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NONLINEAR MODEL OF VERY THIN CONDUCTING FIBERS

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Abstract: *The paper studies static V-A characteristics of very thin conducting fibers. The aim is to find the presence of a negative differential resistance in the characteristic. It is shown that this effect can be found only supposing that the nonlinear dependence of the metal conductivity on temperature is taken into account, i.e., the dependence including the quadratic term. Finally the system of fibers is studied with a proper statistical distribution of their conductivity. The parametric study was performed. The resulting V-A characteristics can be compared with the characteristic measured on the fabricated metal-insulator-metal structure. This comparison shows that the described model can serve as a good way to determine the V-A characteristics of MIM structures.*

Key words: *conducting fiber, metal-insulator-metal structure, negative differential resistance, V-A characteristic*

INTRODUCTION

Structures metal-insulator-metal (MIM) show an interesting behavior. A very challenging task is to design the MIM structure showing a negative differential resistance [1]. There are several models describing their conductivity [2]. One from them is a filamentary model [1]. The metal-insulator interface is not smooth and there are places with high electric field enabling a structural change and creation of conducting filaments. Finally the diffusion of metal, in most cases gold, through the insulator forms conducting paths between two electrodes [2]. According to [1] the number of these filaments is about $5 \cdot 10^4 \text{ mm}^{-2}$, their diameter is of the order of 1 nm. Accepting this model of passing current through the MIM structure, volt-ampere (V-A) characteristic can be calculated as the V-A characteristic of a very thin gold fiber, or more precisely the set of these fibers.

The conductivity of thin layers, and thin fibers as well, is different from that of bulk metal. This happens if the mean electron free path is longer than the layer thickness, or fiber diameter. The conductivity of the layer is therefore remarkably reduced [3] due to electron scattering on the surface.

Equations describing the V-A characteristic of a gold fiber are determined taking into account both the thermal losses inside a fiber and leaking thermal energy from fiber into a surrounding insulator, and the thermal dependence of a fiber resistance. First the model of the

V-A characteristic of a single fiber is derived and the area of a negative differential resistance is found. As it occurs at very high temperatures, not only a single fiber, but the set of fibers has to be taken into account. The particular fibers in this set have, as in a real MIM structure, different resistances defined by a normal distribution. The negative differential resistance is now caused due to overheating of fibers causing their cutting and, therefore, by the reduction of the passing current. At real structures this process is reversible. Here we, however, calculate the V-A characteristics assuming the raised voltage only.

1 V-A CHARACTERISTIC OF A VERY THIN CONDUCTING FIBER

Let us assume a metallic conducting fiber in the shape of a cylinder, l in length, and with radius r_0 , made of material with resistivity ρ . This fiber is inserted into a homogeneous ideally insulating material with thermal conductivity κ . The steady state equilibrium of this system is assumed. Resistance, or more exactly the V-A characteristic of this very thin conducting fiber is determined by power dissipated in the fiber itself and power leaked from the fiber due to cooling by the surrounding material. A temperature distribution in the material around the fiber is determined by a differential equation describing the heat propagation [4]. Its steady state solution is

$$T(r) = A + B \ln(r), \quad (1)$$

where constants A and B are determined from the boundary conditions

$$\kappa \frac{dT}{dr} = \frac{UI}{S}, \quad \text{at} \quad r = r_0 \quad (2)$$

$$T = T_\infty, \quad \text{at} \quad r = r_\infty \quad (3)$$

where r_∞ is a distance from the fiber axis where we have temperature equal to temperature T_∞ of the surrounding material, $S = 2\pi r_0 l$ is a fiber surface area. Finally we have the temperature on the fiber surface, or more precisely difference $T(r_0) - T_\infty$

$$\Delta T = T(r_0) - T_\infty = \frac{UIr_0}{\kappa S} \ln\left(\frac{r_\infty}{r_0}\right). \quad (4)$$

Fiber resistivity taking into account the dependence on temperature is

$$\rho = \rho_{20} \left[1 + \sum_{n=1}^{\infty} \alpha_n (\Delta T)^n \right]. \quad (5)$$

Current passing through the fiber is determined by Ohm's law

$$I = \frac{U}{R}, \quad (6)$$

where the fiber resistance

$$R = \frac{\rho l}{\pi r_0^2}. \quad (7)$$

The fiber temperature, and therefore its resistance (7), (5), depend on the passing current and the applied voltage, and vice versa the current (6) depends on the resistance. Therefore the system of equations (4), (5), and (6) represents the set of nonlinear equations. This set of equations was solved by an iterative procedure applied to the sequentially raising voltage.

