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LP FILTERS WITH REAL TOAS

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Abstract: *One way how to design active filters for wider frequency band is application of transimpedance operational amplifiers based on current mode. Using numerical and symbolical analysis was investigated influence of real parameters of active elements on resulting parameters of active filters. From works was grown an equivalent circuit diagram. Equivalent values of parameters of circuit diagram can be determined using developed empirical formulas, which enable to express the influence of real parameters of transimpedance operational amplifiers on filter performance and to determine behaviour of synthetic elements in higher frequency area.*

Key words: *active filters, transimpedance operational amplifiers,*

INTRODUCTION

Design of active filter is possible to solve by several ways. One way, how to create an active filter directly from an LC ladder prototype low pass (LP) filter, is application of Bruton's transformation which is defined by formula

$$Z_T(s) = \frac{K}{s} Z(s) \qquad (1)$$

where $Z_T(s)$ is transformed impedance, K is constant and Z(s) is impedance of an element of passive LC ladder LP filter.

The multiplication by K/s transforms

$R \Rightarrow 1/sC_T$, $sL \Rightarrow R_T$, $1/sC \Rightarrow 1/s^2 D_T$.

After this transformation the voltage transfer function remains invariant. The element with impedance $1/s^2DT$ is the so-called frequency dependent negative resistor (FDNR). Adding serial or parallel resistor to the FDNR we obtain DCR circuits, which are as for voltage transfer function equivalent to RLC circuits. For illustration the serial combination of elements R_T, C_T, D_T on Fig. 1 is equivalent to serial resonance circuit LRC.

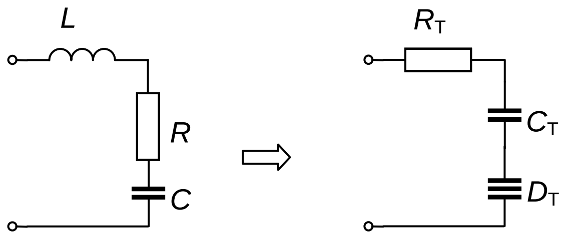

Fig. 1: The equivalent scheme for the circuit RLC.

The FDNR approach is frequently used for design of LP filters. For this aim is widely used the generalized immitance converter (GIC), which is usually based on Antoniou´s two opamp circuit (Fig. 2) [1].

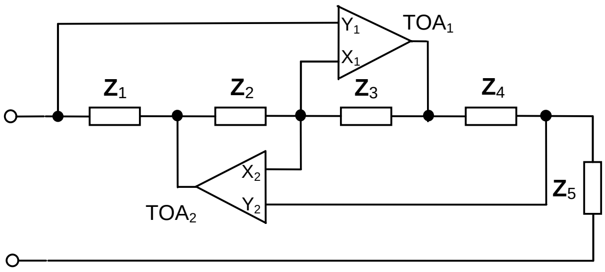

Fig. 2: Generalized immitance converter

Properties of this circuit with classical opamp were described in many references. Nowadays, TOAs can be used for creation of synthetic elements. By application of ideal TOAs in the circuit in Fig. 2 we obtain network with input impedance

$$\mathbf{Z}_{\rm id} = \frac{\mathbf{Z}_1\mathbf{Z}_3\mathbf{Z}_5}{\mathbf{Z}_2\mathbf{Z}_4} \tag{2}$$

Let's turn attention to the FDNR simulation. The FDNR can be obtained by these substitutions:

1. $\mathbf{Z}_1 = 1/sC_1,\ \mathbf{Z}_3 = 1/sC_3,\ \mathbf{Z}_i = R_i,\ i = 2,4,5$ or
   $\mathbf{Z}_3 = 1/sC_3,\ \mathbf{Z}_5 = 1/sC_5,\ \mathbf{Z}_i = R_i,\ i = 1,2,4$
   Interconnection of input X and output of a real TOA by capacitor C_3 is a source of instability; hence this substitution will not further be considered.
2. $\mathbf{Z}_1 = 1/sC_1,\ \mathbf{Z}_5 = 1/sC_5,\ \mathbf{Z}_i = R_i,\ i = 2,3,4$
   In this case the network represents a FDNR with impedance

$$\mathbf{Z}_{\rm idD} = \frac{R_3}{s^2C_1C_5R_2R_4} = \frac{1}{s^2D_{\rm id}} \tag{3}$$

where

$$D_{\rm id} = \frac{C_1C_5R_2R_4}{R_3}\ . \tag{4}$$

An important question is: how properties of real TOAs affect properties of synthetic elements. Analytic expressions describing the effect of real TOAs can be derived only for the simplest cases [3]. This paper will bring simple criteria for optimum selection of TOA type in association with required properties of FDNR.

