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ANALYSIS OF STRESSES OF DISC IN INDUCTIVE DYNAMIC DRIVE

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Abstract: Observations made during the experiment with physical model of inductive dynamic drive showed, that in some extreme cases disc was deformed fundamentally, what was proved by its vibrations. For investigation of state of stress the mathematical model of the magneto-elastic vibrations for circulation plate was used. Results presented by the authors showing the distributions of stresses for the parameters of IDD employed in ultra rapid hybrid circuit breaker, can be useful for constructors.

Key words: *Drive, magneto-elastic vibrations, stresses*

INTRODUCTION

Inductive dynamic drive (IDD) is fundamental element of hybrid circuit breaker. These drives must meet following requirements such as the achievement of the height speed in short time, the stability mechanical characteristics and long-lasting reliability[1],[4]. In order to receive large acceleration of IDD disc we must produce a strong impulsive magnetic field and at the same time we should minimize the mass of the disk. Therefore a large impulsive force acts on the disc. However if the distribution of force along the radius of the disk isn't suitable (uniform enough) and its value is too large for dimensions of the disk used then the disk itself will deform.

Therefore the mathematical model of IDD should allow for the obtaining of distributions of eddy currents in the disk and on this basic state of magneto elastic vibration and stresses in the disk.

The IDD model consists of two parts: electro-dynamical and mechanical. The electro-dynamical (EM) model is based on the solution of Poisson's equation in the area disk (1) and Laplace's equation outside it.

$$\frac{\partial^2 A_\Theta}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial A_\Theta}{\partial \rho} - \frac{A_\Theta}{\rho^2} + \frac{\partial^2 A_\Theta}{\partial z^2} = -\mu_0 j_\Theta \qquad (1)$$

where **A** and **J** have only angular component.

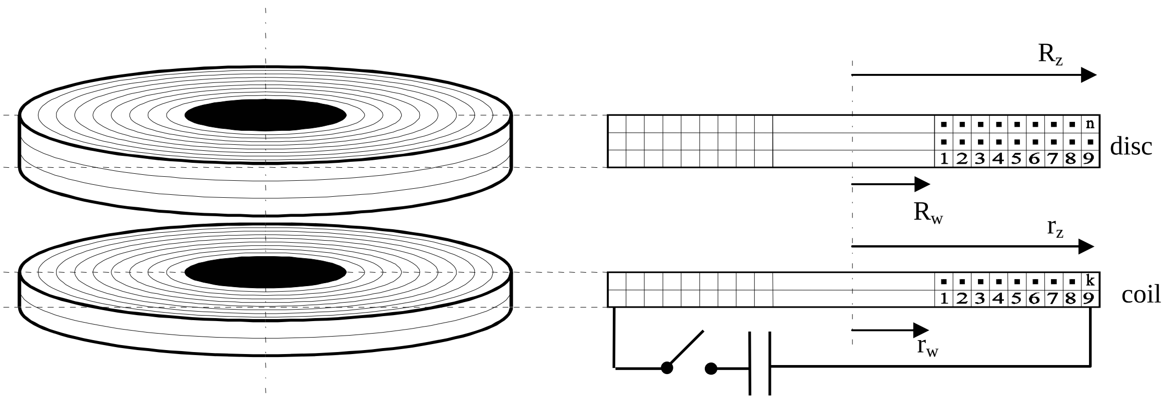

Fig.1 System of disc and coil and their discretization

For that purpose partition of the disc on filaments and the coil into sections was made. Because one can consider every filament of the disc and section of the coil as coaxial rings we can use analytical formula to determine vector potential, and hence inductance of every ring of coil and of disc and mutual inductance between them too. Then one can change the field approach to the circuit approach with large numbers of mutual inductance. However, it must be emphasized that this model assumed that disk is a rigid body. On the basic of EM one obtains the distribution of pressure and force in the whole volume at the disk. The EM model was exactly described in [2].

