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MECHANICAL FORCES ANALYSIS IN ELECTROSTATICAL FIELD OF THE PLATE CAPACITOR

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Abstract: The paper deals with the origin of mechanical forces acting on the boundary of different dielectric materials in the electrostatic field of a plate capacitor. Boundary effects of the plate capacitor field cause the force dragging the dielectric plate into the air capacitor.

Keywords: Forces at the boundary of two dielectrics, plate capacitor, electrostatic field

1 Introduction

It's known, that in an electrostatic field between two dielectrics a force acts. The force value per unit surface area (pressure) equals the difference of the electrostatic field energy density in both materials. It causes (apart from other effects) the drugging of the dielectric into the plate air capacitor.

There was a good question of a student that initiated this work: "*How does the force at the boundary between the dielectric and the air develop, if the capacitor electric field is normal to the force direction? In the electrostatic field there must be force in the direction of the field* **E**." The situation is showed in Fig. 2, where the force **F** is normal to the field **E**, indeed.

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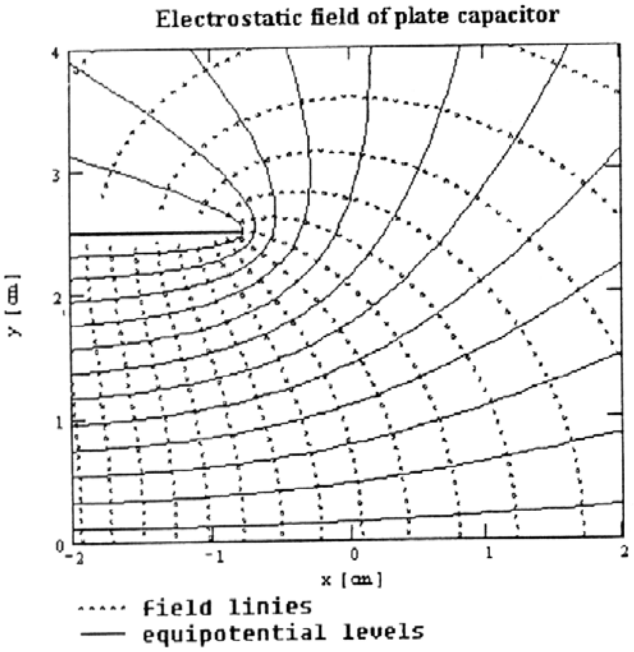

Fig. 1

This paper should answer the question. At the very beginning it is useful to remind that the forces at the boundary of two dielectrics are usually described by thermodynamic principles (law of energy conservation). The thermodynamic method is universal and correct. In this paper we analyse one particular case – the plate capacitor. The conclusions are in accordance with the thermodynamics.

2 Theoretical background

Let us analyse the field of a plate capacitor (without inserted dielectric plate). At quite a long distance ahead of the capacitor edge the field is homogeneous, near the edge there the field is deformed. The commonly known conformal mapping describes the field of an ideal plate capacitor:

$$z = A.(exp(a.w) + aw) \qquad (1)$$

$$z = x + j.y \qquad (2)$$

$$w(x,y) = u(x,y) + j.v(x,y) \qquad (3)$$

where constants A, a must be chosen in accordance with the boundary conditions, x, y are the Cartesian coordinates. The lines of constant value u (v, respectively) represent the field lines (equipotential levels). The field map is shown in Fig. 1.

Because of the symmetry only the upper half of the capacitor edge is shown. Formulas (1) to (3) and the force-line refraction law provide the field analysis at the inserted dielectric plate (Fig. 2). The electric field intensity at the capacitor edge has a vertical and horizontal component as well. The vertical component (normal to dielectric surface) affects the surface charge density of the coupled charge on the dielectric surface (due the polarisation). This charge is found in the field, which has a tangential component as well. Herewith the electrostatic force acting on the charge in the direction equal to resultant tangential mechanical force develops.

$$F_{teor} = \frac{1}{2}\varepsilon_0(\varepsilon_r - 1)E_0{}^2 d\,.1\mathrm{m} \qquad (4)$$

