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EFFECTIVE IMPEDANCE OF A CONDUCTOR AS A FUNCTION OF ITS CROSS-SECTION AND FREQUENCY

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Abstract: Knowledge of the effective impedance of a conductor carrying harmonic current (particularly of high frequencies) is often very important for design of various devices. Although its frequency-dependent values for conductors of circular cross-section are known, not so much is known about this parameter for conductors of more general shapes. The paper presents numerically obtained relevant results for "cross-type" and "band-type" conductors for frequencies ranging from 50 Hz to 500 MHz.

Keywords: Skin effect, effective impedance, numerical analysis, finite element method.

1 Introduction

The skin effect is characterised by nonuniform distribution of the current density within the cross-section of the conductor. It brings about increase of its effective resistance and decrease of its internal inductance. For frequencies to approximately hundreds kHz the distribution of the current density is the less uniform the more complicated is the circumference of the conductor (its highest values can be found at various corners while values near smooth parts are usually much lower). For even higher frequencies (starting from MHz) the depth of penetration becomes very small so that the influence of the shape of the cross-section is practically negligible. The current is now transferred practically along its perimeter and its density depends almost only on its length.

This is of fundamental importance for construction of conductors for high frequency currents. The shape of their cross-section is characterised by higher circumferential length resulting in lower Joule losses and effective resistance. The paper representing a continuation of [1] deals with frequency-dependent impedance for selected, more complex shapes of the conductor.

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2 Formulation of the technical problem

It is necessary to investigate the frequency-dependent effective impedance of conductors in two basic arrangements:

- Compact conductors in the form of crosses or spiders, see Fig. 1, that are mechanically stiff, need only little place, but technology of their production based on cold drawing through diamond dies is expensive.

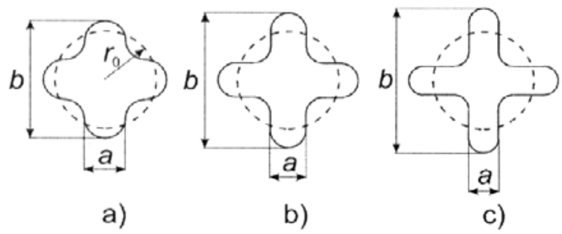

Fig. 1: "Cross-type" conductor
a) $a = 0.94$ mm, $b = 2.81$ mm, $l = 8.82$ mm
b) $a = 0.77$ mm, $b = 3.14$ mm, $l = 11.96$ mm
c) $a = 0.69$ mm, $b = 3.45$ mm, $l = 14.35$ mm

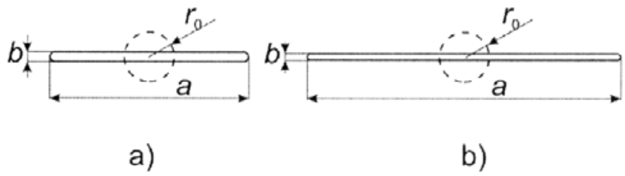

Fig. 2: "Band-type" conductor
a) $a = 9.21$ mm, $b = 0.46$ mm, $l = 19.86$ mm
b) $a = 14.51$ mm, $b = 0.29$ mm, $l = 29.93$ mm

Letter l denotes the length of the perimeter, the surface of all conductors is constant and corresponds to the circular conductor indicated by the dashed line with $r_0 = 1.155$ mm.

- "Band-type" conductors are indicated in Fig. 2. Technology of their production based on cold rolling is much cheaper, but their mechanical properties are substantially worse. They also usually need more place.

3 Mathematical model

Let us consider a long conductor carrying harmonic current I of frequency f. The conductor, whose material is supposed uniform, isotropic and linear (constant magnetic permeability μ_0 and electrical conductivity γ), has an arbitrary cross-section (Fig. 3) of area S_c.