1 MATHEMATICAL MODEL OF MOVEMENT OF THIN PLATE

For investigation of state of stress it was decided to built mathematical model of the magneto-elastic vibrations for circulation plate. Therefore theory of thin plate in method of continuum mechanics was used. Thin plate is such construction element (in comparison with membrane) where nonzero stiffness for bending is assumed. On the other hand radial dimensions must be much larger in comparison to thickness of disc. According to [5] we can use theory of thin plate if thickness of investigate element meet following condition:

$$h \le (0.1 \div 0.2)\min(a,b)$$

where: a,b – dimensions of rectangular plate but for circular plate $a=b=2(R_z-R_w)$ [5].

General equation of crosswise vibrations of the thin plate has formula:

$$\nabla^2(D\nabla^2 w)+ \\ +(1-\nu)\left\{2\frac{\partial^2}{\partial x\partial y}\left(D\frac{\partial^2}{\partial x\partial y}\right)-\frac{\partial^2}{\partial x^2}\left(D\frac{\partial^2}{\partial y^2}\right)-\frac{\partial^2}{\partial y^2}\left(D\frac{\partial^2}{\partial x^2}\right)\right\}w+ \\ +\bar{\rho}\frac{\partial^2 w}{\partial t^2}=p(x,y,t) \qquad (2)$$

In the case where thickness of the plate is constant and material is homogeneity (D(x,y)=const) equation (2) in cylindrical coordinate system can be written as:

$$\frac{D}{\bar{\rho}}\nabla^4 w+\frac{\partial^2 w}{\partial t^2}=\frac{p(r,\varphi,t)}{\bar{\rho}} \qquad (3)$$

where: $\bar{\rho}=h\rho$ mass of plate respects to unit of surface**,**

$w=w(r,\varphi,t)$ - displacement,

$D=\dfrac{Eh^3}{12(1-\upsilon^2)}$ -cylindrical stiffness of plate on bending,

E,h – respectively Young`s modulus and thickness of plate,

υ - Poisson’s coefficient,

p(r,φ,t)-space-timing distribution of pressure acting on surface of disc.

Because of zero initial conditions (circular-symmetric) and also circular symmetry of space-timing electro-dynamics force obtained from EM, equation (3) simplifies to form:

$$\frac{\partial^2 w}{\partial t^2}+\frac{D}{\bar{\rho}}\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}\right)^2 w=\frac{p(r,t)}{\bar{\rho}} \qquad (4)$$

Because of free edge of the plate, boundary conditions on edge of plate determine values of bending moment and of sharing force. For solving thus presented initial-boundary problem, a method separation of variables (Fourier method) was used. It means that particular integrals in product functions form were found:

$$w_i(r,t)=W_i(r)T_i(t) \qquad (5)$$

By substituting (5) to (4) at zero pressure (homogenous equation) and separating variables we obtain system of equation:

$$\ddot{T}+\omega^2 T=0 \qquad (6)$$

$$\left(\frac{d^2}{dr^2}+\frac{1}{r}\frac{d}{dr}\right)\left(\frac{d^2W}{dr^2}+\frac{1}{r}\frac{dW}{dr}\right)=\lambda^4 W \qquad (7)$$

where: $\lambda^4=\dfrac{\omega^2\bar{\rho}}{D}$.

Solution of equation (7) satisfying boundary conditions is solved of eigen problem, where functions W_i are eigenfunctions and values λ_i are eigenvalues.