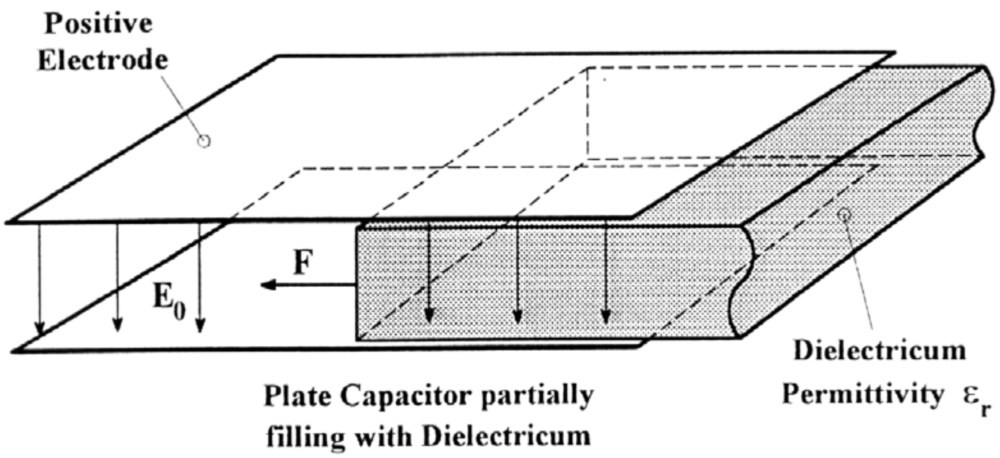

Fig. 2

The result of the capacitor-fringing field are showed in Fig. 3, 4. In Fig. 3, there is a dependence of the tangential field component on x coordinate.

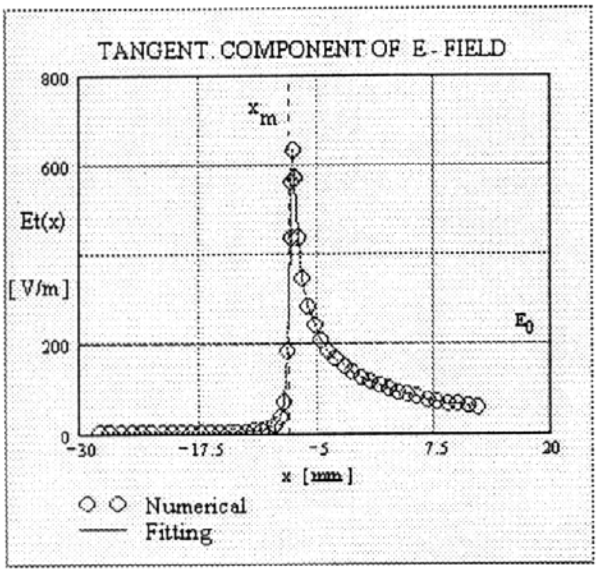

Fig. 3

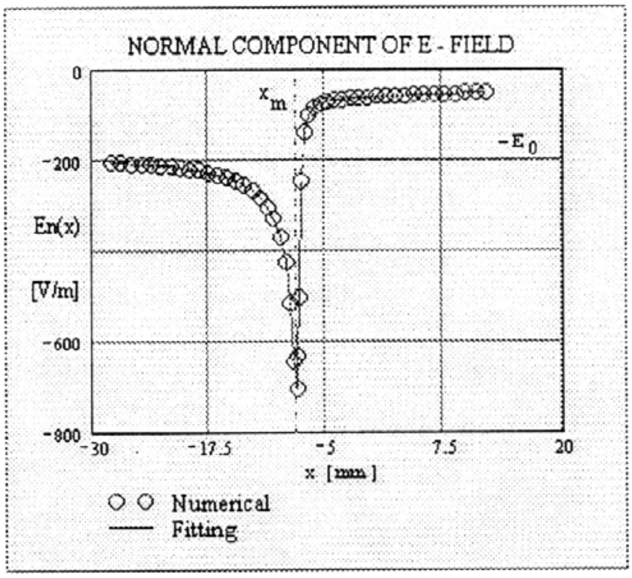

Fig. 4

In Fig. 4, there is the same dependence of the normal field component. The mechanical force is calculated numerical according formula (5):

$$F_{num} = 2\,\varepsilon_0\left(\varepsilon_r - 1\right)\int_{x_{min}}^{x_{max}} E_{nor}(x).E_{\tan}(x)dx \qquad (5)$$

We choose the integration limits symmetric to the capacitor edge. The capacitor edge is in Fig. 3 and 4 marked with a dashed line.

The results obtained by this procedure differ to a certain extent from the ideal value. The disproportion is given by the enumeration errors, because we evaluate a formula values of which near the capacitor edge change significantly. The number of points in which we calculate the field and the manner of the continuous approximation influence the result as well.

3 Conclusion

The mechanical forces at the boundary of two dielectrics are caused by the field forces.

In the analysed particular case the coupled charge density at the surface of the dielectric is due to the tangent field component at the capacitor edge. The field, that has the tangential component as well, acts on the coupled charge. The resultant tangential force is given by the by integration of the force acting on the surface differential element. The numeric result conforms the theory within the scope of the computing error. The ratio of numerically computed force and the theoretical value is within 70 – 120 % (in dependency on the parameters choice). The inaccuracy is caused by the numerical field computing method and by the numerical force integrating.

In the case of infinitely large capacitor and the dielectric plate of a finite length, there does not exist a force acting in the tangential direction.

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