85	59 882	611
	SGL180		198.45	-0.75	184.90	8.20	7 818	6
XCR	XCR120		115.95	-2.25	111.10	0.30	2 666 539	3 328 342
	XCR130		127.90	-1.70	120.40	1.00	339 473	649 754
	XCR155		153.20	1.40	135.70	2.80	26 873	47 000
	XCR170		167.10	1.10	143.90	-1.10	8 534	13 119

Tab. 1. Used experiments and prediction results.

CSN 41 1523 structural steel								
R_e	R_m	K'	n'	σ_f'	ε_f'	b	c	E
[MPa]	[MPa]	[MPa]	[-]	[MPa]	[-]	[-]	[-]	[MPa]
334.5	550	1020	0.141	1009	0.922	-0.093	-0.659	210000
f_{-1}	w_f	$N_{c,f}$	f_0 *	w_p *	$N_{c,p}$ *	t_{-1}	w_t	$N_{c,t}$
[MPa]	[MPa]	[MPa]	[-]	[MPa]	[-]	[-]	[-]	[MPa]
220	21.21	6.015e6	311.3	27.31	6.015e6	134.3	15.04	6.015e6

Tab. 2. Material parameters set for the CSN 41 1523 steel. The parameters marked with the asterisk (*) were not set directly but were derived from other parameters as described in the text.

4. Fatigue Computation

4.1. Load History Decomposition

Some load paths are relatively more complicated to be evaluated as a single load cycle. We expected that they should be split in halves so that the individual load cycles could be evaluated. There are several methods implemented in PragTic, which have to take care of load history decomposition. The methods usually use the rain-flow decomposition either of some reduced uniaxial load history, of normal stress/strain on some plane or of shear stress in some specified direction.

The spectrum of methods is relatively broad, nevertheless none of them is recognized as the right one. We decided therefore to remove this point from our questions. Some of the paths are clearly related to only one cycle (PRO, CRC, EPS, RBL), some of them seem to be better described as two independent cycles (CRS, XCR) and the decision is indefinite by two of them (SGL and L paths). The number of more complex load paths is too small unfortunately to allow us real examination of the quality of those different decomposition procedures.

The decomposition was therefore ordered by an intervention to the source code of PragTic. The two questionable load paths were tested in both ways – as being only one cycle or two of them. The L path provides reasonable prediction results, when separated to two cycles. The case of the SGL path is much more problematic, since both options lead to odd results, nevertheless the variant, where the path defines only one cycle, is a bit closer to realistic values. As can be understood from fig. 3, the SGL path forms the most problematic obstacle for any PCF calculation anyway.

4.2. Results Analysis and Discussion

The results of the use of the PCF criterion can be observed in fig. 3 and in tab. 1. The results are not very good at the first sight, nevertheless the data set is worth of a more detailed evaluation.

