

# System model reduction for MBS optimization

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## Abstract

A disadvantage of optimization of flexible multibody systems (MBS) is a computing time, mainly for large systems, especially designed by FEM. The computing time rises with the complexity of the model significantly. A reduction techniques allow decreasing of degrees of freedom and it contributes to the reduction of the computing time. These techniques can be used for the reduction from thousands and more degrees of freedom to tens, but some limits exist. A reduction degree (ratio between number of DOFs before and after the reduction) is the most important feature because it predicts the final accuracy of the model. The next one is the selection of master and slave degrees of freedom that play an important role in connecting all bodies together within the MBS (e.g. by joints). There are many reduction methods, but they differ in available accuracy, speed, efficiency and suitability for the same reduction degree. A dimension of the original system is decisive for the reduction method suitability, many methods require an inversion matrix from the part of the stiffness matrix. The inversion matrix are than large and the computing time grows up. This paper deals with the reduction techniques, their disadvantages, suitability and applicability.

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## 1. Introduction

Optimization of structures and multibody systems is very important in mechanical design. The optimized parameters can be stiffness, eigenfrequencies, eigenmodes, acceleration, accuracy and other properties generally used in mechanics. The best properties of the proposed system are demanded. Optimizations lead to the best solutions but the way towards them is not easy. Optimized models are generally produced from the models modeled by the Finite Element Method (FEM). However, the precise model requires more details, elements and also degrees of freedom (DOF) naturally, too. From such a model the generated matrices are large and it is uneconomical to solve them. There are many types of reduction methods which decrease the dimension of the model. They are based almost all of them on the static (Guyan) reduction [4], but another advanced reduction methods exist that are not based on the static reduction. For the model it is necessary to choose the master (will be leaved) and the slave (will be left out) degrees of freedom. These DOFs are selected either automatically by selection criteria or manually. The quality of the reduced system is largely influenced by the selection procedure (see fig. 8 and fig. 9).

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## 2. The reduction methods

The reduction method most known and used is so-called static reduction (Guyan reduction). It was introduced in 1965. This method is used widely in many FEM programs (e.g. ANSYS). Static reduction is very simple to use, but it has one significant disadvantage – model reduced by static reduction is accurate only at zero frequency. This disadvantage is improved by the dynamic reduction, where the accurate frequency is selected and for every frequency a new transform matrix is generated.

More satisfying method is IRS (Improved Reduced Systems) introduced in [3]. This method comes out from the static reduction, where the expressions are substituted by the expansion up to the fourth order. An expression is also obtained more accurately than by using static reduction, but the model is accurate at the zero frequency, too. Small modification makes from the IRS method the dynamic IRS reduction method.

A very good results are obtained using the iterated IRS reduction method [3]. The iterations improve the reduced matrices coming from the static reduction. This method is time-consuming but the results are the most accurate ones from all previous methods.

Another reduction approach is based on the Krylov subspaces [5] that is used in product `mor4ansys` [6].

### 2.1. Comparison of methods

Firstly, are compared methods based on the static reduction. The reduction was done for the body in fig. 4 modeled in ANSYS by the PLAIN42 elements.

The comparison of the reduction methods based on the static reduction is in fig. 1 for the Guyan method, fig. 2 for the IRS method and fig. 3 for the iterated IRS. The horizontal axis represents the reduction degree that means a ratio between the size of the original system and the reduced system in percents. The vertical axis shows the deviation between eigenfrequency

