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EQUIVALENT CIRCUIT OF HALL EFFECT GENERATOR

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Abstract: *Now Hall Effect devices are used in many technical applications, but the full effect description is only in special literature. In the paper the microscopic theory based on classical electron gas model is presented and linear equivalent circuit of Hall generator is derived from the model. Extensive numerical simulation of practical devices results in a full set of device parameters and a lot of characteristics. Numerical results reveal that Hall generator is a soft source of low voltage and produces very small output current and power. It follows from the characteristics that driving constant current source in not necessary at standard conditions and can be replaced by a more convenient constant voltage source. It is shown that the heat production in probe of small size limits the value of electrical parameters and is the main problem in applications. Presented graphs show that the parameters of Hall probe depend strongly on semiconductor impurity doping.*

Key words: *Hall Effect, Electron gas theory, Linear equivalent circuit, Hall Effect probe, Semiconductor parameters*

INTRODUCTION

Hall Effect, discovered more than one hundred years ago [1], finds at last twenty years a lot of practical applications. All of them are based on the measurement of magnetic field. At present time, Hall probes appear as the best instrument for measurement of technical magnetic field. Other applications, especially in sensors, are derived from Hall probes. List of typical practical application is in Ref. [2], for instance.

Irrespective of important practical use, Hall Effect theory is not a subject of typical textbooks. In specialised books only basic theory is outlined [3]. On the other hand in the design of Hall Effect devices more exact circuit theory should be very useful, at least for their analysis and optimum design. To our knowledge such simple practical theory is not available at typical technical literature. Therefore, the subject of this paper is to give a practical theory for better description of Hall devices using simple circuit theory.

In the theoretical part basic linear equations are derived from elementary microscopic model of charge carriers in conductors and semiconductors. Then the equations are transformed to integral ones and basic linear circuit equations for Hall probes are shown and their simple equivalent circuit derived. As there was not a possibility of experimental verification, this model was simulated on a sample with parameters near to those that are in practical use. Typical results are shown both in numerical and graphical forms and their practical consequences are discussed.

1 THEORY

Hall Effect is due to the action of magnetic field on moving current carriers in the conductor or semiconductor. Very simply theory is outlined first. Let us suppose that the magnetic field with flux density $\boldsymbol{B}$ is perpendicular to the drift velocity $\boldsymbol{v}$ of moving carriers of charge q. The direction of drift velocity is along the conductor axis. The magnetic force $\boldsymbol{F}_{\mathrm{m}}$ in both the vector and scalar form is given by formula

$$\mathbf{F_m} = q(\mathbf{v} \times \mathbf{B}) \qquad F_m = qvB \qquad (1)$$

The magnetic force is perpendicular to both the field and charge carrier velocity. Its effect is that the charge carriers have velocity component perpendicular to the drift one, Fig. 1. Charge carriers reach the conductor boundary and charge it. On the opposite conductor side there is a charge of the same value but of opposite sign. These charges create an electric field of strength $\boldsymbol{E}_{\mathrm{h}}$, which is perpendicular to the drift velocity (and magnetic field). The electric field produces a force $\boldsymbol{F}_e$ to the carriers

$$\mathbf{F_e} = q\mathbf{E_h} \qquad F_e = qE_h \tag{2}$$

The electric force $\boldsymbol{F}_e$ is opposite to the magnetic one $\boldsymbol{F}_m$. In equilibrium the both forces are equal, which leads to the vector and scalar formula

$$q\mathbf{E_h} = q(\mathbf{v}\times\mathbf{B}) \qquad E_h = vB \tag{3}$$

Usually, the right hand side of formula (3) is presented as Hall Effect. The field E_h produces a measurable voltage, termed Hall voltage, between opposite sides of conductor.

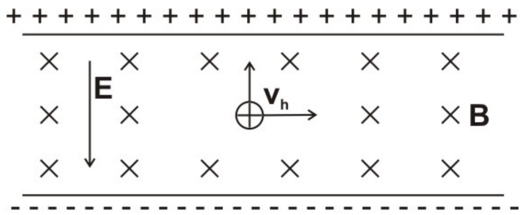

Fig.1: Principle of Hall Effect

More exact theory is based on the model of electron gas in metals. Charge carriers, or particles in general, are supposed to be ideal elastic spheres that are moving among ions in metal lattice. A lot of collisions between particles and lattice appear. The movement of particles is ideally chaotic. However, when external electric field is applied, particles are accelerated by the field in time between collisions and lose all its momentum during a collision. This process repeats again and again resulting in the mean drift velocity $\boldsymbol{v}$ of the particle in the direction of the field.