1 Investigation of FDNR properties

The aim of investigation is derivation of formulas for element values of FDNR equivalent scheme. In the first step the SNAP program was used. It performs the symbolic analysis of circuit functions [2]. SNAP program, however, delivers a very complicated analytic formula of FDNR impedance, of which it is hard to derive relations for element values of FDNR equivalent scheme. At last it was chosen an approximation of PSPICE program numerical results to the relation for element values. Ideal TOA in PSPICE program can be modeled by controlled voltage and current sources with unity gains [4].

Influence of real TIAs properties on FDNR parameters will be investigated by the equivalent circuit (Fig. 3). This equivalent circuit consists of ideal TOAs and added parts. Values of added parts are accessed through catalogues of TOAs. Additional elements R_{Y1}, R_{Y2}, C_{Y1}, C_{Y2}, R_{T1}, R_{T2}, C_{T1}, C_{T2} simulate a finite value of input Y impedance and transimpedance Z_T of TOA_1, TOA_2 and their frequency dependencies. Non-zero input X and amplifier output impedances are modeled by resistors R_{X1}, R_{x2}, R_{out1}, R_{out2} and capacitor C_{X1}, C_{x2}. Model elements of the first TOA are here marked by index 1 and model elements of the second TOA by index 2.

For determination of equivalent circuit values of FDNR scheme will be below in text used these parameters of TOAs and operational network:

$R_{\rm t1o} = R_{\rm t2o} = 1.10^6\ \Omega$, $R_{\rm y1o} = R_{\rm y2o} = 1.10^6\ \Omega$,

$R_{\rm x1o} = R_{\rm x2o} = 50\ \Omega$, $R_{\rm out1o} = R_{\rm out2o} = 15\ \Omega$,

$C_{\rm t1o} = C_{\rm t2o} = 5$ pF, $C_{\rm x1o} = C_{\rm x2o} = C_{\rm y1o} = C_{\rm y2o} = 2$ pF,

$R_{2o} = R_{3o} = R_{4o} = 1{\rm k}\Omega$, $C_{1o} = C_{5o} = 10$ nF.

For these original values we bring here results that were obtained from simulation of FDNR circuit. The obtained frequency dependence of absolute value of FDNR impedance is shown in Fig. 4.

From Fig. 4 it is evident that FDNR behaves as a resistor in low frequency range, a parallel resonance circuit in interface of low and middle frequency range, a FDNR in middle frequency range and a serial circuit in the range of high frequencies.