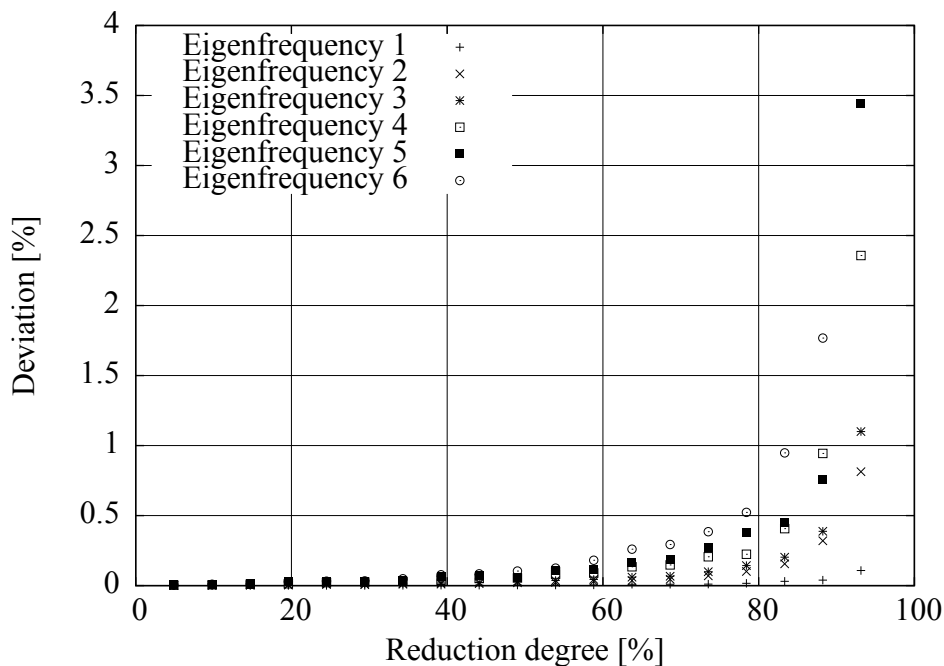


Fig. 1. Deviations of eigenfrequencies (static reduction).

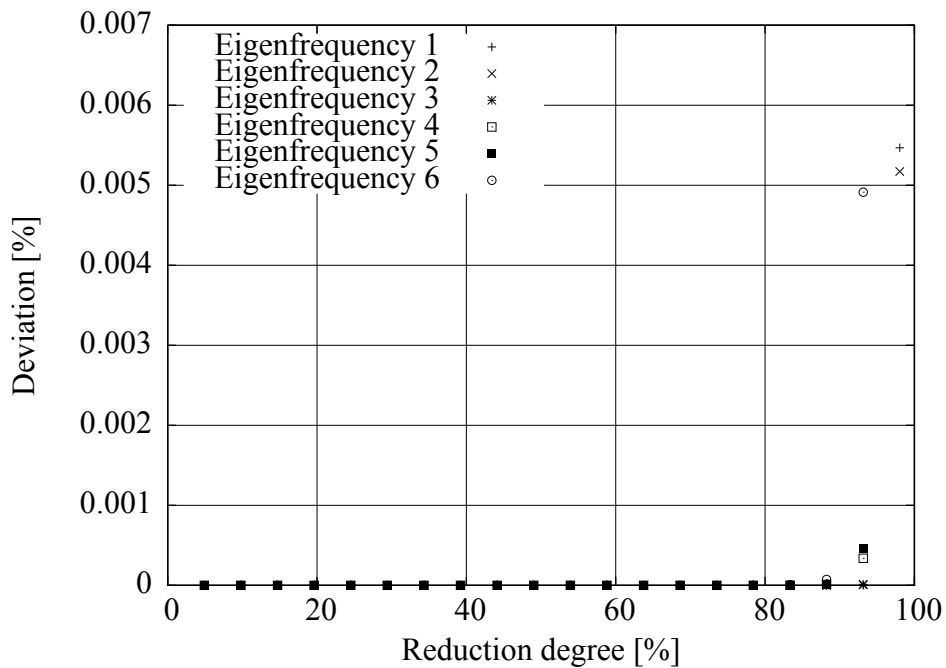


Fig. 2. Deviations of eigenfrequencies (IRS).