If we suppose that particle carrier lose all its momentum, the momentum conservation law for a collision can be written in the form

$$\Delta\mathbf{p} = 0 - m\mathbf{v} = \int_0^{t_c} \mathbf{F_c}(t)\mathrm{d}t \tag{4}$$

where m is particle mass, $\boldsymbol{v}$ its velocity, $\boldsymbol{F}_c$ collision force and t_c time of collision duration. The integral is the impulse of collision force. The momentum difference is from final state (zero momentum after the collision) to initial state, momentum $m\boldsymbol{v}$ before the collision. Since both the collision time and the time dependence of collision force are unknown, another approach is necessary.

Simple macroscopic explanation of momentum loses is possible by the application of friction force. Let constant friction force $\boldsymbol{F}_f$ acts on the particle during the time τ between subsequent collisions. The friction force ensures that the average particle momentum does not increase by the effect of electric field. The momentum conservation now requires that the impulse of friction force between collisions compensates the increase of particle momentum in the same time. Momentum conservation (4) has now form[1]

$$-m\mathbf{v} = \int_0^{t_c} \mathbf{F_f}\,\mathrm{d}t = \mathbf{F_f}\tau \quad \Rightarrow \mathbf{F_f} = \frac{m\mathbf{v}}{\tau} \tag{5}$$

The time τ is the particle mean free time between collisions. From the last equation the friction force can be given by particle parameters, see (5). Unfortunately, the mean time is not known. The equilibrium requires that friction force $\boldsymbol{F}_f = m\boldsymbol{v}/\tau$ in (5) has the same value as electric force $\boldsymbol{F}_e = q\boldsymbol{E}$ in (2), which leads to equation

$$\frac{m}{\tau}\mathbf{v} = q\mathbf{E} \tag{6}$$

For low values of electric field strength $\boldsymbol{E}$ the drift velocity $\boldsymbol{v}$ is proportional to $\boldsymbol{E}$

$$\mathbf{v} = \mu\mathbf{E} \tag{7}$$

where the proportional constant μ is the mobility. Its physical meaning is the charge carrier velocity in the field of unit strength. Important relation for carrier parameters follows from last two equations

$$\mu = \frac{q\tau}{m} \tag{8}$$

If the magnetic field of flux density $\boldsymbol{B}$ is present too, the equation of motion (5) has the general form

$$\frac{m}{\tau}\mathbf{v} = q\mathbf{E} + q(\mathbf{v}\times\mathbf{B}) \tag{9}$$

If we use energy conservation law, we can define means free path of charge carrier that is more convenient and used more frequently. Unfortunately, energy conservation law results in nonlinear formula, which is reason why it is not used.

Let us suppose that the magnetic filed is in the direction of axis Z in Fig. 2, $\boldsymbol{B} = (0, 0, B_z)$. The electric field responsible for drift has the direction of X axis, Fig. 2. The electric field due to the Hall Effect has the direction of Y axis. The electric field vector has components $\boldsymbol{E} = (E_x, E_y, 0)$. The same is valid for charge carrier velocity; the drift velocity is in the direction of X axis, while the Hall Effect velocity is along Y axis. The components of velocity are $\boldsymbol{v} = (v_x, v_y, 0)$. After the substitution of these vector components into vector equation (9) we will get two important component equations for all field and charge carrier velocities

[1] Strictly speaking the average momentum in the time of collision should be $2m\boldsymbol{v}$ in order to ensure average drift velocity $\boldsymbol{v}$. Since mean free time τ is not neither measurable nor calculable, the simplification is possible.

$$\frac{m}{\tau}v_x = qE_x + qv_yB_z$$
$$\frac{m}{\tau}v_y = -qE_y - qv_xB_z \tag{10}$$

The minus sign of the first term of the second equation in (10) respect the fact that the motion of charge carriers produces electric field. This charge carrier motion due to the external force produces the electromotive voltage. This part of sample is an electric source. If a load is connected to corresponding sides of the sample the direction of charge carrier velocity and electric field agrees in the load.