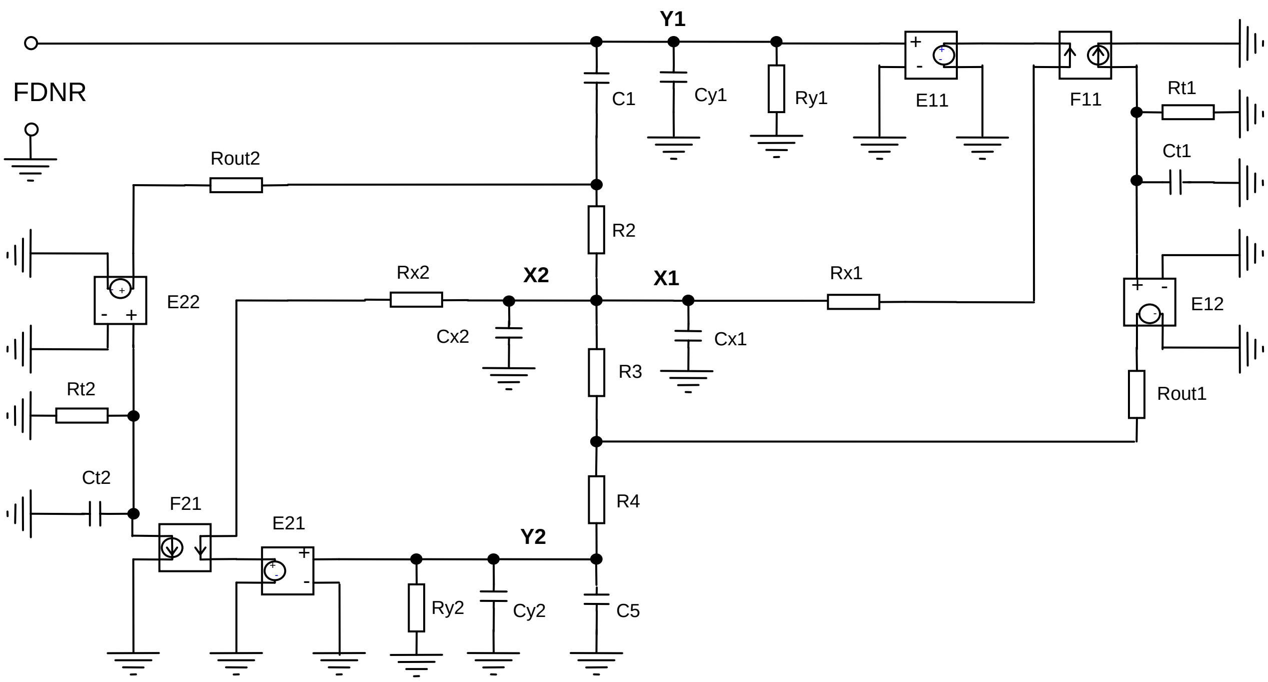

Fig. 3: Model of FDNR with real TOAs

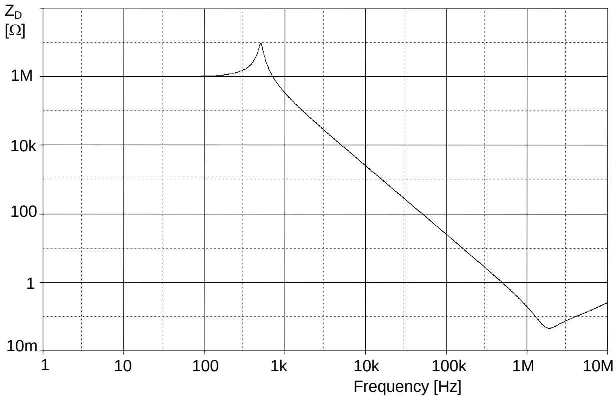

Fig. 4: The frequency dependence of FDNR impedance

On the base of Fig. 4 we can draw an equivalent scheme for the FDNR shown in Fig. 5.

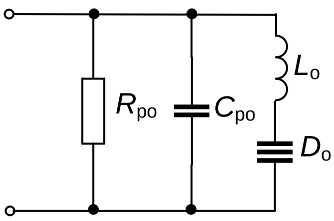

Fig. 5: An equivalent scheme for the FDNR

It consists of parallel combinations of elements

$$R_{po} = 1\text{M}\Omega, \quad C_{po} = 33\,\text{pF},$$

and serial connected elements

$$D_o = 1.10^{-13}, \quad L_o = 4\text{ nH}.$$

Because the PSPICE program works on a base of numerical values it is needed to provide restriction in values of FDNR model elements. This restriction is given by possibilities of FDNR realisation in technical practice. The result of investigation is a high quantum of data. For illustration we introduce here some partial results.

1.1 Determination of the area of a resistor and parallel resonance circuit FDNR behavior

Considering ideal TOAs the FDNR impedance is infinite for zero frequency

$$Z_{idD} \to \infty \,, \tag{5}$$

when using real TOAs impedance value is limited to

$$Z_D = R_p \,. \tag{6}$$

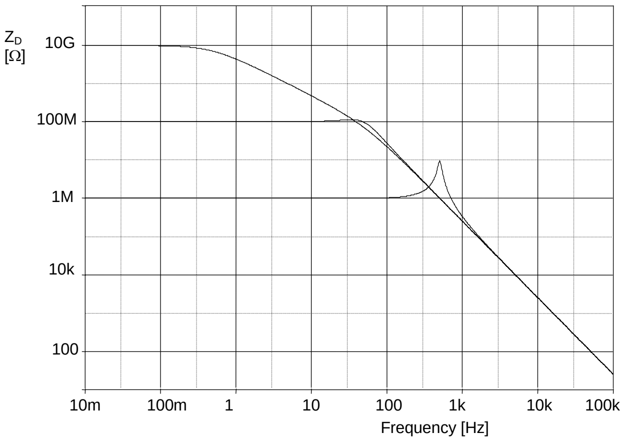

Fig. 6: Influence of input resistance R_{y1}

As can be seen in Fig. 3, input resistor Ry1 and capacitor Cy1 are connected parallel to synthetized FDNR. The influence of a finite value of resistor Ry1 illustrates Fig. 6 and Tab. 1.