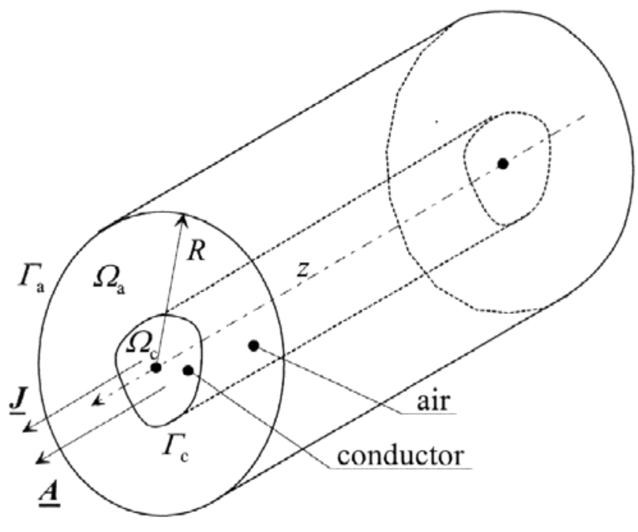

Fig. 3: The basic arrangement

The definition area of the problem (in 2D) is chosen with respect to disability of determining the boundary condition along the circumference Γ_c of the conductor. That is why the area of the conductor was surrounded by sufficiently large air cylinder (Ω_a) with radius R, along whose circumference Γ_a the field distribution may be estimated with relatively good accuracy.

As all electromagnetic quantities are harmonic, they may be expressed in terms of their phasors. Now the phasor of the current density $\underline{J}_z$ consists of

$$\underline{J}_z = \underline{J}_{pz} + \underline{J}_{ez}, \tag{1}$$

where $\underline{J}_{pz}$ is the potential component given as $\underline{I}/S_c$ and $\underline{J}_{ez}$ the eddy current component. Within the conductor (area Ω_c) distribution the phasor $\underline{A}_z$ of the vector potential is described by the Helmholtz equation [2]

$$\Delta \underline{A}_z - \mathrm{j} \cdot \omega\gamma\mu_0 \underline{A}_z = -\mu_0 \underline{J}_{pz} \tag{2}$$

while in the air (area Ω_a) by the Laplace equation

$$\Delta \underline{A}_z = 0. \tag{3}$$

The phasor of the eddy current density $\underline{J}_{ez}$ within the conductor is then expressed as

$$\underline{J}_{ez} = -\mathrm{j} \cdot \omega\gamma \underline{A}_z. \tag{4}$$

The boundary conditions may be expressed as follows:

- Boundary Γ_a: $\underline{A}_z = \text{const.}$
- Boundary Γ_c: continuity of both $\underline{A}_z$ and its normal derivatives $\dfrac{\partial \underline{A}_z}{\partial n}$.

After solution of system (2) and (3) it is possible to determine:

- The effective resistance R'_{eff} per unit length

$$R'_{\text{eff}} = \frac{\int_{\Omega_c} \underline{J}_z \cdot \underline{J}_z^* \cdot \mathrm{d}S}{\gamma \cdot I_{\text{eff}}^2}. \tag{5}$$

- The internal effective inductance L'_{eff} per unit length

$$L'_{\text{eff}} = \frac{\int_{\Omega_c} \left(\left(\frac{\partial \underline{A}_z}{\partial x} \right)^2 + \left(\frac{\partial \underline{A}_z}{\partial y} \right)^2 \right) \mathrm{d}S}{\mu_0 \cdot I_{\text{eff}}^2}. \tag{6}$$

The resultant effective impedance Z'_{eff} per unit length is given as

$$\underline{Z}'_{\text{eff}} = R'_{\text{eff}} + \mathrm{j} \cdot \omega L'_{\text{eff}}. \tag{7}$$

Computation of the described mathematical model was realised by professional code QuickField 5 [3] in combination with several single-purpose user procedures developed and written by the authors in Borland Delphi.