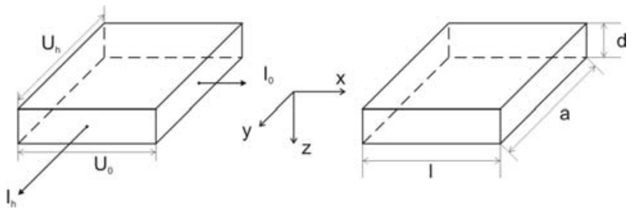

Fig.2: Circuit description of Hall Effect and sample geometry.

The application of circuit theory requires the use of macroscopic values as voltage, current, resistance etc. The transition between microscopic and macroscopic values is the Ohm's Law in differential form

$$\mathbf{i} = nq\mathbf{v} = nq\mu\mathbf{E} = \gamma\mathbf{E} \tag{11}$$

where n is the charge carrier concentration, q is charge, $\mathbf{v}$ is velocity, μ is mobility, see formula (7), and γ is the specific conductance (macroscopic parameter).

According to Fig. 2 we define geometrical parameters of the sample, length l, width a, and thickness d. There are two circuits in the sample, the driving or source circuit that will be denoted by index 0 and Hall or response circuit of index h. The sample cross sections of both the circuits are given by formula

$$S_0 = ad \qquad S_h = ld \tag{12}$$

The source and Hall voltages are given by simple formulae

$$U_0 = lE_x \qquad U_h = aE_y \tag{13}$$

The formulae

$$I_0 = S_0 i_x = S_0 nqv_x \quad I_h = S_h i_y = S_h nqv_y \tag{14}$$

are derived for source and Hall currents using differential Ohm's Law (11) and definitions (12).

Using the equations (12) to (14), the material equations (10) are transformed into circuit equations

$$I_0 = \frac{U_0}{R_0} + K_0 I_h$$
$$I_h = -\frac{U_h}{R_h} - K_h I_0 \tag{15}$$

where R_o and R_h are source and Hall resistances, respectively, given by well-known formulae

$$R_0 = \frac{l}{S_0 nq\mu} = \frac{l}{S_0\gamma}$$
$$R_h = \frac{a}{S_h nq\mu} = \frac{a}{S_h\gamma} \tag{16}$$

and

$$K_0 = \frac{S_0}{S_h}\mu B \qquad K_h = \frac{S_h}{S_0}\mu B \tag{17}$$

are dimensionless driving and Hall transfer constants.

In circuit equations (15) there are four electrical quantities, it can be treated as a two-port. However a more illustrative approach is to use one of circuit quantities as a driving one and calculate suitable characteristics from remaining quantities. A little bit simpler approach is to suppose the driving of the sample by ideal current source I_o. The second independent quantity should be the Hall current I_h. After simple rearrangement the equations (15) have the form

$$U_0 = R_0 I_0 - R_0 K_0 I_h$$
$$U_h = -R_h K_h I_0 - R_h I_h \tag{18}$$

Last set of equations have a simple explanation if we use more comprehensible symbols

$$U_0 = U_{0m} - R_e I_h$$
$$U_h = U_{h0} - R_i I_h \tag{19}$$

In the first equation the U_{om} is the source voltage when no Hall current flows (open Hall circuit condition). The second term is a small correction of source voltage due to the Hall current.

The second equation of the set (19) is more important, since it describes the linear equivalent circuit of Hall voltage, see Fig. 3. The Hall circuit consist of ideal voltage source U_{ho} and internal resistance R_i. From equations (18) and definitions (16) and (17) we can obtain practical formulae for these two circuit parameters

$$|U_{h0}| = R_h K_h I_0 = \frac{a}{l}\frac{1}{R_0}\mu B I_0$$
$$R_i = R_h = \frac{a}{S_h\gamma} \tag{20}$$

The minus sign for Hall open circuit voltage U_{ho} can be explained by the opposite direction of current and voltage in the sample, see Fig. 1.

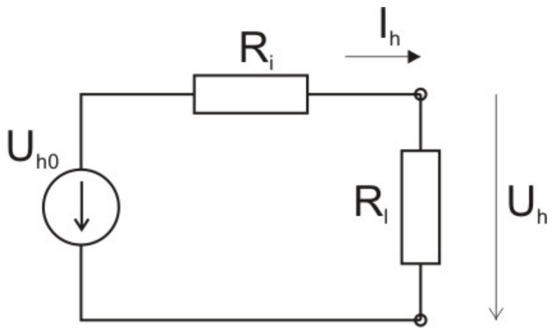

Fig.3: Hall Effect linear equivalent circuit.