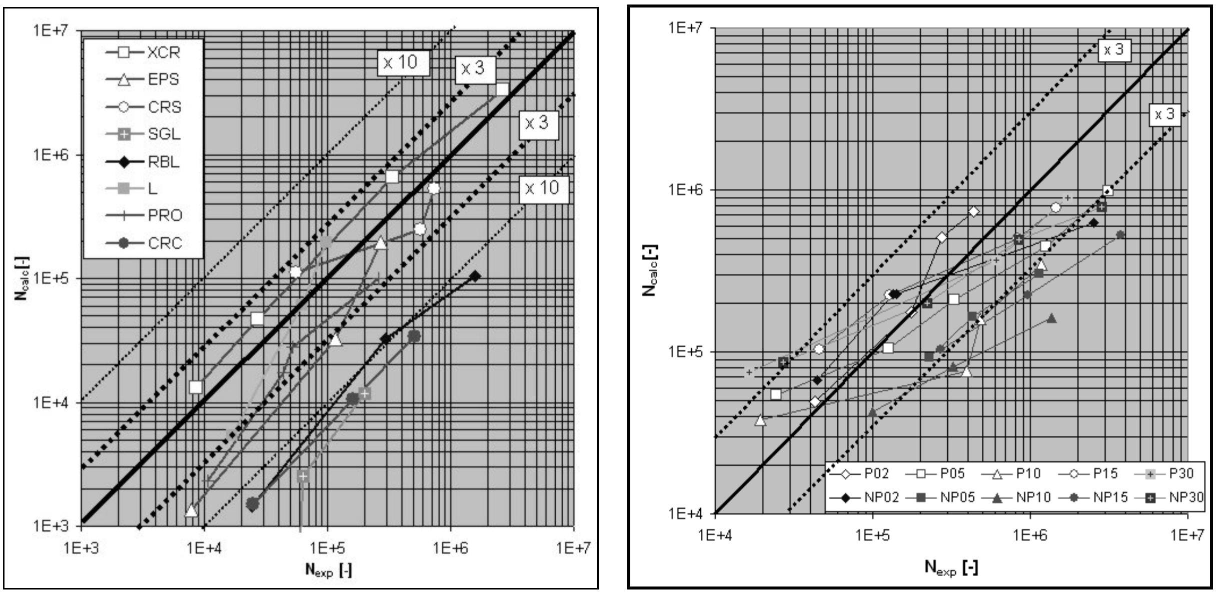

Fig. 3. Results of the PCF criterion for the original data set described here (left) and for the data examined in sec. 4.2 (right), where results on load paths are similar to PRO (P here) and CRC (NP here). The load ratios of the test sets P10 and NP10 are the closest ones to the load paths tested in the main text (PRO and CRC).

The majority of joins have similar direction as the axis of the quadrant marking the ideal position. This finding can be interpreted as confirmation that the replacement of the complex local load situation by a damage parameter utilizing two or three independent S-N curves could work even in the low-cycle area.

The lines, which are more or less within the range of the multiple 3 of the experimental lifetime to the conservative or non-conservative sides are the XCR, L, CRS, PRO. Note that all these load paths do not enclose any area in the σ-τ space.

The rectangular RBL, sandglass SGL and circular CRC paths are all shifted to the conservative side by multiples higher than 10. They are relatively close one to another. The results of the SGL path are the oddest ones here. They pursue the trends of the other load paths for the higher lifetimes, but deviate strongly for lower lifetimes. The prediction is extremely conservative for the highest load, i.e. the criterion says that the specimen should be broken sooner than it really happened. No explanation concerning flaws in material etc. works here, since this is an inverse situation. We were not able to explain this trend in any way until now.

The elliptical EPS path is only slightly more shifted to the conservative side, than it is by the proportional PRO path, but the circular CRC path is shifted even by multiple of 20 to the same side. This is a very problematic point. If the rest of results and previous paragraphs are compared, one would expect that the clue could be hidden somewhere in a necessity to involve some additional parameter related to hardening caused by multiaxiality of loads [1]. If the material hardens, then its fatigue strength increases – the computational results would not lead to such low (conservative) lifetimes. Such a proposal could have its weak point in the fact that the original PCr criterion was tested on a large number of fatigue limits with different phase shifts and load paths similar to PRO, EPS, CRC. No similar trend was observed there until now.

The objection is not completely sustained by experimental data. The usual position of a fatigue limit is somewhere near 1e6-1e7 number of cycles. The end points of the relevant joins here are at maximum 5e5 number of cycles. There is a gap uncovered by experimental data, but it does not seem reasonable to assume that a leap substantiated by an impact of a plastic deformation could shift the original acceptable prediction to a lifetime multiple of 20 on such a short space.