4 Illustrative example

Investigated was the dependence of the effective impedance per unit length $Z'_{\text{eff}} = R'_{\text{eff}} + \text{j} \cdot \omega L'_{\text{eff}}$ on frequency f within the range 50 Hz – 500 MHz for copper ($\gamma = 5.7 \cdot 10^7$ S/m)

- "cross-type" conductor (Fig. 1) and
- "band-type" conductor (Fig. 2).

The results are compared with the similar dependence for the reference copper conductor of the same circular cross-section ($r_0 = 1.155$ mm).

Fig. 4 shows the definition area for computation of the "cross-type" conductors. The letter R denotes radius of the artificial boundary Γ_a (the value was chosen such that energy of the magnetic field does not change more than by 5% while increasing R to $R + 0.1$ m).

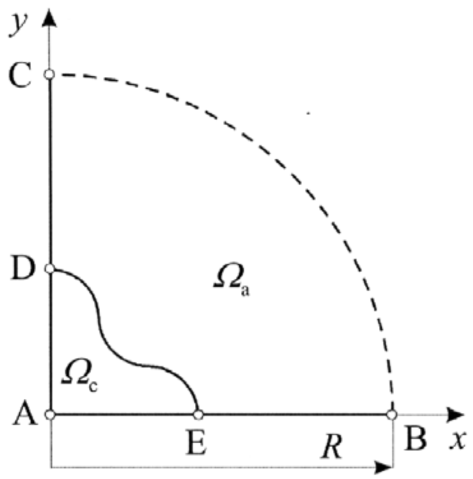

Fig. 4: Definition area of the conductor in Fig. 1a

Tab. 1 summarises all relevant results for "cross-type" conductors and Tab. 2 summarises the same results, but for "band-type" conductors.

Tab. 1: Convergence of the value of the magnetic field energy for "cross-type" conductors

	type a)	type b)	type c)
R (m)	W_m (J)	W_m (J)	W_m (J)
0,3	1,212E-06	1,215E-06	1,197E-06
0,5	1,324E-06	1,327E-06	1,310E-06
0,6	1,326E-06	1,367E-06	1,350E-06
0,7	1,398E-06	1,401E-06	1,384E-06

Tab. 2: Convergence of the value of the magnetic field energy for "band-type" and circular conductors

	type a)	type b)	circular
R (m)	W_m (J)	W_m (J)	W_m (J)
0,3	1,064E-06	9,713E-07	1,299E-06
0,5	1,176E-06	1,084E-06	1,362E-06
0,6	1,216E-06	1,124E-06	1,402E-06
0,7	1,250E-06	1,158E-06	1,436E-06

Figs. 5 and 6 show distribution of the phasor $\underline{Z}$ of impedance in dependence on frequency.

