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ESTIMATION OF PARAMETERS OF FERROMAGNETIC OBJECTS

DR. ING. MIROSŁAW WOŁOSZYN

Abstract: *Ferromagnetic object placed in the earthly magnetic field causes additional disturbance of the distribution of this field all around itself. The determination of the position and the value of the extremes of the magnetic field of the object allows to calculate the approximate localization and identification of the object. This paper presents the results of analysis of the position and the value of the extremes of the magnetic field and new method of localization and calculation of magnetic moment of the object. The numerical calculations have been made on the basis of MatLab.*

Key words: *localization, identification, ferromagnetic object, magnetic moment*

INTRODUCTION

A ferromagnetic object (e.g. mine, missile) placed in the earthly magnetic field undergoes magnetization. In spite of the magnetizing with a homogeneous filed the object is subject to inhomogeneous magnetization. Such an object undergoes so-called induced magnetization. It also has permanent magnetism. Both types of magnetization produce an object's own resultant field, which in certain positions weakens the earth's magnetic field, while in some cases it reinforces its value. In the case of a relatively large distance from the object (the distance three times bigger from the highest object's value) a sufficiently accurate tool for the analysis of an object's magnetic field is the dipole model [3, 4].

The resultant magnetic field in the object's surround is the superposition of the earth's fields and an object's own field (1):

$$\mathbf{B}_\mathrm{t} = \mathbf{B}_\mathrm{e} + \mathbf{B}_\mathrm{o} \tag{1}$$

where: $\mathbf{B}_\mathrm{e}$ – the Earth's magnetic flux density vector, $\mathbf{B}_\mathrm{o}$ – an object's magnetic flux density vector

The module of object's own magnetic field equals:

$$\left|\mathbf{B}_\mathrm{o}\right| = \left|\mathbf{B}_\mathrm{t}\right| - \left|\mathbf{B}_\mathrm{e}\right| \tag{2}$$

Expanding of the B_e function in the Taylor series and assuming single components (B_x, B_y, B_z) of the object's own magnetic field, as increments of the values of components (B_{xe}, B_{ye}, B_{ze}) of the $\mathbf{B_e}$ vector, we get magnetic field of the object of the approximate value:

$$\mathrm{B_o} \approx \frac{B_{xe}}{B_e}\cdot B_x + \frac{B_{ye}}{B_e}\cdot B_y + \frac{B_{ze}}{B_e}\cdot B_z \tag{3}$$

where: B_{xe}, B_{ye}, B_{ze} – components of the Earth magnetic field, B_x, B_y, B_z – components of the object's magnetic field.

Assuming Cartesian coordinate system with x axis paraller to B_{xe} (x axis paraller to N-S magnetic direction) the formula (3) changes into:

$$\mathrm{B_o} \approx B_x \cdot \cos(\mathrm{I}) - B_z \cdot \sin(\mathrm{I}) \tag{4}$$

1 EXTREME VALUES OF THE FIELD

In the analysis of object's magnetic field an arbitrary direction of the magnetic moment vector and the constant value of the module M_o were assumed (fig.1). Ferromagnetic object's magnetic field for the θ angle of the approximate value of 0° and 180° has one of two significant extrema. Object's magnetic flux density module is the non-linear function of 6 variables. In this paper the analysis of the extreme values of the magnetic flux density module and their location was performed on the basis of numerical calculations, for the two variables φ and θ, on the xy plane with the depth z above the object and with the magnetic inclination I = 70°(magnetic inclination in Poland). Ferromagnetic object's magnetic field for the inclination I=70° has two significant extreme values: the positive and negative extrema. For the low values of θ angle the location of the negative extrema (x_{min},y_{min}) is relatively close to the location of the object (x_o,y_o). The same property can be observed for the angle of approximate 180°, but refers to the positive extrema. The results of the analysis of the ferromagnetic object's magnetic field reveal an interesting ratio of the location of the extreme values to the angle φ. The angle between the axis reaching the extreme values and the magnetic direction N-S approximately equals φ angle.

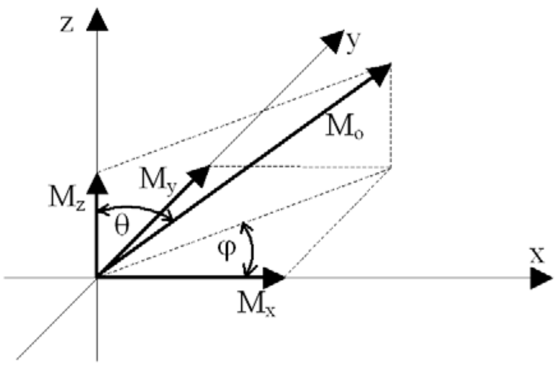

Fig.1 The components of the magnetic moment vector with the constant value of M_o