Furthermore it is assumed that solution of non-homogenous equation (4) and pressure function it can be presented as expansion in a series according to eigenfunctions.:

$$w(t,r)=\sum_{i=0}^{\infty}T_i(t)W_i(r) \qquad (8)$$

$$p(r,t)=\sum_{i=0}^{\infty}\Phi_i(t)W_i(r) \qquad (9)$$

By substituting (8),(9) to (4) once again and separating variables we obtain two equations. Once of them is equation for eigenvalues and the other has form:

$$\ddot{T}_i+\omega_i^{\,2}T_i=\frac{1}{\bar{\rho}}\Phi_i \qquad (10)$$

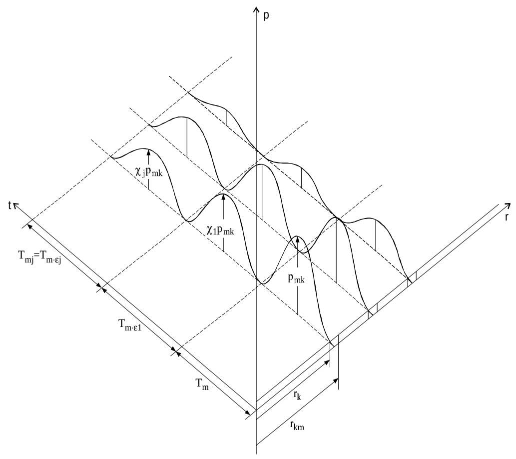

Fig.2 Functions of pressures acting on filaments of disc

To determine coefficients of series (9) we take advantage of orthogonality of eigenfunctions:

$$\int_{r_0}^{r_k} p(r,t) r W_j(r) dr = \sum_{i=0}^{\infty} \Phi_i \int_{r_0}^{r_k} r W_i(r) W_j(r) dr \quad (11)$$

and because:

$$\int_{r_0}^{r_k} r W_i \cdot W_j dr = \begin{cases} \gamma_i^{\ 2} \text{ gdy } i = j \\ 0 \ \text{ gdy } i \neq j \end{cases} \quad (12)$$

Then Φ_i functions for following eigenvalues are determined by:

$$\Phi_i = \frac{1}{\gamma_i^2} \int_{r_0}^{r_k} p(r,t) r W_i(r) dr \quad (13)$$

Observing pressure and force functions obtained from EM (fig.2,5) one can note, that every impulse of force cosinusoidal function approximates very well. Therefore curve of pressure acting on k-filament is determined by following analytic function:

$$p_k(t) = \chi_j p_{mk} (1 - \cos\frac{2\pi}{T_{mj}} t) \quad (14)$$

For a force acting on single filament, p(r,t) is described by (see fig.2):

$$p(r,t) = \begin{cases} 0 & r < (k-1)s \\ p_k(t) & (k-1)s \le r \le ks \\ 0 & ks \le r \end{cases} \quad (15)$$

Therefore for single filament Φ_i function has form:

$$\Phi_{ijk} = \frac{\chi_j p_{mk} (1 - \cos\frac{2\pi}{T_{mj}} t)}{\gamma_i^2} \int_{(k-1)s}^{ks} r W_i(r) dr \quad (16)$$

Then equation of movement (10) gets form:

$$\frac{d^2 T_{ijk}}{dt^2} + \omega_i^{\ 2} T_{ijk} = c_{ijk} \left(1 - \cos\frac{2\pi}{T_{mj}} t \right) \quad (17)$$

where: $c_{ijk} = \dfrac{\chi_j p_{mk}}{\gamma_i^2} \displaystyle\int_{(k-1)s}^{ks} r W_i(r) dr$

and: $i = 0,1,2....l_f$ – index of mode of vibrations,
$j = 0,1,2....l_t$ – index of load period,
$k = 0,1,2....l_p$ – index of disc filament.

Solution of problem for a completely load of disc one treats as superposition of solutions for each filament. Solution of movement of vibrating disc and it's computer realisation were based on ideal elasticity and its homogeneity and isotropy.