The general dependence of fiber resistivity on temperature (5) has the form of the sum of a power series. For practical computations this series must be limited to the final number of terms. Mostly we assume (5) in the form including only linear term α_1 . To get the negative resistance in the fiber V-A characteristic, we have, however, to take at least two terms in series (5), i.e., α_1 and α_2 . Material constant α_1 can be taken from literature, e.g., from [5]. Constant α_2 was determined by fitting formula (5) to a measured resistance temperature dependence. Using this dependence for gold published in [5] and [6], we got $\alpha_2 = 3.3037 \cdot 10^{-7} \text{ K}^{-2}$ valid in the temperature interval from 0 to 500 °C. The value of α_1 is taken from [5] $4.1 \cdot 10^{-3} \text{ K}^{-1}$. The resistivity of gold at the room temperature taken from [5] is $0.023 \mu\Omega\text{m}$. This

value is valid for a bulk material. The resistivity of very thin layers and fibers, where the average free path of electrons is comparable with a layer thickness, is higher. As a reasonable guess we can take value on order higher, i.e., $\rho_{20} = 0.23 \mu\Omega\text{m}$ [7]. Assuming polymethylmetacrylate as an insulating material we have from [5] $\kappa = 0.08 \text{ Wm}^{-1}\text{K}^{-1}$.

The parametric study of the V-A characteristics of a single gold fiber has been performed. The form of these characteristics depending on all parameters was studied. Fig. 1a shows the V-A characteristics of the fiber with $r_0 = 2.5 \text{ nm}$, $r_\infty = 2.5 \mu\text{m}$, and $\rho_{20} = 0.23 \mu\Omega\text{m}$, calculated for different lengths l . The value of the negative differential resistance is higher at shorter fibers, that have at the same time the lower value of the voltage of the current maximum. Fig. 1b plots the corresponding temperature depending on voltage. Similarly Fig. 2 presents the V-A fiber characteristics and temperature via voltage calculated for the different fiber radii, and taking $l = 200 \text{ nm}$ and other parameters the same as in Fig. 1. The current maximum increases with the raised value of r_0 , and the simultaneously the area of the negative differential resistance is more apparent. The temperature corresponding to the existence of the negative differential resistance is however unrealistically high, see Fig. 1b and Fig 2b. This temperature value can be partly reduced using a thinner and longer fiber. This reduction is not substantial as we cannot change the fiber parameters arbitrarily, but we have to take into account the realistic MIM structures designed to show the negative differential resistance. Due to this we have to consider not only one fiber but the system of fibers. This is treated in the next paragraph.

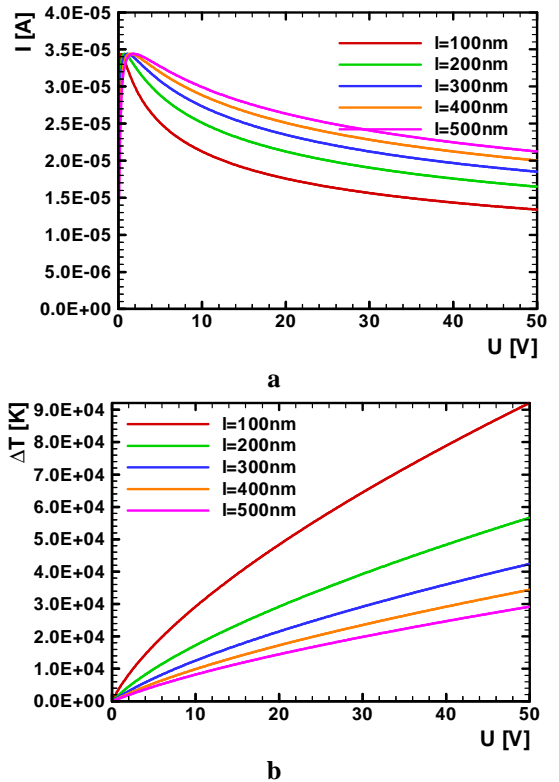


Fig. 1: V-A characteristics of the fiber with different length (a), the dependence of fiber temperature on voltage for different fiber lengths (b).