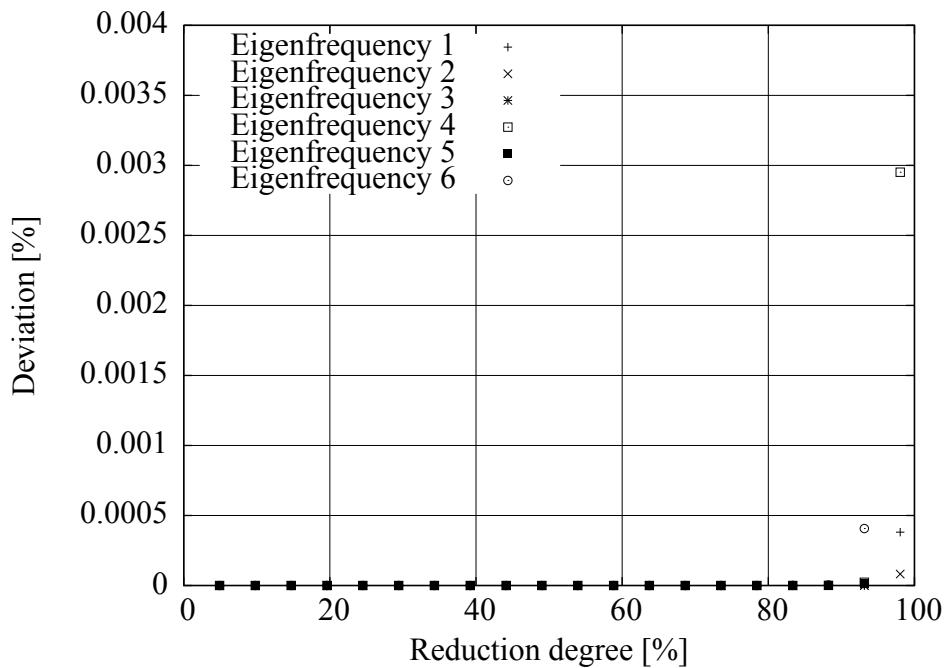


Fig. 3. Deviations of eigenfrequencies (iterated IRS).

of the original and the reduced system. Obviously, the iterated IRS method [3] is the best one. Let us compare iterated IRS method and the very good method based on the Krylov subspaces [5] built-in `mor4ansys` product [6]. The testing was done with the body from fig. 7. The body is fixed on the short side (triangular signs) and excited by the harmonic function on the opposite size (arrow). The original system has 4368 DOFs and the number of differential equations is the same. The system is reduced to only 60 chosen degrees of freedom (reduction degree is

1.37 %). The results of the numerical integrations are in fig. 5. It is very remarkable that the selection of the same master–slave points for the reduction by the Krylov subspaces and by the iterated IRS leads to the quite different results. The reduction by the iterated IRS results into lower deviations from the original system.

The original system requires for the integration time from 0 to 1 second about 50.14 hours computing time in MATLAB, the system reduced by `mor4ansys` 0.32 hours (0.64 %), but the system reduced by iterated IRS only 0.16 hours (0.32 %) (AMD Athlon 64 3000+, 2 GB RAM). The time also depends on the integrator type, `ode45` is used.

### 2.2. Selection of master and slave degrees of freedom

The selection of the master and slave degrees of freedom for the reduction methods is an important decision and it influences the output accuracy. For the selection several approaches can be used. The first one is based on the experience, the second one selects the points with large masses and the third one is an automatic algorithm – e.g. based on the ratio between mass and stiffness of the diagonal values of the mass and stiffness matrices [2], [7].

The automatic selection has one significant disadvantage (especially for the connection of bodies), the important and interesting points are often omitted from the selected points – and

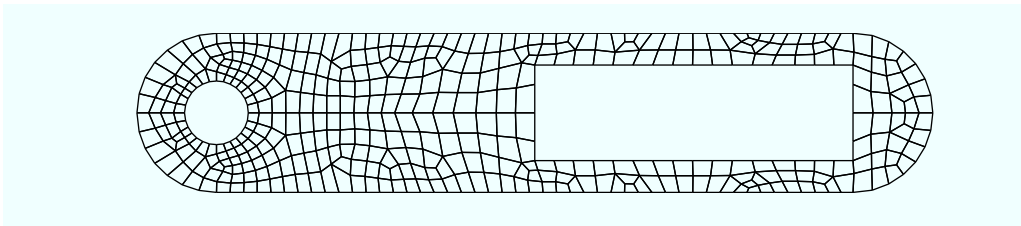


Fig. 4. The mesh of the reduced body.