To identify the conditions in which dangerous state of stresses appears hypothesis of Huber-Misess-Henecky was used. Therefore in order to determine effort of materials reduced stress was taken and has got following form:

$$\sigma_{red} = \sqrt{\sigma_r^{\ 2} + \sigma_\varphi^2 - \sigma_r \sigma_\varphi} \quad (18)$$

Where: σ_r, σ_φ -normal stresses as on fig.3

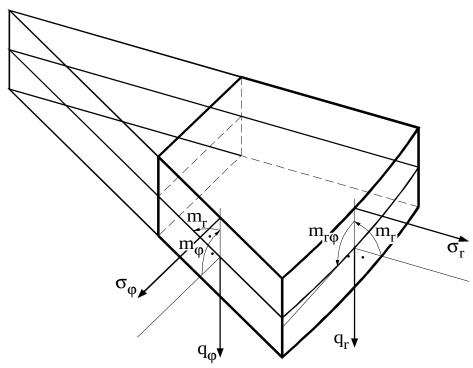

Fig.3 Normal stresses and bending moment in circular sector

The above mechanical model (MM) was implemented in Delphi language. In the first stage eigenvalues are counted and orthogonality of eigenfunctions are checked. The screen with established modes is presented in fig.4

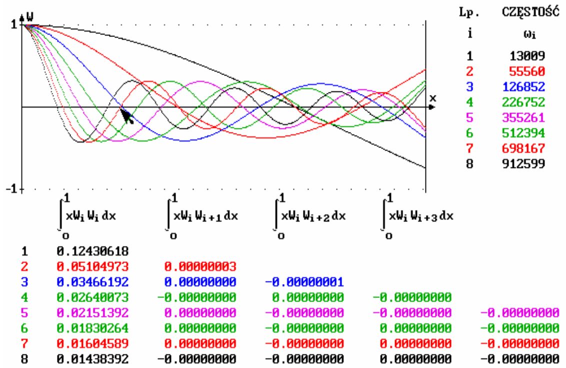

Fig.4 First screen of programm MM with finding eigen functions

2 Results of Investigations

Results of the investigations presented in the paper were obtained for two objects. Parameters of one of them in table1 are presented:

Initial energy of capacitor: E=450J	Capacitance of capacitor: C=100µF
Outer radius of disc: Rz=8 cm	Outer radius of coil: r_z=6 cm
Inner radius of disc: Rw=0 cm	Inner radius of coil: r_w=2.4 cm
Add mass: 0 kg	Winding of coil z_w=12
Material of disc: duralumin h of disc 4,6,8,10,12,14mm	Thickness of coil: hc=0.3 cm

Tab. 1 Parameters of IDD (full disc)

As a result of simulation in EM program (for parameters from tab.1) time-functions of force acting on chosen filaments were obtained (fig.5). As one can see character of every curves is similar (attenuation, periods of following pulses), which justifies approximations of these numerical forces by analytic cosinusoidal functions.

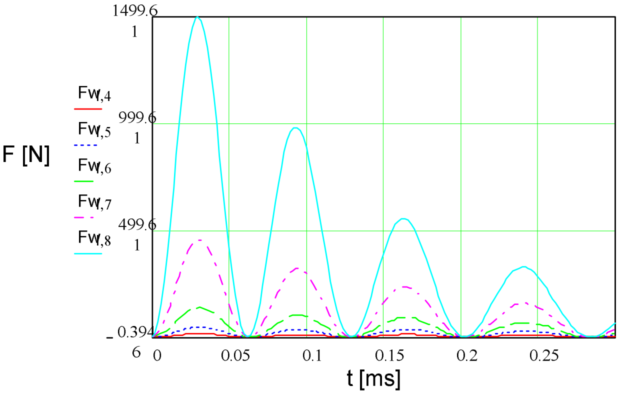

Fig.5 Curves of force in time function acting on filaments

Distribution of maximal values of force and pressure in radius function is presented on fig.6. These maximal values of force for program MM are loaded as amplitude analytic cosinusoidal function acting on the following filaments. That process in second real screen of MM program is shown on fig.7.

