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AN INTRODUCTION TO THE WAVELET TRANSFORM

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Abstract: The article deals with comparing the Wavelet Transform with the Fourier Transform. In the article is shown difference between the Fourier Transform and the Wavelet Transform in an example. The article describes continuous wavelet, as an introduction to the Wavelet Transform, followed by discrete version of the transform.

Keywords: Wavelet Transform, Fourier Transform, signals, distortion

1 Introduction

For an analysis of a signal the most well-known is the Fourier analysis, which breaks down a signal into constituent sinusoids of different frequencies. The Fourier analysis has a serious drawback. When a signal has been transformed by Fourier Transform, then time information is lost. When looking at a Fourier Transform of a signal is not possible to say when a particular event happened. As the signal contains numerous non-stationary or transitory characteristics as drift, trends, abrupt changes etc. the Fourier Transform is not very sufficient for analysis of this signal. In 1946 Dennis Gabor improved the Fourier Transform. The point of this improved analysis is to analyse only a small section of a signal at a time; this technique is called *windowing* the signal today. This improved process, called the *Short-Time Fourier Transform (STFT)*, maps a signal into a two-dimensional function of time and frequency. The drawback of this method is that once you chose a particular size for the time window, you have to work with the same window all the time for all frequencies. We need a more flexible approach-one where we can vary the window size to determine more accurately either time or frequency. Wavelet

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analysis allows the use of long intervals where we want more precise low frequency information, and shorter regions where we want high frequency information.

2 Fourier Analysis

Fourier analysis converts time domain waveform into their frequency components and vice-versa. When the analysed waveform is periodical, the Fourier series can be used to calculate the magnitudes and phases of the fundamental and its harmonic components.

The Fourier series a periodic function *x(t)* is defined by:

$$x(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos(\frac{2\pi nt}{T}) + b_n \sin(\frac{2\pi nt}{T}))$$

In this equation, a_0 is the average value of the function *x(t);* a_n and b_n are the rectangular components of the *n*th harmonic.

The Fourier transformation is convenient for slow changing signals. It is not possible to determine the point where the signal has been changed (spikes, noise, notches etc.) That means the Fourier Transform is not convenient for an analysis of fast changing signals.

3 Wavelet Transform

By using the Fourier Transform to a signal is impossible to tell when a particular event happened. This is a serious drawback of Fourier Transform. For stationary signals (a signal that does not change so much) the Fourier transform is convenient, but for transitory or non-stationary characteristics (drift, trends, abrupt changes, e.t.c) the Fourier transform is unsuitable. That is the reason why the wavelet transform is implemented into praxis more frequently then ever.

The wavelet Transform provides a fast effective way of analysing non-stationary voltage and current transformation. Basically the wavelet Transform is comparing a disturbed signal with a mother wavelet function. The mother function can be defined by

$$g(t) = e^{-\alpha t^2} e^{j\omega t} \tag{3.1}$$

By changing the width, magnitude and time shifting of the mother wavelet a disturbed signal function is approximated. Subsequently, by using this analyse it is possible to obtain the frequency spectrum of the disturbed signal.

Relation 3.2 showed below represents any wavelets depending upon parameters (a),(b),(t). The parameter (a) in relation 3.2 represents the width of wavelet; the parameter (b) represents a shifting in time; the parameter (t) represents time.

$$g'(a,b,t) = \frac{1}{\sqrt{a}} g(\frac{t-b}{a}) \tag{3.2}$$

3.1 Continuous Wavelet Transform

The wavelet Transform of a continuous signal *x(t)* is defined by

$$\mathrm{WT(a,b)}=\frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} f(t) g(\frac{t-b}{a}) dt \tag{3.3}$$

Notice that mother wavelet has $a = 1$, hence the wavelets with a<1 are contracted and wavelets with a>1 are expanded. Term $\frac{1}{\sqrt{a}}$ in equation 3.2 represents normalisation of energy, therefore all wavelets have the same energy.

In Figure 3.1 a shifted mother wavelet in time is shown. This figure is shown as an example, therefore the shifting very is rude. In general the shifting of wavelet in

time is very fine and mostly wavelets are surfaced. By using this process frequency spectrum of the distorted signal is obtained.

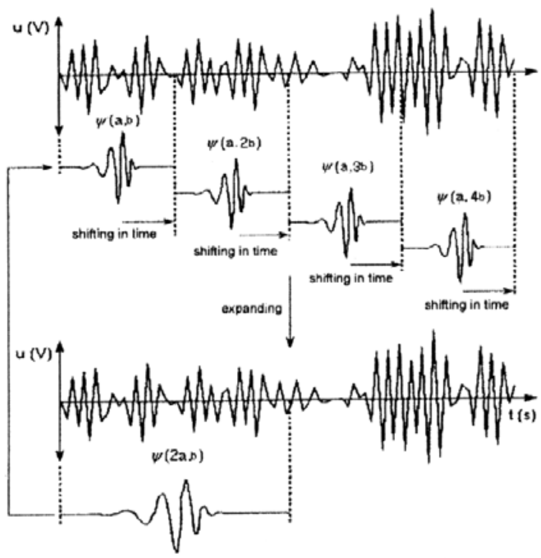

Figure 3.1 Process of calculating the frequency spectrum in the distorted signal

When parameter (a) is large (expanded wavelet) then it is convenient to analyse slow changing signals, therefore low frequencies. When (a) is small (contracted wavelet) then it is convenient to analyse transient states, therefore high frequencies.