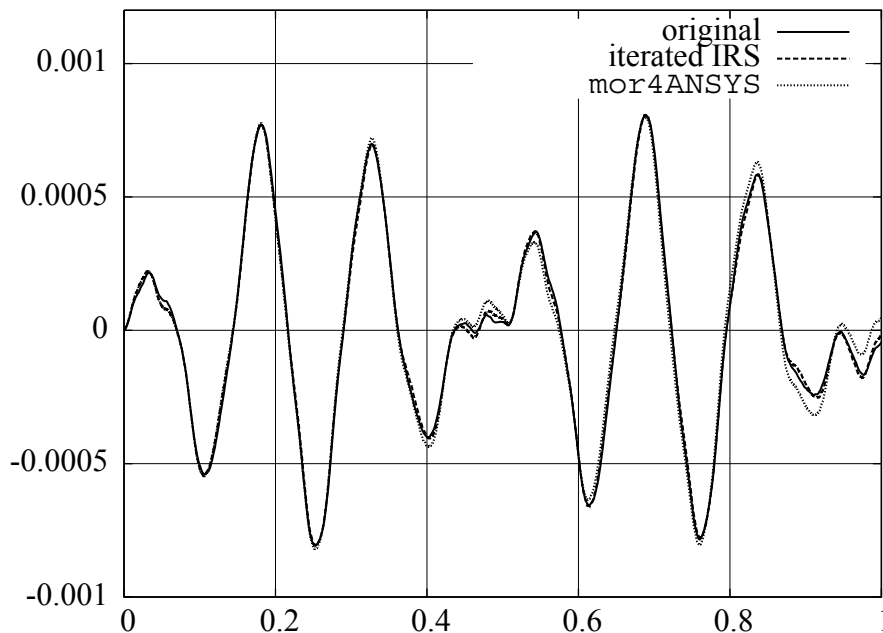


Fig. 5. A comparison of original and reduced models by iterated IRS and `mor4ansys` method.

these points (DOFs) are missing for connecting the bodies or applying the forces. Then these points must be selected additionally and than the results are usually inaccurate.

We discover that a very good result practically non-sensitive at additional selections gives the selection approach when the boundary points (nodes) are selected. The results are in figs 8 and 9.

### *2.3. Automatic connection of reduced flexible bodies*

The reduction process has another big problem for the mechanisms that move in some workspace (e.g. machine tools or robots). The investigation of the mechanical properties must be done in the whole workspace. In each position within the workspace the reduced models of flexible bodies of particular elements of the mechanisms must be interconnected according to their connection by kinematical joints. This finishes the creation of the investigated model of the mechanism and the analysis can be done only after its completion. This is traditionally done by hand and the solving time increases enormously. Therefore an automatic procedure for the connection of reduced flexible bodies has been developed [8], [9]. The procedure is based on the automatic generation of connecting springs according to the particular type of the kinematical joint (e.g. rotational joint in fig. 6).

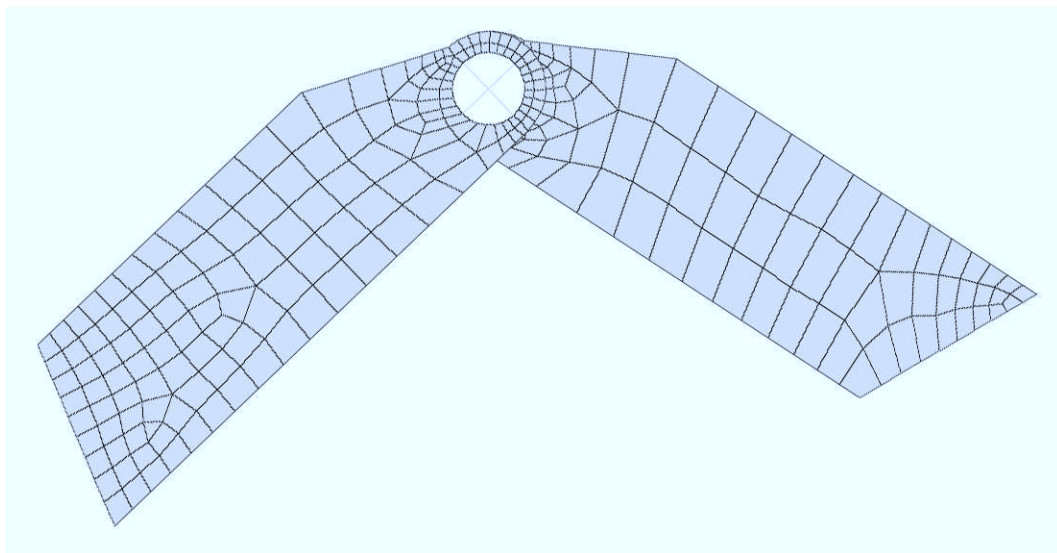


Fig. 6. Automatic connection of two flexible bodies by rotational kinematical joint.