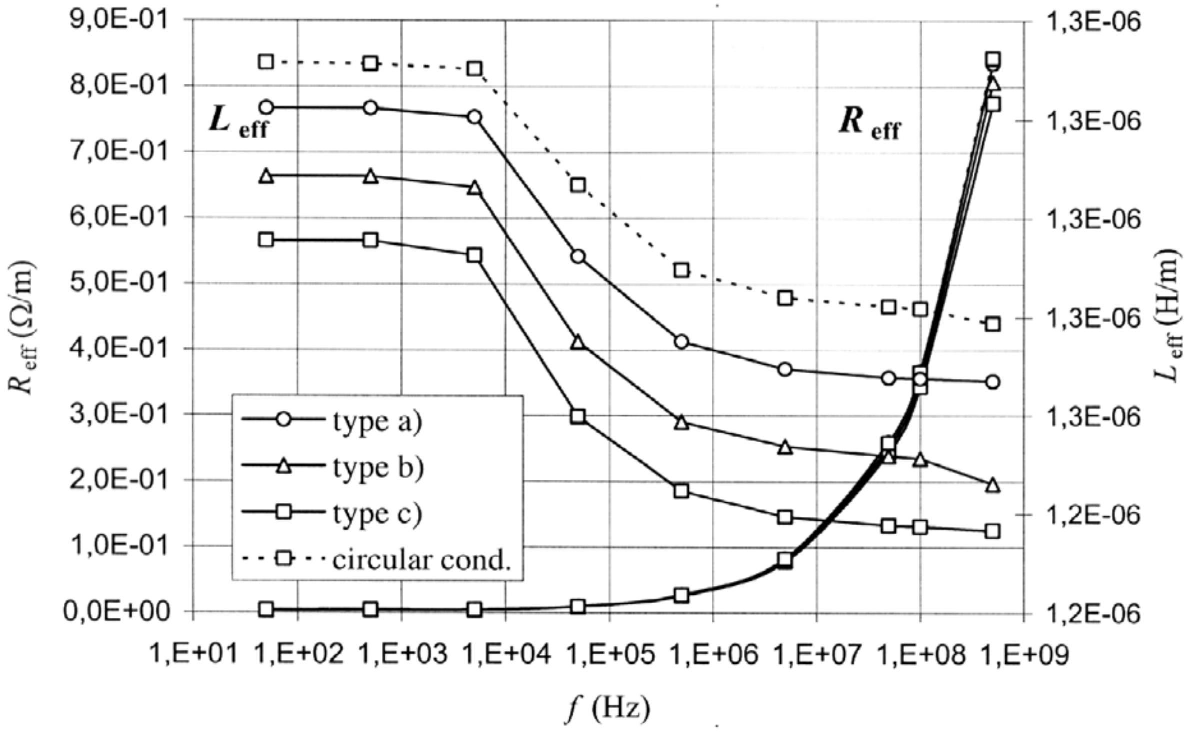

Fig. 5: Frequency dependence of the resistance and inductance for the "cross-type" conductor

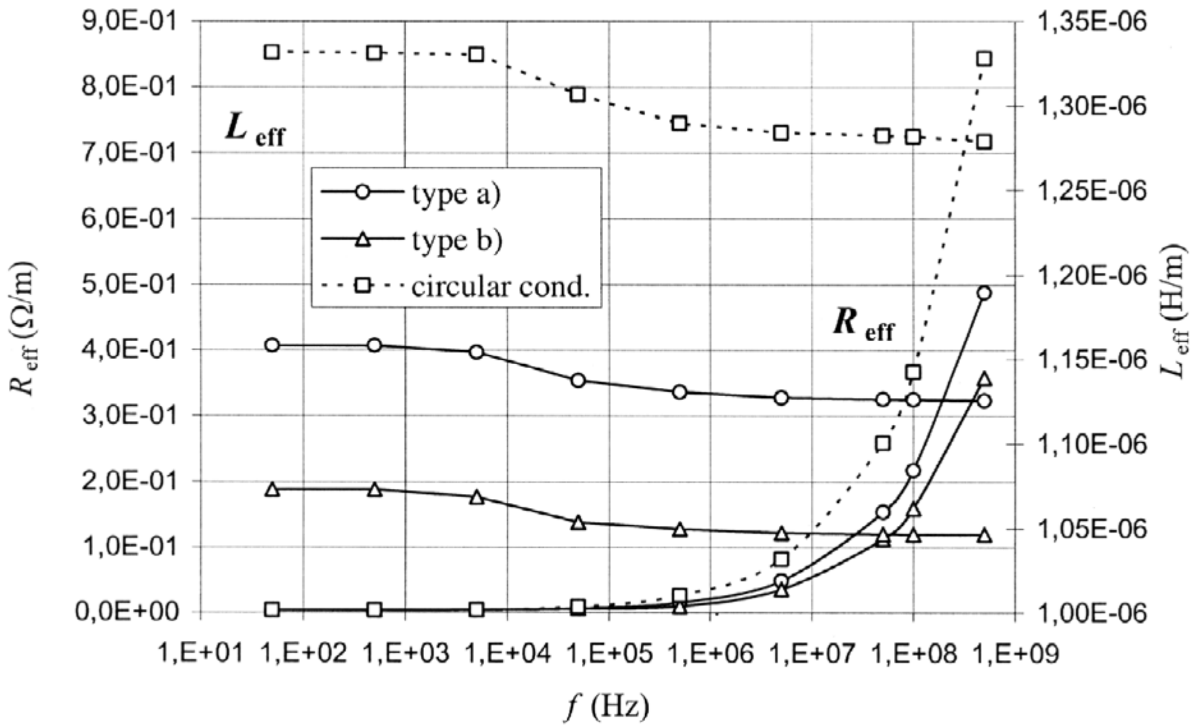

Fig. 6: Frequency dependence of the resistance and inductance for the "band-type" conductor