Practical meaning of the first equation of the set (18) or (19) is the influence of Hall circuit to the driving circuit. Since the current source is supposed, applied source voltage will decrease if Hall current increase.
If we suppose the ideal voltage source, we can get the same system of equations (19) with interchanged quantities I_o and U_o. The formulae for coefficients are little bit complicated in this case.

2 SIMULATION

The experimental verification of Hall Effect theory is difficult at least from two reasons:

1. The output Hall voltage and current are of low values. Their measurement by typical instruments is difficult and of low precision. However specialized instruments for low level measurement exist, therefore this problem can be solved.
2. The Hall probes are not practically available. There are a lot of Hall sensors, which are sophisticated integrated circuits. The sensing element is not available directly. Integrated circuit produces constant current to flow through the probe and the Hall voltage is amplified or otherwise processed. The production of unique Hall sensing element with connection should be very expensive.

The above items are the reason why we use simulation of Hall Effect probe instead of its direct measurement. The simulation will be made with parameters that are close to those used in practical devices.

Probe excitation, material parameter and geometrical dimensions are in Table 1. We suppose driving with ideal source current of 10 mA and typical magnetic field of 0.2 T. The material is silicon doped with arsenic, material values are taken from Ref. [4]. Main performance limitation is probe heating. We suppose natural air cooling (no ventilator). The probe temperature is supposed to be by 100 K higher than its surrounding air and practical value of convection coefficient 15 $Wm^{-2}K^{-1}$ is used. This value is recommended for technical calculations.

Parameter	Symbol	Value	Unit
Flux density	B	0.2	T
Driving current	I_o	10	mA
Carrier Mobility	μ	0.139	$m^2V^{-1}s^{-1}$
Carrier density	n	10^{21}	m^{-3}
Carrier charge	q	$1.6x10^{-19}$	C
Specific resistance	ρ	0.046	Ωm
Probe length	l	2	mm
Probe width	a	1	mm
Probe thickness	d	0.5	mm
Heat convection coefficient	α	15	$Wm^{-2}K^{-1}$
Temperature difference	$\Delta\zeta$	100	K

Tab. 1: Input electrical, environmental, material and geometrical parameters

3 RESULTS

Using parameters from Tab. 1 several probe characteristics are calculated. The load characteristic, given by the second equation of the set (19), is in Fig. 4. A relatively low open circuit voltage and short circuit current follow form the graph. The output power dissipated in Hall circuit is in Fig. 5. Maximum output power is for the case when the load resistance is equal to the inter resistance of equivalent circuit in Fig. 3. Since the constant current source is used, the source voltage can depend on the Hall current; see the first equation in the set (19). The dependence is very small, as it follows from Fig. 6. The Hall generator efficiency shown in Fig. 7 is also very low, and has maximum at the same conditions as output power.

Important output parameters of Hall probe are given in Tab. 2. Part of them is more correct values from graphs in Fig. 4 to 7. Equivalent circuit in Fig. 3 has open circuited voltage of 25 mV and internal resistance of 46 Ω. Short circuited current is 540 µA and maximum output power has value only of 3.4 µW. Efficiency is very low; its maximum is less than 0.02 %.

Other parameters in Tab. 2 are related to equations (15) and the rest of Tab. 2 shows a very small decrease of source parameters due to the Hall current rising from zero to maximum value.