We wanted to discuss this part of results with another data. Other experimental results obtained on the same CSN 41 1523 material have already been published in another report [6]. The results compiled into fig. 3 (right) correspond to a multiaxial testing of smooth unnotched specimens by tension and torsion under proportional (P) or out-of-phase (NP) loading with the phase shift 90°. Results cover different ratios between amplitudes of nominal stresses in the torsion and tension (0.2; 0.5; 1.0; 1.5 and 3.0). Only the final results are presented in fig. 3. The prediction quality is much better than it is by the previous data. The best results are obtained for load ratios 0.2 and 3.0, the rest of the data set seems to observe a trend similar to data measured at CDM. The non-proportional path leads to more conservative prediction than the relevant proportional paths. The results of experiments with the load ratio 1.0 (similar load ratio is applied in the experiments presented here previously) tend to shift way in both P and NP variants.

Another our test performed with results of the original PCr criterion can verify anybody on http://www.pragtic.com/experiments.php. Select two different modes of loading there:

- PRO - with the phase shift between both load channels being zero, all mean stresses zero and loading only by first axial channel and torsion;

- CRC – the same but with the phase shift 90° - the load path will not be exactly circular, but still covers the largest possible area for this kind of harmonic two-channel loading.

The FatLim database works with comparison of computational results by the fatigue index error ΔFI. This parameter describes a relative deviation of the load applied σ_{eq} and experimentally set uniaxial fatigue limit $f_{-1,exp}$:

$$\Delta FI = \frac{\sigma_{eq} - f_{-1,\exp}}{f_{-1,\exp}} . \tag{9}$$

The loads presented in the FatLim database should correspond to multiaxial fatigue limits – the fatigue index error has to be zero there.

The currently gathered experimental data set offers statistical comparison of 132 tests of the PRO category and 28 of the CRC category. The difference between the mean values of each group is 2% of the fatigue index error ΔFI. Although the slope of the S-N curve of the material presented in this report is unusually small, such a difference cannot generate so large deviation in lifetimes.

Nevertheless, when the data is examined according to the ratio of applied torsion to axial load (see fig. 4), a glimpse of a trend similar to previous results in finite lifetimes can be found. We call it only a glimpse, since the data set in the non-proportional category is clearly insufficient. More results falling into the category of the load ratio higher than one should be examined.

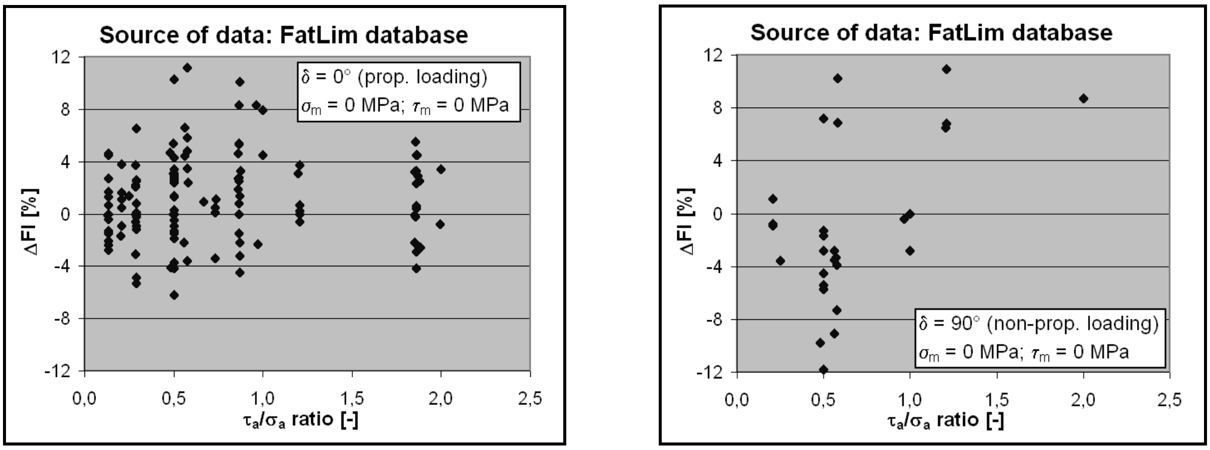

Fig. 4. Comparison of result data retrieved by the PCr computation on experiments with load paths similar to PRO and CRC load paths.