### *2.4. Qualities of reduction methods*

All reduction methods are applicable, but there are some limitations. The static reduction is very simply applicable but it is efficient only above the reduction degree of 30 percent. A higher reduction degree leads to the results for this method inaccurate and practically unusable. And if the eigenfrequencies are enough accurate then the eigenmodes are almost always poor. The reduction IRS and iterated IRS are suitable for a higher reduction degree. These methods give good results and they are sufficiently accurate and eigenmodes are usually in a good agreement with the original system. These three reduction methods work with an inverse matrix and they are also time-consuming and they need large memory. The method based on the Krylov subspaces used in mor4ansys is faster. The comparison between iterated IRS method and

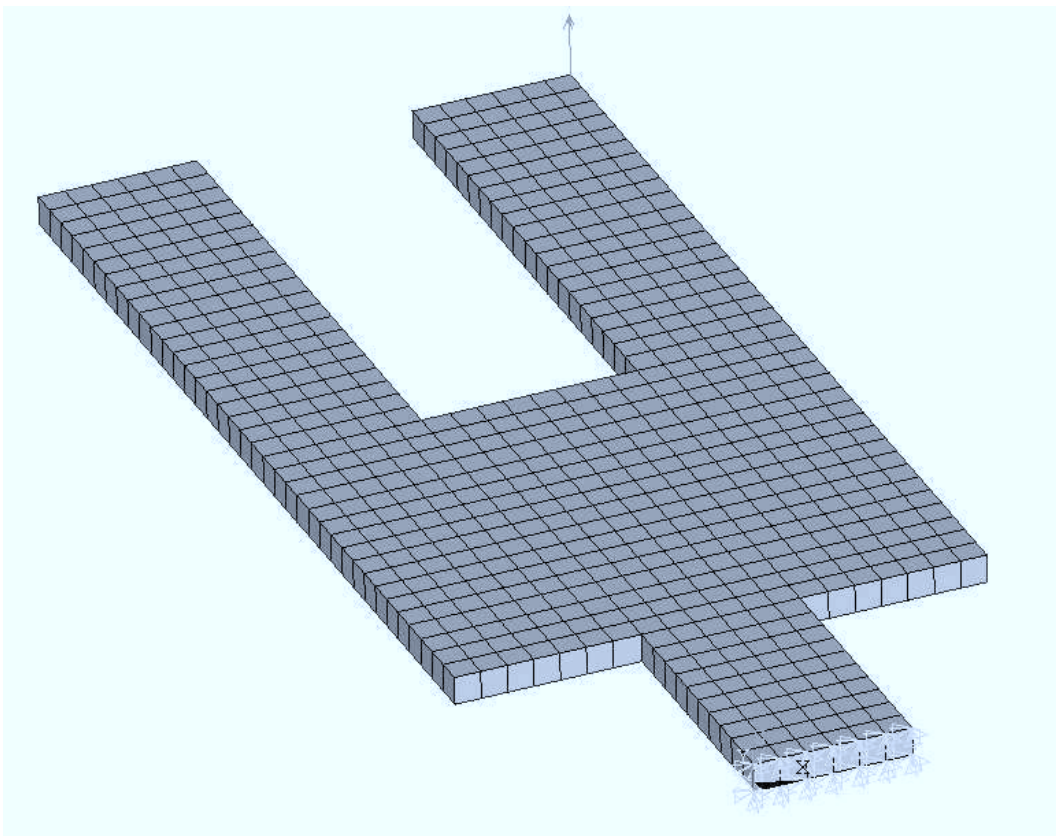


Fig. 7. Body for comparison between iterated IRS and mor4ansys.