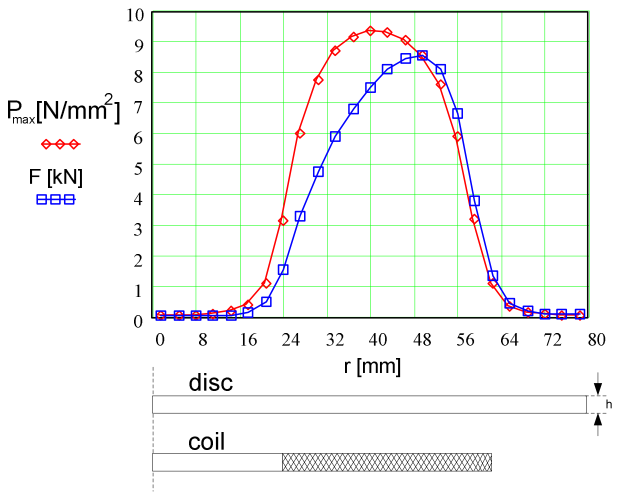

Fig.6 Distribution of pressure and force for dates from tab.1

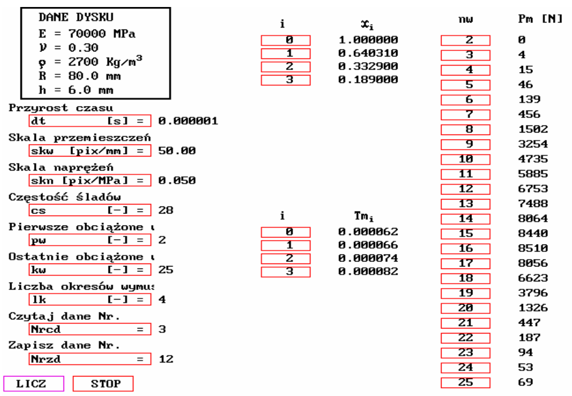

Fig.7 Second screen of program MM for full disc

The figure 8 presents a screen of the program realizing the model MM, where in the top screen one can see the traces of the middle surface of the vibrating disc (half of the diameter) and the state of stresses for the parameters from tab.1 for h=6mm. In the right bottom hand corner of the fig.8 one can optionally display sub-screen which presents maximal reduced stress in the distance function from centre of the disc. As one can expect the extreme value of stress is produced in centre of the disc. The result obtained for h=6mm (σ_{zr}=1192.5MPa) shows that disc can deform.

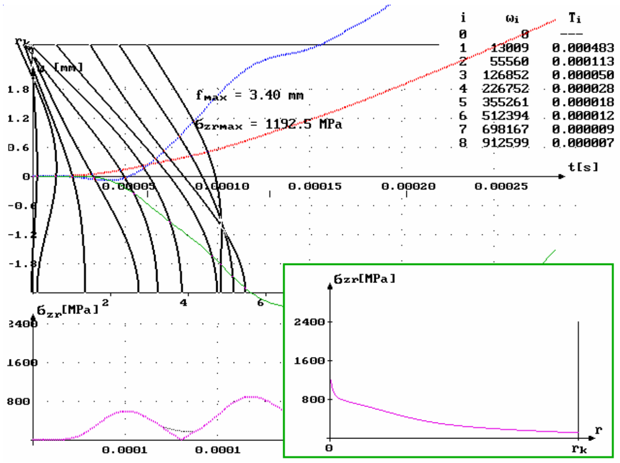

Fig. 8 Screen of program solving movement equation of vibration of the disk and state of stresses (dates on fig.7)

Top screen: black lines- deformed central surfaces of disc in any moment,

Red line: displacement of centre of mass,

Blue line displacement of midpoint of disk,

Green line: vibration of midpoint of disc in relation to midpoint of mass

Bottom screen: maximal reduced stresses in time function

Sub-screen: maximal reduced stress in the distance function from centre.

Fig.9 presents result of simulation for h=12mm. For this case the maximal amplitude of the disc surface deflection was 0.59mm and the maximal reduced stress (520.3MPa) did not exceed the admissible value of the used duralumin alloy. However because of non uniform of horizontal and vertical scales it seems that surfaces of disc are impossibly deformed.