Fig. 2 shows the relation of the maximum (5) and minimum (6) ratio of the value of the negative extrema B_{min} to the value of the positive extrema B_{max} in the function of the θ angle (7).

$$\beta_{max} = \left.\frac{B_{min}}{B_{max}}\right|_{\varphi=0^\circ} \tag{5}$$

$$\beta_{min} = \left.\frac{B_{min}}{B_{max}}\right|_{\varphi=180^\circ} \tag{6}$$

$$\beta = \left.\frac{B_{min}}{B_{max}}\right| \tag{7}$$

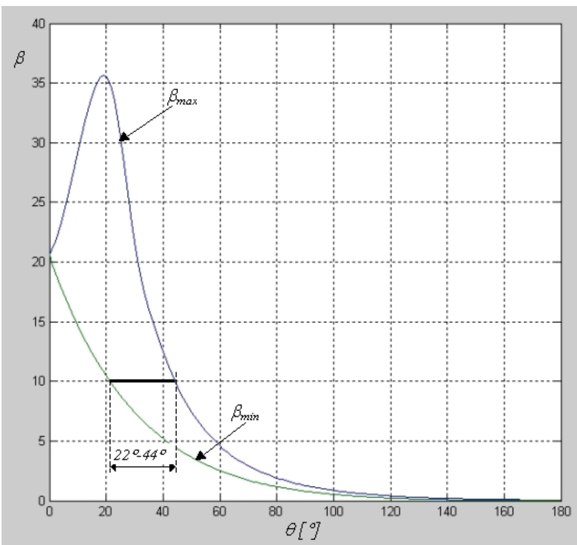

Fig.2 Graph of the β coefficient (9) in the function of θ angle

What can be concluded from the graph (fig.2) is that for the high values of β the range of the possible values of the θ angle is between 0° and 30°. The lower value of the β coefficient results in the reduction of the range of possible values of the θ angle.

2 LOCALIZATION OBJECT'S

On the basis of the ratios derived from the analysis of the magnetic field, with the use of the least square method the formulae for estimating object's location (x_o, y_o, z_o) have been specified. It is a new method of the localization of an object, which is similar but easier than the one presented in the paper [5], in which a simplified assumption ($B_o \approx B_{oz}$) was implemented. The approximate location z_o of the object [speto] equals (8). The relative error of the estimation z_o is below 5%.

$$z_o = r \cdot f(\beta) \tag{8}$$

where:

$$f(\beta) = \frac{\beta}{-0.679 + 1.23 \cdot \beta + 0.017 \cdot \beta^2} \tag{9}$$

$$r = \sqrt{\Delta x^2 + \Delta y^2} \tag{10}$$

$$\begin{aligned} \Delta x &= |x_{min} - x_{max}| \\ \Delta y &= |y_{min} - y_{max}| \end{aligned} \tag{11}$$

The approximate location x_o and y_o of the object equals (12,13):

$$x_o = \left| \begin{array}{ll} x_{min} - \Delta x \cdot g(\beta) & \beta \geq 10 \\ x_{min} - \Delta x \cdot g(\beta) + 0.1 \cdot |\Delta y| \cdot |\sin(\varphi_e)| & 1 \leq \beta < 10 \\ x_{max} + \Delta x \cdot g(\beta^{-1}) + 0.1 \cdot |-\Delta y| \cdot |\sin(\varphi_e)| & 0.1 < \beta < 1 \\ x_{max} + \Delta x \cdot g(\beta^{-1}) & \beta < 0.1 \end{array} \right. \tag{12}$$

$$y_o = \left| \begin{array}{ll} y_{min} - \Delta y \cdot g(\beta) & \beta \geq 1 \\ y_{max} + \Delta y \cdot g(\beta^{-1}) & \beta < 1 \end{array} \right. \tag{13}$$

where:

$$g(\beta) = g_{max}(\beta) - (g_{max}(\beta) - g_{min}(\beta) \cdot \frac{\varphi_e}{\pi} \tag{14}$$

$$g_{max}(\beta) = \frac{1}{1.235 + 0.559 \cdot \beta + 0.002 \cdot \beta^2} \tag{15}$$

$$g_{min}(\beta) = \frac{1}{1.422 + 0.902 \cdot \beta + 0.02 \cdot \beta^2} \tag{16}$$

Relative errors δ_x (17) and δ_y (18) of the x_o and y_o estimation are below 10%.