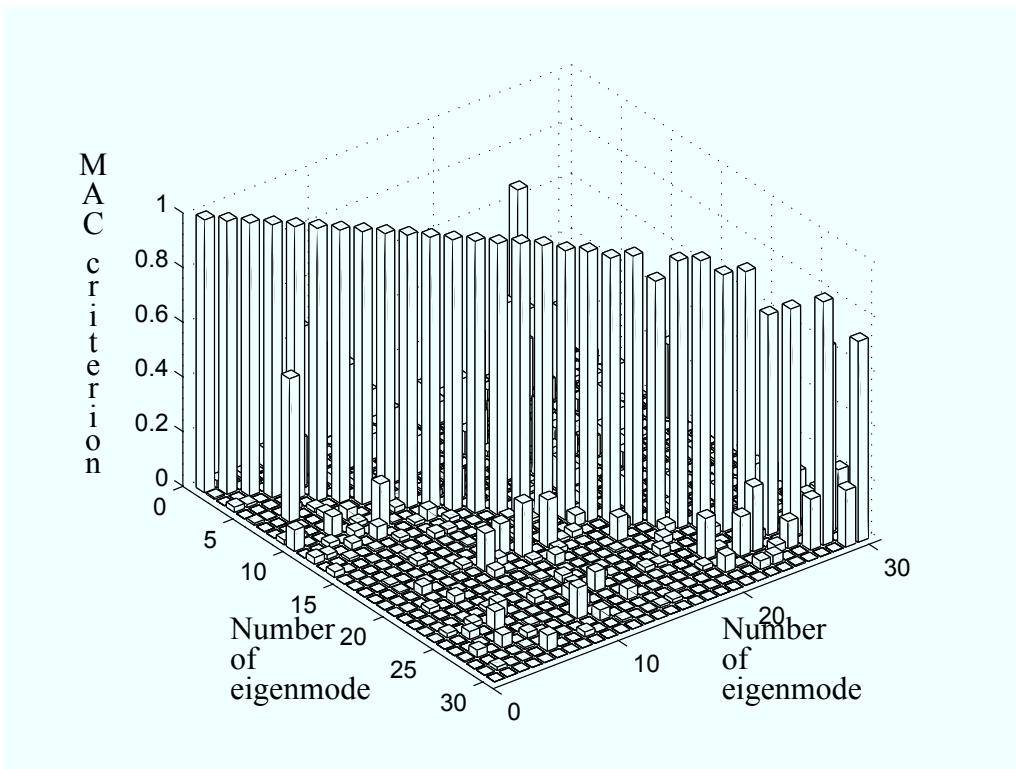


Fig. 8. The MAC criterion for a poor selection of master and slave degrees.

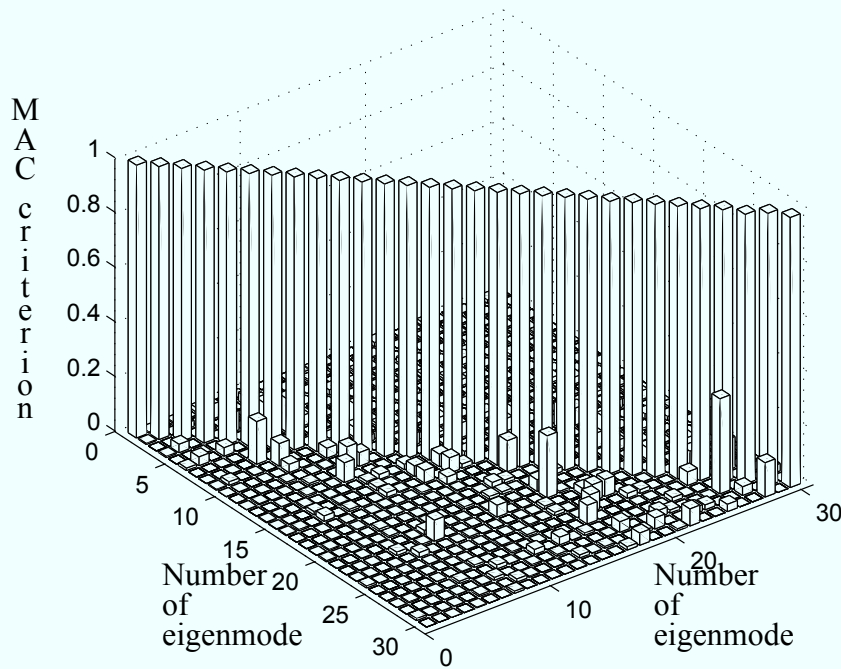


Fig. 9. The MAC criterion for a good selection of master and slave degrees.

mor4ansys is in fig. 5. The speed of the method is not a constraining factor because the reduction is done for the optimization only once and then only the reduced matrices are used.

The selection of the master and slave degrees of freedom influences largely the final result, especially the eigenmodes. A MAC criterion (Modal Assurance Criterion) [1] shows large differences in fig. 8 and in fig. 9. The results of poor and convenient selections are in fig. 8 and fig. 9. The main diagonal must be equal one in case of perfect agreement and the other components must be equal zero.

### 3. Conclusions

The applications and usage of the reduced models are wide. They can be used for testing, real-time applications or optimizations. The main advantage is a very small resulting system with the almost identical behavior to the original one. The reduction is done only once, later only the reduced matrices are used and all following computations are very fast. Therefore, the reduction techniques provide a very powerful tool especially for the design and optimization.

This paper has summarized the properties of different reduction techniques and the development of several additional critical procedures that increase the reduction accuracy and efficiency significantly. These are the procedure for the selection of master–slave DOFs and the procedure for the automatic connection of reduced flexible bodies. Based on that a powerful global optimization procedure (e.g. optimization of mechanical properties of machine-tool in its whole workspace).

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