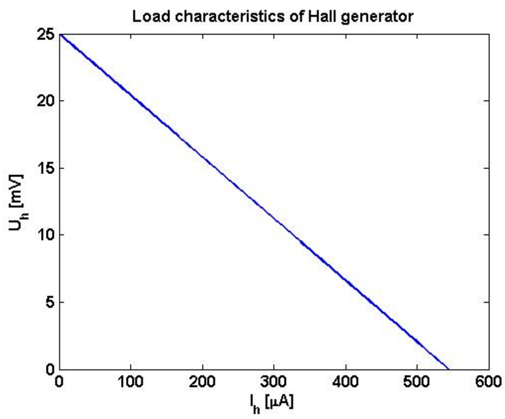

Fig.4: Load characteristics of Hall generator

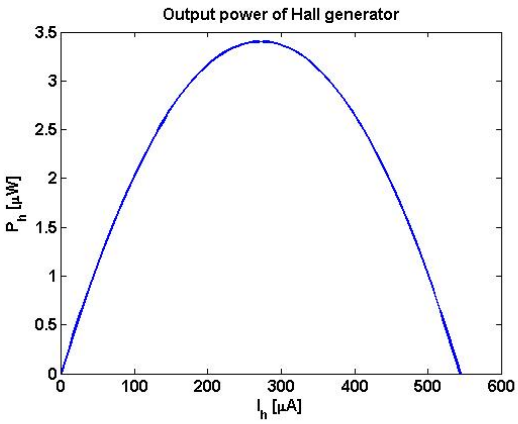

Fig.5: Output power dependence of Hall generator

The semiconductor electrical parameters are influenced strongly by the impurity density. We can therefore expect that Hall probe parameters will rapidly change, if the charge carrier density changes. Dependence of mobility and specific resistance is in Fig. 8 and 9, respectively. In these and following figures the decade logarithm is used on the scale for concentration. The polynomial approximation by low order polynomial is used in these two graphs.

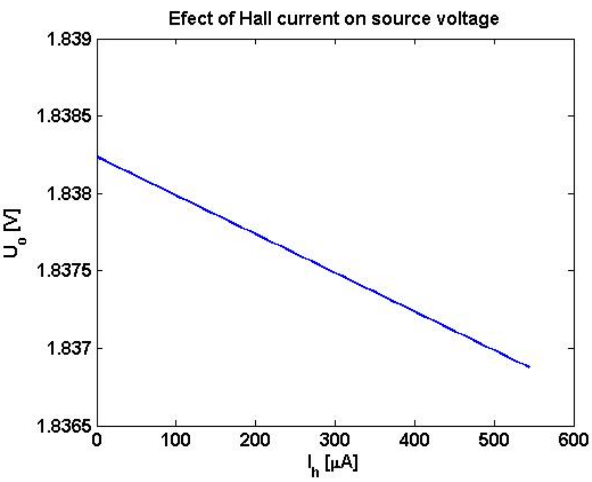

Fig.6: Input characteristics of Hall generator

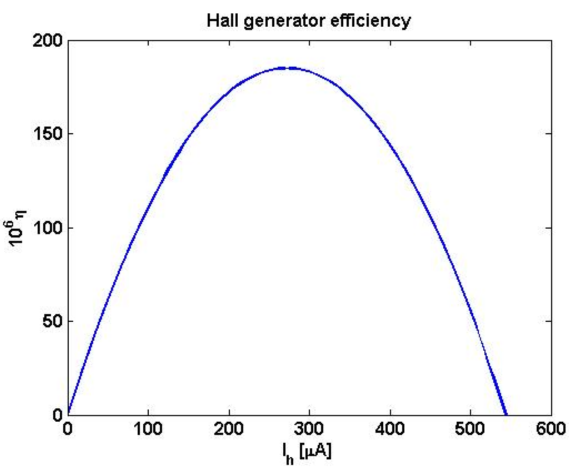

Fig.7: Efficiency of Hall generator

Parameter	Symbol	Value	Unit
Open circuited Hall voltage	U_{ho}	25	mV
Short circuited Hall current	I_{hm}	540	μA
Output Hall power	P_h	3.4	μW
Internal resistance	R_i	46	Ω
Source circuit resistance	R_o	184	Ω
Hall circuit resistance	R_h	46	Ω
Transfer coefficient K_o	K_o	0.0136	
Transfer coefficient K_h	K_h	0.0544	
Maximum source voltage	U_{om}	1.84	V
Maximum source power	P_o	18.4	mW
Maximum efficiency	η	0.019	%
Decrease of source voltage	ΔU_o	1.36	mV
Decrease of source power	ΔP_o	13.6	μW