R_{y1} [MΩ]	0.1	1	10	100	1000
R_p [MΩ]	0.100	1.000	10.00	100.0	1000

Tab. 1

As the influence of any other circuit element to R_p value has not been detected, we can formulate relationship

$$R_p = R_{y1} \tag{7}$$

Resistor R_p and element D forms parallel resonant circuit with resonant frequency

$$f_{pr} = \frac{1}{2\pi\sqrt{R_p \cdot D}} = \frac{1}{2\pi\sqrt{R_{y1} \cdot D}} \tag{8}$$

with quality

$$Q_p = \frac{Z_{Cpr}}{Z_{Dr}} = \frac{2\pi f_p D}{C_p} \,. \tag{9}$$

Here the index r indicates the resonant element values

Impedance shows significant D character above frequency f_{pr} under these conditions

$$Z_{Cpr} \geq Z_{Dr} \quad \Rightarrow \quad Q_p \geq 1 \,. \tag{10}$$

Otherwise if

$$Z_{Cpr} < Z_{Dr} \quad . \tag{11}$$

resonant state newer occurs and FDNR has resistive and capacitive area in his frequency characteristic. Boundary frequencies for this areas are

$$f = 0, \quad f_{RC} = \frac{1}{2\pi R_p C_p}, \quad f_{CD} = \frac{C_p}{2\pi D} \,. \tag{12}$$

1.2 Determination of D element value

At first, let’s turn attention to R_{t1}, R_{t2} values influence on D element value. We will sweep values of R_{t1}, R_{t2} independently and $R_{t1} = R_{t2}$ together. Impact of finite values of transfer resistances we will assess using coefficient

$$p = \frac{D}{D_{id}}, \tag{13}$$

where D is simulated value and D_{id} is ideal value obtained from (4). Results are shown in Fig. 7.

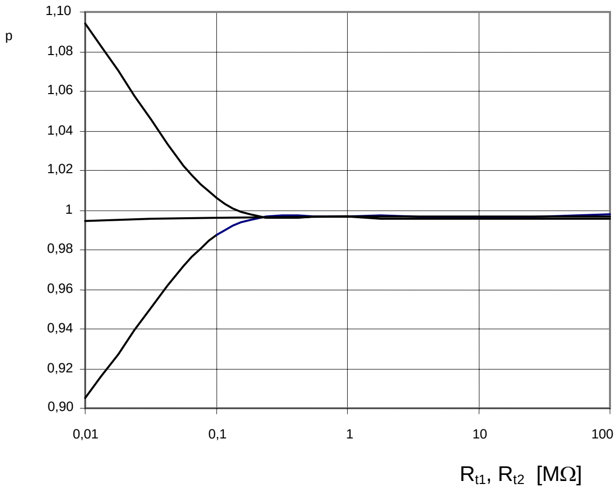

Fig. 7: Influence of R_{t1}, R_{t2} values

From Fig. 7 is evident, that optimal results are for $R_{t1} = R_{t2}$. So, further we will use the same TOAs in FDNR circuit.

The resistor R_{y2} and capacitor C_{y2} are connected in parallel with the capacitor C_5, increasing its capacity

$$C'_5 = C_5 + C_{y2}. \tag{14}$$

so it is necessary take it into account calculating D value from (4). The resistor R_{y2} value is relatively high in comparison with the capacitor C'_5 reactance, therefore its effect is practically insignificant and the element D value can be calculated from the relation

$$D = \frac{C_1 C'_5 R_2 R_4}{R_3}. \tag{15}$$

1.3 Determination of C_p element value

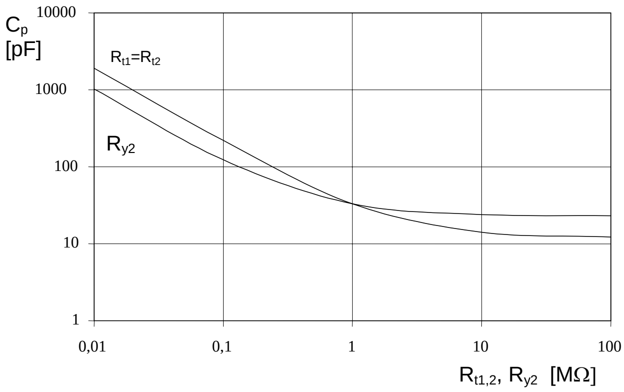

Fig. 8: Influence of $R_{t1,2}$, R_{y2} values

The value of C_p is affected by many of FDNR circuit elements.

The capacitor C_{y1} is in parallel to realized FDNR so we can write

$$C_p = C_D + C_{y1}, \tag{16}$$

where C_D means contributions of other circuit elements.

A finite value of impacts on loses in FDNR. The same effect has an input resistance of TOA_2.