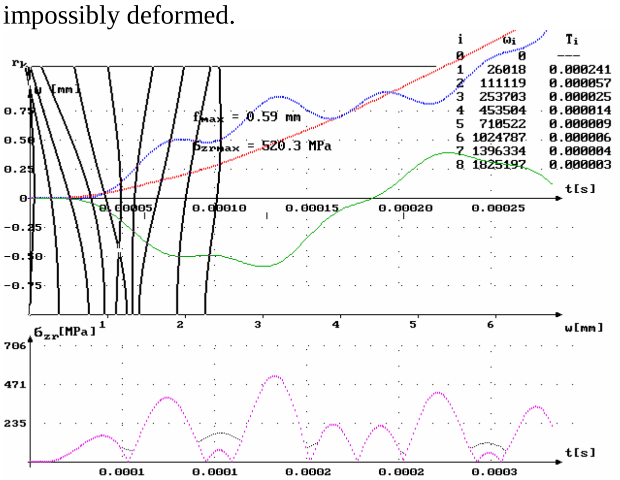

Fig.9 Result for thickness of disc h=12mm

On the basis of the IDD model (MM), we have conducted simulations determining distributions of stresses in the disc in the function of its thickness, for the unchanged parameters of IDD (fig.10).

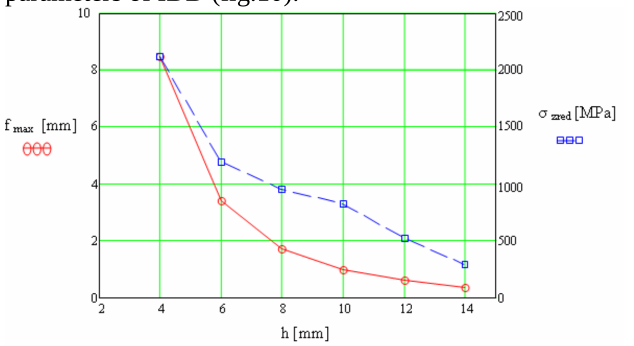

Fig.10 Trajectories of maximal reduced stress and deflection for full disc in the function of its thickness

Then, research for parameters of IDD from tab.2 was conducted. In this case distribution of pressure in radius function was more uniform (fig.11) than previously (fig.6). At that moment for the same thickness (h=4mm) reduced stress σ_{zr} decreased a lot (from above 2000MPa (fig.6) to 250MPa (fig.14)).

Initial energy of capacitor: E=450J	Capacitance of capacitor: C=100μF
Outer radius of disc: Rz=8 cm	Outer radius of coil: r_z=6 cm
Inner radius of disc: Rw=0, 1, 2, 3, 4, 5, 6 cm	Inner radius of coil: r_w=0.8 cm
Add mass: 0 kg	Winding of coil z_w=24
Material of disc: duralumin h=4mm	Thickness of coil: hc=0.3 cm

Tab. 2 Parameters of IDD (h=const)

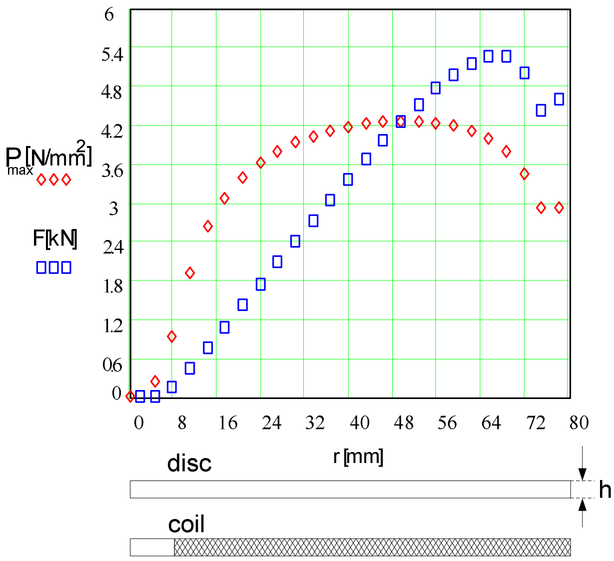

Fig.11 Distribution of pressure and force for dates from tab.2

Because results of investigations of disc without a whole show that extreme σ_{zr} is in the centre of disc, so we decided to perform simulation for various inner radius (R_w). Fig.12.13 show the original screens of the program realizing the case Rw=Ro=10mm