Tab. 2: Output parameters of Hall generator

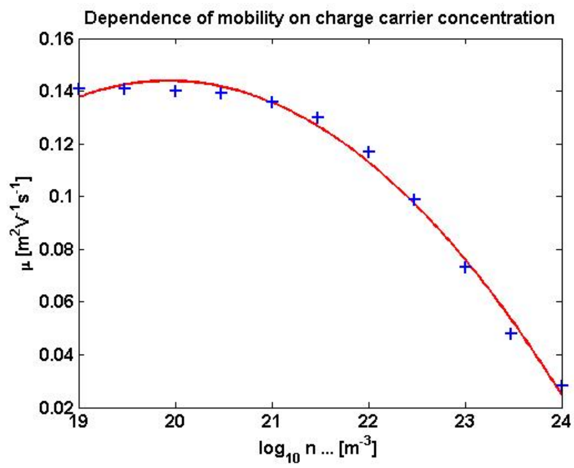

Fig.8: Dependence of semiconductor mobility on charge carrier concentration, polynomial approximation

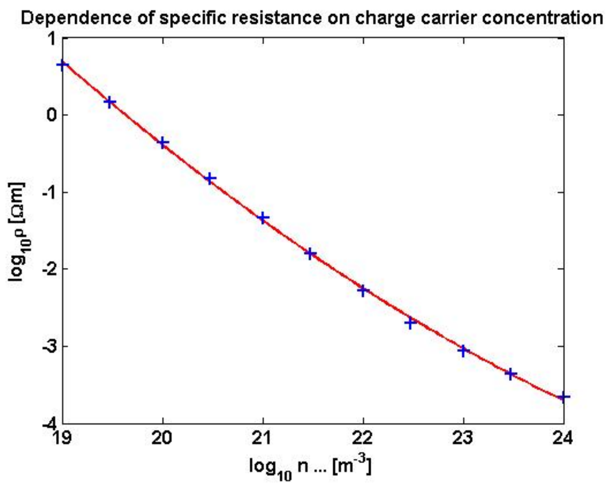

Fig.9: Dependence of semiconductor specific resistance on charge carrier concentration

The dependence of Hall voltage on charge carrier concentration is in Fig. 10. Necessary driving voltage can be read from Fig. 11. The element cooling is the main factor that limits the electrical parameters. Ration of power really dissipated to maximum allowable power is in Fig. 12. Another form of last three graphs uses logarithmic scale on both the axes.