Impact of transfer resistances R_{t1}, R_{t2} and input resistance R_{y2} to C_p is shown in Fig. 8.

Moreover, the value of C_p is affected by operational network elements, which is illustrated in Fig. 9 and Fig. 10.

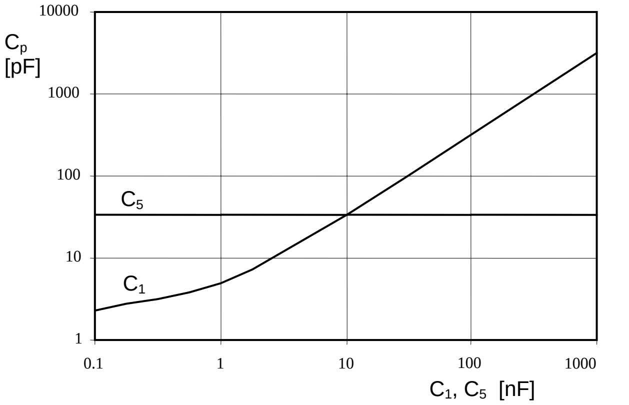

Fig. 9: Influence of C_1, C_5 values

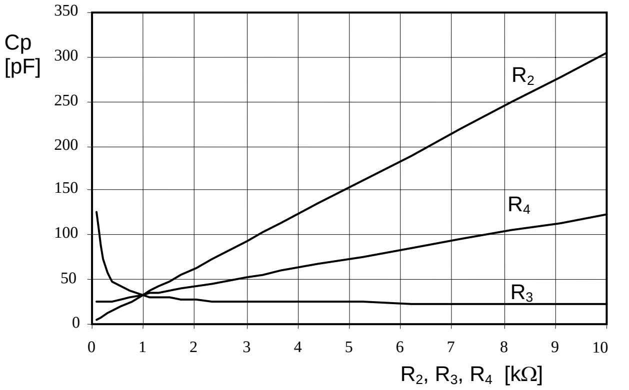

Fig. 10: Influence of R_2, R_3, R_4 values

1.4 Determination of D working area limits

Investigation of FDNR behaviour on high frequencies shows, that serial combination L, D in Fig. 6 needs to be append by resistor R_s and capacitor C_s (Fig. 11). The reason is in serial resonance, which occurs in some cases. Now, calculation of element values of substitute circuit in Fig. 11 is little bit complicated.

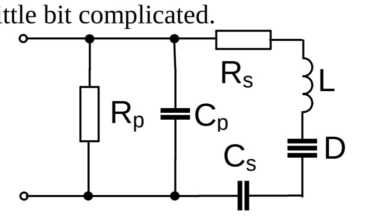

Fig. 11: An equivalent scheme for the FDNR

In modeled circuit in Fig. 3 are real TOAs replaced by simple models, consisting from two functional blocks (current conveyor and voltage follower) This model does not respect frequency dependence of amplification and input impedance of TOA, output inductance of TOA is not respected, too. But, this model is sufficiently suitable for criteria investigation, how real TOAs basic parameters impact on FDNR parameters.

Mutual impedance relations of R_s, C_s, L. D determines the frequency f_m, when impedance Z_D has minimal value. In Tab. 2 is illustrated effect of $C_{t1} = C_{t2}$ to the value f_m. Analogical effects of $R_{x1} = R_{x2}$ and $R_{out1} = R_{out2}$ are shown in Tab. 3 and Tab. 4, respectively.

$C_{t1, 2}$ [pF]	1	2	5	10	20
f_m [MHz]	3.589	2.754	1.884	1.396	1.023

Tab. 2

$R_{x1, 2}$ [Ω]	10	20	50	100
f_m [MHz]	3.428	2.660	1.844	1.413

Tab. 3

$R_{out1,2}$ [Ω]	10	20	50	100
f_m [MHz]	2.018	1.778	1.413	1.148

Tab. 4

2 CONCLUSION

Real TOAs influence the characteristics of synthetic elements. Analytic expressions describing the effect of real TOAs can be derived only for the simplest cases.

In this paper there were derived relations of FDNR calculation on the base of an FDNR model with real TOAs equivalent circuit. For this aim there was used PSPICE v. 9.2 simulator, which generated amount of data. Obtained data were a base for approximate relations and graphical dependences investigation, which enabled real FDNR parameter calculation from TOAs catalogue data. Relations presented here offers simple criteria for optimum selection of TOA type in association with required properties of FDNR.

3 ACKNOWLEDGEMENT

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