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## DISTRIBUTION OF THE ELECTROMAGNETIC FIELD IN INHOMOGENEOUS STRIPLINE

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**Abstract:** In this paper investigation of influence of the inhomogeneous distribution of the electric conductivity on the distribution of current density and electric fields in stripline, ground plane, dielectric substrate is described. Moreover influence of the strip width and dielectric thickness on field parameters is examined.

**Keywords:** PCB stripline, nonhomogeneous material, finite element method.

### 1 Introduction

The increasing importance in fabrication and development of planar integrated circuits has renewed interest in the use of the open microstrip geometry as transmission line in planar microwave integrated circuits. This form of transmitting structure is useful in both a monolithic integrated circuits on a semiconductor substrate as well as in hybrid microwave circuits on ceramic substrate. The thickness of the metallic strips in the first case is between 1 and 3  $\mu\text{m}$  and in the second case between 5 and 20  $\mu\text{m}$  [1,2].

In burning process of the manufacturing of the dielectric substrate when burning temperature is not uniformly distributed electric conductivity  $\sigma$  can be special dependent. In this publication it is considered simplified situation when conductivity is dependent only from one coordinate, namely perpendicular to the substrate surface. Such problems were signalled in previous publications and analytically calculated, but under assumption that problem is one dimensional. In this article field equations are formulated for the case of inhomogeneous dielectric substrate and then solved by finite element method. Moreover influence of the strip width and dielectric thickness on field parameters is examined. At the end some illustrative example will be given where influence of different construction parameters on field distribution is explored [3,4].

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## 2 Main equations

Since the transfer components of the current density vector can be neglected compared with the longitudinal component for the case when  $w > \lambda_0$ , we assume that the current density vector in conductors has only a  $z$  component [2].

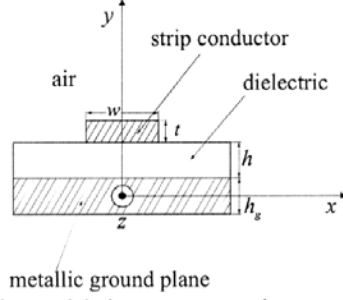


Fig.1. Stripline with its cross section and dimensions.

Maxwell's equations for the problem under discussion describe electric field in and around a strip line structure:

$$\frac{1}{\mu} \nabla \times \mathbf{E} = -\frac{\partial \mathbf{H}}{\partial t} \quad (1)$$

$$\nabla \times \mathbf{H} = \mathbf{J}_e + \sigma(y)\mathbf{E} + \varepsilon \frac{\partial \mathbf{E}}{\partial t} \quad (2)$$

where electric conductivity is assume to change linearly with  $y$  coordinate [3]

$$\sigma(y) = a_1 y + a_0 \quad (3)$$

and  $a_0$  and  $a_1$  are given material constants. After calculation of rotation of the first equation and substituting the second, we get:

$$\nabla \times \frac{1}{\mu} (\nabla \times \mathbf{E}) + \sigma(y) \frac{\partial \mathbf{E}}{\partial t} + \varepsilon \frac{\partial^2 \mathbf{E}}{\partial t^2} = -\frac{\partial \mathbf{J}_e}{\partial t} \quad (4)$$

Equation describing distribution of longitudinal component  $E_z$  both inside and outside the stripline structure can be derived from (3):

$$\nabla \times \frac{1}{\mu} (\nabla \times \mathbf{E}) + \sigma(y) \frac{\partial \mathbf{E}}{\partial t} + \varepsilon \frac{\partial^2 \mathbf{E}}{\partial t^2} = -\frac{\partial \mathbf{J}_e}{\partial t} \quad (5)$$

In complex domain both vectors can be written as [5]:

$$E_z = \text{Re} \left[ \underline{E}_z(x, y) e^{i\alpha z} \right] \quad J_{ez} = \text{Re} \left[ \underline{J}_{ez}(x, y) e^{i\alpha z} \right] \quad (6)$$

In complex domain we get following equations:

$$\begin{cases} \nabla_t \left( \frac{1}{\mu} \nabla_t E_{zr}(x, y) \right) + \omega \sigma(y) E_{zi}(x, y) = -\omega J_{ezi}(x, y) \\ \nabla_t \left( \frac{1}{\mu} \nabla_t E_{zi}(x, y) \right) - \omega \sigma(y) E_{zr}(x, y) = \omega J_{ezr}(x, y) \end{cases} \quad (7)$$

This set of equations was solved by finite element method in two dimensions.