Both indicated characteristics (Fig. 5, Fig. 6) are compared with analogous characteristics of the cylindrical conductor of circular cross–section.

5 Conclusion

Numerical computation of the task took a long time because of necessity to generate a mesh with an adequate density, consisting of more than 10^5 nodes. Otherwise, it wouldn't be possible to guarantee the convergence of results. Fig. 7 shows the distribution of phasor of the impedance in the dependence on frequency.

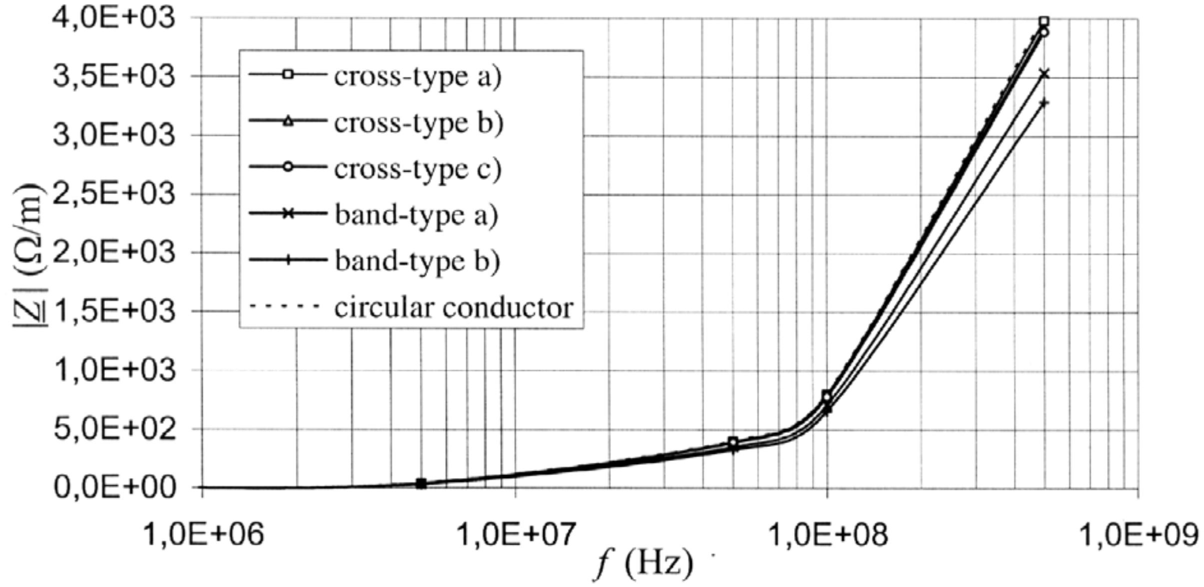

Fig. 7: Frequency dependence of the module of the impedance

For frequencies up to 1 MHz, the values of impedance of profile conductors are comparable with impedance of the reference conductor with circular cross-section. For higher frequencies, however, these impedances decrease.

6 Acknowledgement

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References

[1] Benešová, Z., Doležel, I., Dvořák, P., Mayer, D., Ulrych, B.: Effective Resistance of a Conductor as a Function of the Shape of its Cross-Section and Frequency. Proc. IC SPETO'2003, Niedzyca, Poland, 2003, pp. 79–82.

[2] Chari, M. V. K., Salon, S. J.: Numerical Methods in Electromagnetism, Academic Press, San Diego, 2000.

[3] www.quickfield.com (February 2003).