3.2 Discrete Wavelet Transform

The calculating of continuous wavelet Transform is difficult for all scales and shifting in time to do. Therefore, shifting in time and scales are based on powers of two : so-called *dyadic* scales and positions. In general the term 3.2 becomes

$$g^{'}(m,n,t) = \frac{1}{\sqrt{a_0^m}} g(\frac{t - nb_0 a_0^m}{a_0^m}) \tag{3.4}$$

$$g^{'}(m,n,t) = \frac{1}{\sqrt{a_0^m}} g(ta_0^{-m} - nb_0) \tag{3.5}$$

where m,n define step in scale and translation; a_0, b_0 are the segmantion step sizes for the scale and translation.

Hence, the discrete wavelet coefficients are given by

$$DWT(m,n) = \int_{-\infty}^{\infty} \frac{1}{\sqrt{a_0^m}} f(t) g(a_0^{-m} t - nb_0) dt \tag{3.6}$$

The wavelets' representation is discrete, because transformation is made in steps, which are given as different combinations of m and n. Basically, *DWT* is implemented using shunt connected high-and low-pass filters as shown in Figure 3.2. The signal $x[n]$ is decomposed (filtered) into small segments of frequency spectrum and subsequently analysed. The down-sampling by half is made due to modification of the number of samples. The filtering is shown is implemented in Figure 3.2 ; in this figure the reconstruction of the decomposed signal is implemented.

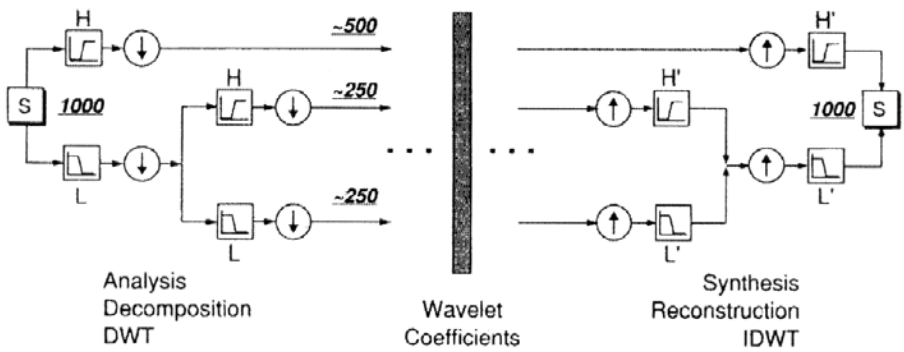

Figure 3.2 Decomposing and reconstruction of signal S

Let us consider a noised sine wave as signal S. Figure 3.2 shows how it works. You may observe that actual lengths of the detail and approximation data are slightly more than half the length of the original signal; this is caused by filtering process, which is implemented by convolving the signal with a filter. The convolution 'smears' the signal, introducing several extra samples into results.

3.3 An Example of Using Wavelet Transform

Figure 3.3 shows a disturbed signal about time 30ms. To remove the noise present in the waveform the Squared Wavelet Transform coefficients (SWTC) at scales $m=1,2,3$ and 4 are used. The signal contains a rapid oscillation disturbance before 30ms and is followed by a slow oscillation disturbance after a time of 30ms.

The SWTCs at scales 1,2 and 3 catch the rapid oscillations. TheSWTCs at Scale 4 catches the slow oscillations, which occurred after a time of 30ms.

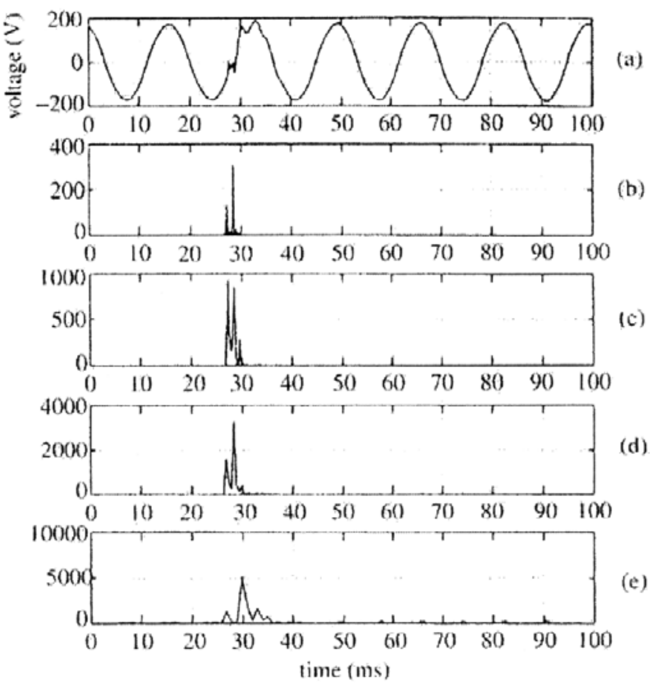

Figure 3.3 Wavelet disturbance detection using Daub4: (a) the voltage disturbance signal; (b),(c), (d) and (e) the SWTCs at scales 1.2.3 and 4. Respectively Copyright ©1993 IEEE

4 Conclusion

It has been shown that the Wavelet Transform is appropriate to detect transients, which can be contained in an analysed signal. The drawback of the Wavelet Transform is in the difficulty of interpreting the results of Wavelet Transform. In addition, the Wavelet Transform take a lot of time to calculate wavelet coeficients.

The Fourier Transform is a faster analysis than the Wavelet analysis, but it is lost information when a particular event has taken place. It has been shown as an improved Fourier analysis, called windowing, but this has unlimited frequency and time resolution; that is the reason why the Fourier analysis is more sufficient to use for signals which do not contain any transients.The last paragraph is focused on an example, which shows the advantage of the Wavelet Transform. In this example it is shown that, by using the Wavelet Transform it is possible to detect the transients.

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