$$\delta_x = \frac{\Delta x}{z} \cdot 100\% \tag{17}$$

$$\delta_y = \frac{\Delta y}{z} \cdot 100\% \tag{18}$$

3 Estimation of magnetic moment vector

The value of the β coefficient enables determination of the range of the value of the θ angle. The average value of this range can be chosen for estimation of the θ_e angle. The angle between the axis reaching the extreme values and the magnetic direction N-S approximately equals φ angle, so this angle can be estimated from formulae:

$$\varphi_e = \mathrm{arctg}(\frac{\Delta y}{\Delta x}) \tag{19}$$

The magnetic moment can be calculated using the greater extremum from the formula:

$$M_e = \frac{r_i^5 B_i}{100 \cdot (A_x \cos(I) - A_z \sin(I))} [Am^2] \tag{20}$$

where: $B_i = B_{max}$ or B_{min} [nT] and

$$A_x = (2x^2 - y^2 - z^2)\sin(\theta_e)\cos(\phi_e) + \\ + 3xy\sin(\theta_e)\sin(\phi_e) + 3xz\cos(\theta_e) \tag{21}$$

$$A_z = 3xz\sin(\theta_e)\cos(\phi_e) + 3yz\sin(\theta_e)\sin(\phi_e) + \\ + (2z^2 - x^2 - y^2)\cos(\theta_e) \tag{22}$$

$$r = \sqrt{x^2 + y^2 + z_o^2} \tag{23}$$

$$x = x_{ekstr} - x_o \tag{24}$$

$$y = y_{ekstr} - y_o \tag{25}$$

4 Measuring method

The magnetic field can be measured using a vector (SQUID, fluxgate) magnetometer and a scalar (proton, optically pumped) magnetometer. The scalar magnetometer measures the modulus of the magnetic flux density vector. The scalar magnetometers are independent of the variations of the position of the mobile platform regarding to Earth's magnetic field vector. The sensitivity of a modern optically pumped magnetometer is 1pT. Such high sensitivity can be applied only with two magnetometers, which work in differencial configuration. One of them works as the base station magnetometer and second as the measure magnetometer. The position of the measure magnetometer system is determined with GPS.

5 Examples of estimation

The distribution of the magnetic fields of three ferromagnetic objects are presented in fig.3. After the selection of the distribution of the magnetic field of the chosen object (fig.3) it is possible to localize and calculate magnetic moment vector. On the table tab.1the parameters of the three objects are presented.

Object	xyz [m]	M[Am²] θ [°] ϕ [°]	B_{min} B_{max} [nT]	x_{min} x_{max} [m]	y_{min} y_{max} [m]
I	-9.5 11.9 6.2	17.8 110 30	-4.59 8.22	-6.50 12.00	13.75 10.75
II	5.5 -10.75 5.3	8.45 125 140	-12.89 4.38	12.00 9.50	13.00 14.50
III	11.5 13.5 2.9	2.5 70 -30	-1.63 8.37	1.25 6.00	-7.50 -11.75

Tab. 1: Parameters of the three objects

Object	xyz [m]	M[Am²] θ [°] ϕ [°]
I	-9.96 11.78 6.27	18.87 110 30
II	5.24 -10.81 5.39	8.27 140 140
III	11.43 13.40 2.78	2.1 70 -30

Tab. 2: Estimated parameters of the three objects

In the table tab.2 the calculated parameters of the objects are presented. The error of the localization is less than 10% and identification of magnetic moment vector is less than 20%.

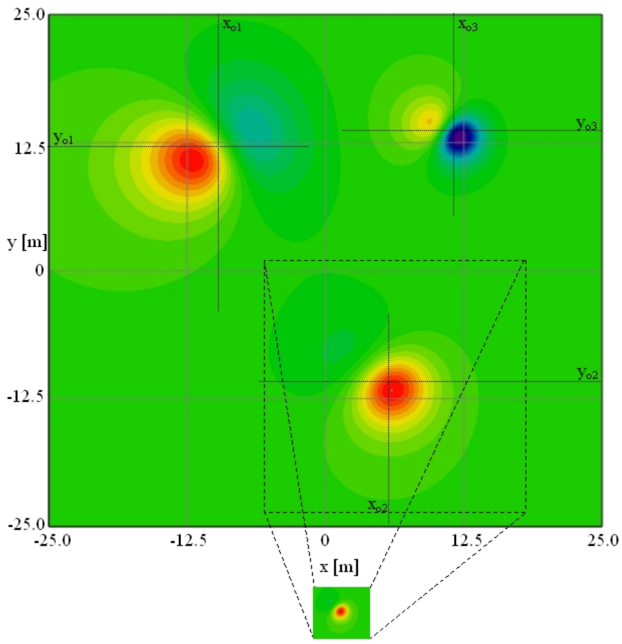

Fig.3 Distribution of magnetic field of the three ferromagnetic objects

6 SUMMARY

Characteristic features of the extreme values of the magnetic field enable approximate localization and identification of the magnetic moment vector of the ferromagnetic object. The new method of localization of an object presented in the paper enables estimation of the object's location with the error below 10 % and moment magnetic vector with the error below 20 %.

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Mirosław Wołoszyn, dr ing., Faculty Electrical and Control Engineering, Gdańsk University of Technology, G.Narutowicza 11/12, 80-952 Gdańsk, Poland
email: mwolosz@ely.pg.gda.pl