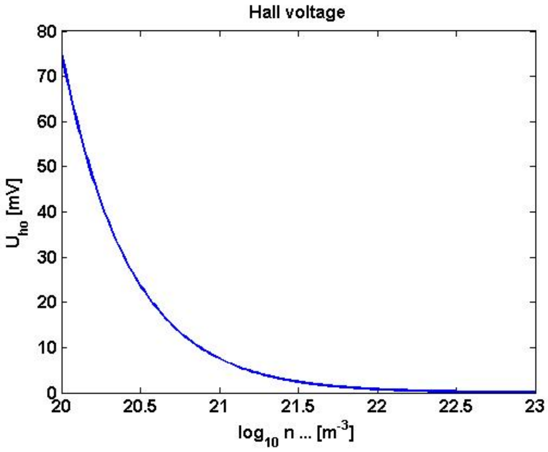

Fig.10: Dependence of Hall voltage on charge carrier concentration

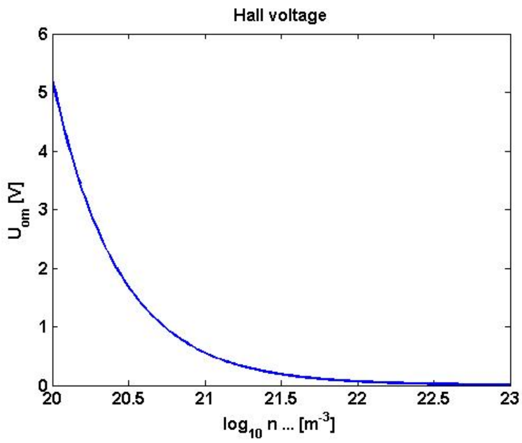

Fig.11: Dependence of source voltage on charge carrier concentration

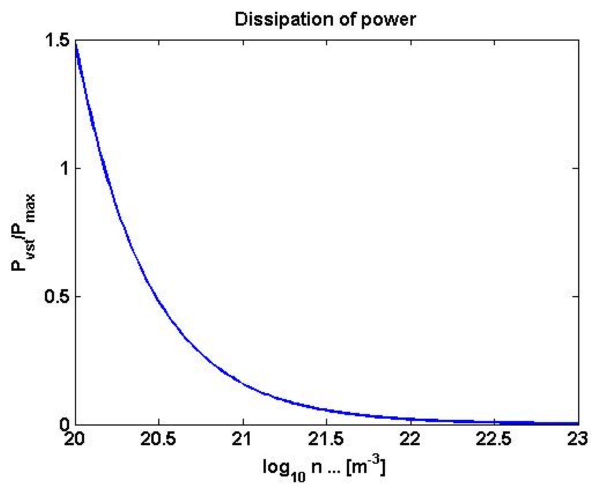

Fig.12: Heat development dependence on charge carrier concentration

4 DISCUSSION

As the results shown above are self-explaining, ion discussion we will focus to the general evaluation of the theoretical and simulation results. Nevertheless, the practical meaning of results will be mentioned too.

Semi conducting devices exhibit a large nonlinearity, in general. In the description of Hall Effect we have used simple microscopic linear theory and a linear equivalent circuit was derived. This is only approximation of practical devices, but we hope that the approximation will be useful as in other modeled devices. Main sources of nonlinearity are dependence of charge carrier mobility on electric field strength. Also the charge carrier concentration depends on electric field.

Another external parameter that strongly influences the semiconductor behavior is the temperature. Since the dimensions of Hall probes are very small, the heat convection is low and their temperature can be high. In the simulation we have expected the devices work at power limit, the temperature increase was supposed 100 K. The reason was to get limiting parameters. The limiting parameters are for ideal case of no temperature dependence and linear device. Real values will be low. By the decrease of the driving current, the more realistic parameters of actual devices can be found. Because of linear dependence on driving current, their values will be considerably lower.

Another approach how to improve the power dissipation is to use a forced heat convection by ventilator or to use heat convection by water. In the first case the driving current increases at least 5 times, in the second case the increase may be 100 times. However in both cases the main advantage of Hall devices, the very small dimensions, is loosed. Probably, they are useful only in academic research.

As it follows from equations (19) and (20), ideal Hall generator is a voltage source controlled circuit quantity, current I_0, and by external parameter, magnetic flux density B. Formula (20), and similar ones that are not given here, shows that the device parameters are given by product μB, where mobility μ is the material parameter and B is the magnetic flux density. The higher product μB, the better parameters can be expected. Since the magnetic field is measured quantity, materials of high mobility are the best choice. The silicon used in simulation was not the optimum solution, but the data were easily available. The effect of sample geometrical parameters is low and the dimensions are controlled by practical use first of all.

From the practical point of view, the load characteristic in Fig. 4 shows that Hall generator is a soft voltage source. Its internal resistance is typically tens or hundreds of Ohms. The short circuited current is therefore very low, maximum values may be mA. Typical Hall voltages are units of mV. Opposite to the thermocouple, which is also voltage source in mV range and exhibits high short circuited currents, the Hall devices cannot work with currents. Also the available optimum power is very low, several μW.

Since the Hall device is a soft voltage source, the voltmeter of high input impedance is necessary for correct measurement. However, the internal resistance is tens of Ohms, typical commercial voltmeter or amplifier with input resistance of MΩ range (or little bit lower one) is an ideal instrument.

The graph in Fig. 7 describes the influence of Hall circuit on driving circuit. The increase of Hall current leads to the decrease of applied voltage, since constant current excitation is supposed. However, the decrease is very low, less than 0.1 % of initial value for zero Hall current. The condition of the use constant current driving source is not necessary. Constant voltage source may be used, which is practically simpler solution.

5 CONCLUSION

In the paper a simple linear microscopic theory of Hall Effect was derived and linear equivalent circuit of Hall devices was presented. As there was no possibility of experimental verification, a simulation on the model near to real devices was made. From the simulation typical parameters and basic characteristics of Hall generators were found. The linear theory is only approximation of real devices, but we can expect that the approximation is good for low excitation current, which is necessary in practical applications.

The equivalent circuit has relatively high internal resistance, which means that Hall devices are soft source of small voltage. The device used for measurement of Hall voltage should have high input resistance. This practical condition is valid for typical commercial instruments and elements at present time. Other modes of Hall devices operation (high currents or powers) are practically impossible.

Characteristics derived from equivalent model can be used in the optimum design of both the Hall Effect sensing element and external circuits that control them.

More correct approach is to use the nonlinear theory. Also the temperature effects should be included since it strongly limits performance of real Hall devices.

6 ACKNOWLEDGEMENT

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