Note please another important point, if a comparison of both data sets presented in fig. 3 is done. The second set with P and NP paths works with the S-N curve in reversed push-pull with the w_f exponent being 9.93. The more than two times lower slope of the S-N curve found by the main experimental data here ($w_f = 21.2$) largely influences the effect of any error found in the N number of cycles. If 10% error in the determination of the load level generates multiple of 2.5 in a number of cycles by the P and NP data, the same error with the less inclined S-N curve provides a multiple 7.5.

5. Conclusion

The PCF method based on the successful PCr criterion was defined for a use in the finite life and its implementation was described. The main experimental data set, to which the prediction results correspond, was obtained from tests performed by our colleagues from CDM IT ASCR with smooth unnotched specimens manufactured of CSN 41 1523 structural steel. The loads applied led to lifetimes ranging from the low-cycle fatigue to the high-cycle fatigue regions.

The load paths XCR, CRS, L, PRO result in an acceptable prediction. The results of SGL, RBL and CRC load paths are shifted largely to the conservative side of the prediction. The most attractive idea to apply a multiaxial fatigue factor to their results was at first set aside. The use of the original PCr method was proved to provide good results for a number of cycles at the fatigue limit and the load paths similar to PRO, EPS and CRC paths used here. There would not be any substantiation for such a large deviation of the CRC results.

To further broaden sources for our discussion, we involved also another set of experiments done with the same material but only with load paths similar to PRO and CRC. The results of PCF criterion are much better there. One of the reasons relates to the fact that this set of experiments was accompanied by a much steeper S-N curve and the potential deviation in set stress values would not be reflected so strongly into the multiples of fatigue lifetimes. As regards the load ratio between tension and torsion channels similar to PRO and CRC experiments, a trend similar to the previous data could be observed, but not so pronounced one. If the ratio of nominal torsion and tension loads approaches unity, results of the proportional load path shift to the conservative side (here the multiple 3) and the results for the non-proportional load paths go even farther from the ideal position.

The last analyses thus focused on performance of the original PCr criterion under similar conditions as the PRO and CRC paths are. Unfortunately, the number of experiments of the CRC path was too small to ensure validity of the extractable argument, that the same significant deviation from the ideal value can be found for cases where the load ratio between torsion and tension approaches 1. Further expansion of the FatLim database is necessary in order to compare more data points of a similar character.

Acknowledgements

Czech Science Foundation sponsored the research presented here by GACR 101/05/0199 project. The works on the FatLim database used here were financially covered by GACR 101/07/P350 project.

References

[1] J. A. Bannantine, D. F. Socie, A multiaxial fatigue life estimation technique. In: Advances in Fatigue Lifetime Predictive Techniques, ASTM STP 1122. Red M. R. Mitchell a R. W. Landgraf. Philadelphia, American Society for Testing and Materials 1992, pp. 249-275.

[2] K. Doubrava, M. Růžička, Z. Hrubý, J. Papuga, M. Balda, Experiment And Prediction of the Lifetime for combined Tensile and Torsion Loading of Material 11 523. In: Proc. of Degradácia konštrukčných materiálov 2007, Žilinská univerzita v Žilině 2007.

[3] J. Papuga, Mapping of Fatigue Damages - Program Shell of FE-Calculation. [PhD Thesis], CTU in Prague 2005. available on http://www.pragtic.com for download.

[4] J. Papuga, M. Růžička, Two new multiaxial criteria for high cycle fatigue computation. Int J Fatigue 2007. In Press - doi: 10.1016/j.ijfatigue.2007.02.015.

[5] J. Papuga, M. Růžička, Proposal for a new high-cycle multiaxial fatigue criterion (Revised version for ULM 2006 conference). In: Únava a lomová mechanika 2006. Škoda Výzkum s.r.o., Plzeň 2006. (in Czech)

[6] F. Plánička, V. Vacek, J. Svoboda, M. Václavík, Fatigue of structural parts under multiaxial loading with synchronized and phase-shifted stress components – Part II. ZČU a CDM UT AVČR, Plzeň 2001. (in Czech)