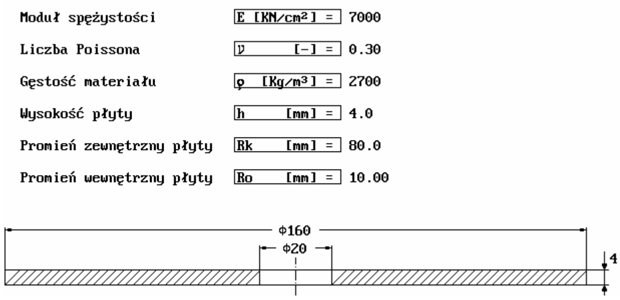

Fig.12. First screen of MM program for various inner radius

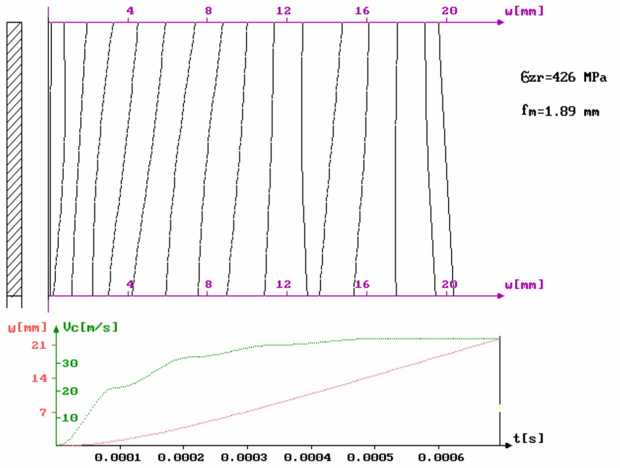

Fig.13. Second screen of MM program for various inner radius.

The results of simulations determining in distribution of stresses in disc (for various inner radiuses) with unchanged resultant force acting on the disc was also calculated (fig.14).

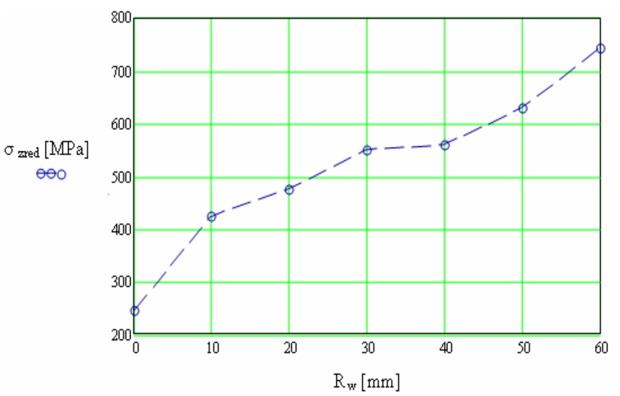

Fig.14 Maximal reduced stress in the inner radius function

Fig.14 shows that with rise of inner radius, maximal reduced stress rises although distribution of stresses along radius of the disc indicates, that maximal stress is located in the centre of the disc.

3 CONCLUSIONS

Increase of reduced stress with increase Rw radius obtained in investigated case doesn't rule out of the possibility of finding optimal disc with Rw=0. One ought to remember that the main purpose is achievement of the largest dynamic of system which depend not only on force but also on mass. Therefore investigations with discs (R=variab) achieving assumed displacement at the same time will be found and then for thus determined disc and real forces acting on them, strength analysis will be performed. On the basis of investigations conducted one can say that state of stress mainly depend on three things:

- distribution of pressure in radius function acting on disc,
- dimensions of disc,
- frequency of force acting on disc.

Results of investigations of influence that the force frequency has over strength, were not presented in this article.

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