### 3 An illustrative example

Analysed in this section conductor microstrip line is composed of three layers with following dimensions [4]:

- stripline line height  $t = 4 \mu\text{m}$ ,
- strip width  $w = 40 \mu\text{m}$ ,
- dielectric thickness  $h = 20 \mu\text{m}$ .
- finite width of a ground plane and dielectric  $w_{dg} = 200 \mu\text{m}$
- ground plane have conductivity  $\sigma(y) = 5.99 \cdot 10^7 (1 + 0.2 y/h) \text{ S/m}$
- dielectric permeability was  $\varepsilon = 5\varepsilon_0$
- excitation current  $I_z = 1 \text{ mA}$  ( $J_{ez} = 6.25 \cdot 10^6 \text{ A/m}^2$ )

Distribution of a current on the ground plane-dielectric surface for a frequency range from 1 GHz to 7 GHz is shown in figure 2. It is the image current induced in bottom metallic layer (and thus also on plane-dielectric surface) because it is assumed that in ground plane exciting current is not present. As can be seen also in figure 2, this current is mostly concentrated under the corresponding stripline. Because with frequency increasing, a line impedance is also increasing and the maximal value of this current decreases. But what is more important, it concentrates more under a strip and decreases more rapidly outside. This is of practical importance, because it allows on denser development of multi strip lines.

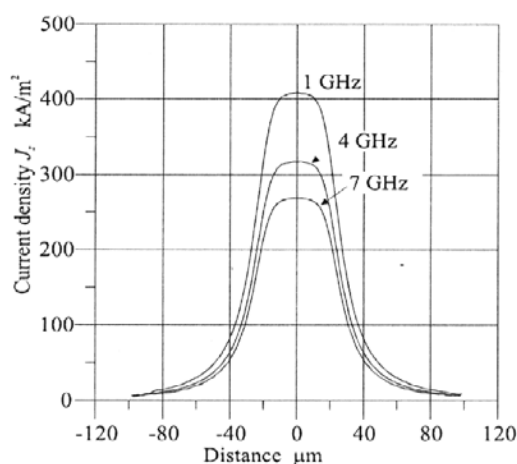


Fig.2. Current density on the boundary between ground plane and dielectric for different values of frequency

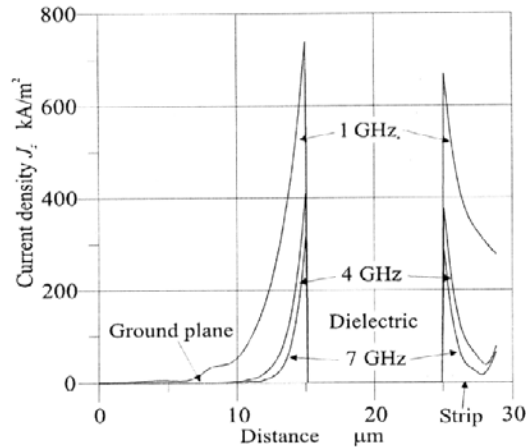


Fig. 3. Current distribution along  $y$  axis.

A total current consisting of exciting and conducting currents is distributed within a skin deep  $\delta$ . This in turn required generation of very dense finite element grid within a skin-deep region and caused that iteration processes were rather slow. The time of computations was increasing substantially with frequency. The only parameter in these simulations, which was subject to change, was frequency. In figure 3 three plots of current density along  $y$  axis are shown. We see that increasing of frequency causes decreasing skin deep and concentration of the current on dielectric-metal surfaces.

## Conclusions

Current density and electric field distribution properties are evaluated by means of numerical solution of field equations dependent on  $\mathbf{E}$  using finite element method. There exists easy way to extend a procedure of computation of electromagnetic waves in PCB stripline to the inhomogeneous case, when finite element method is used.

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