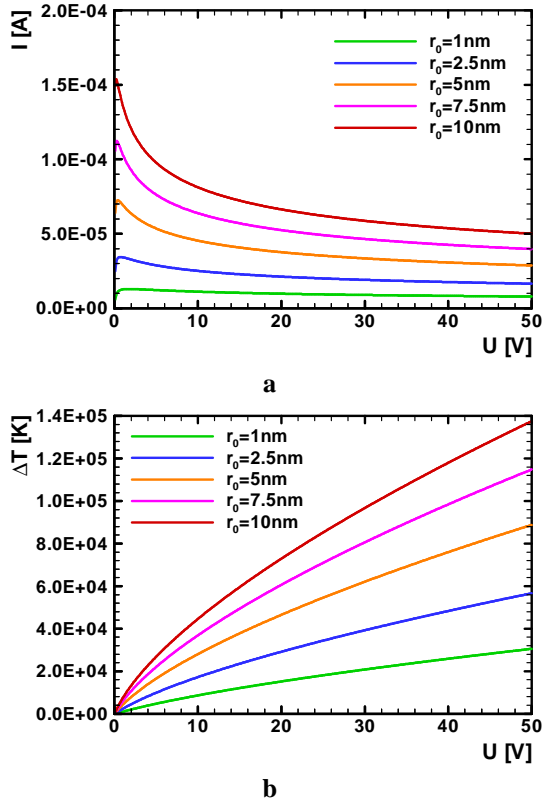


Fig. 2: V-A characteristics of the fiber with different radii (a), the dependence of fiber temperature on voltage for different fiber radii (b).

2 V-A CHARACTERISTIC OF A FIBER SYSTEM

Let assume a system of very thin metal fibers created due to the diffusion of metal into a nanolayer of polymethylmetacrylate as a result of electroforming. The V-A characteristic of this fiber system is calculated in the process of increasing voltage. The corresponding current increases and so the fiber temperature is raised. The fibers are overburnt when reaching their cutoff temperature and consequently the current decreases. At this area the negative differential resistance appears. For simplicity the fibers have the same radius and length. They differ by resistivity ρ_{20} . Its values are spread over the fiber system according to a normal distribution. We use $n = 1000$ fibers in the system. The temperature distribution in the vicinity of each fiber is assumed to be unperturbed by the presence of neighboring fibers. Under these conditions the shape of the V-A characteristic does not depend on the number of fibers. This number determines only the current value, as shown in Fig. 3 where the two V-A characteristics are plotted for 100 and 1000 fibers. The cutoff temperature, at which the particular fibers are overburnt is chosen 300°C , $r_0 = 2.5\text{ nm}$, $l = 200\text{ nm}$, other parameters are defined above. The normal distribution of the fiber resistance was normalized as the minimum value was the value $\rho_{\min} = 0.023\ \mu\Omega\text{m}$ taken from [5], and the maximum value two orders higher, i.e., $\rho_{\max} = 2.3\ \mu\Omega\text{m}$, to cover the spread of these values possible for very thin fibers

$$\rho_{20} = \rho_{\min} + (\rho_{\max} - \rho_{\min})Y, \quad (8)$$

where Y represents the distribution of the fiber resistance in dependence of its number x ,

$$Y(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, \quad (9)$$

The position of the maximum, i.e., the mean value, of the resistance distribution μ is taken equal to the number of fibers. Standard deviation σ influences significantly the calculated V-A characteristic shape and, therefore it represents the stage of freedom in defining the characteristic of a given system. This is documented in Fig. 4.

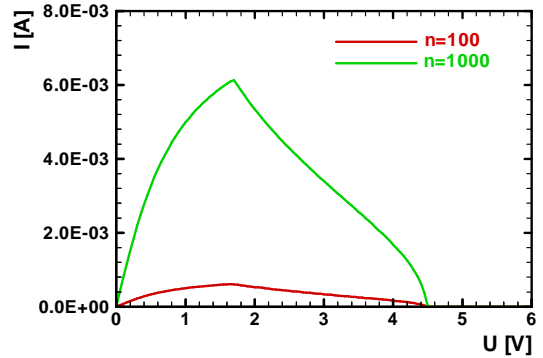


Fig. 3: V-A characteristics of the system 100 and 1000 fibers, parameters are defined in the text.

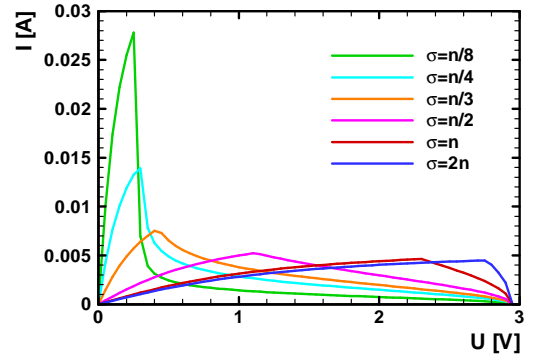


Fig. 4: V-A characteristics of the system of 1000 fibers in dependence on the standard deviation.

We have performed the parametric study of the system of fibers similarly as in the case of a single fiber. The same fiber parameters were used as defined in Chapter 1. Reducing fiber radius r_0 shifts up the voltage at which the current reaches its maximum, but at the same time reduces the current maximum. Fig. 5 shows this behavior of the V-A characteristic. On the other hand, changing the fiber length has practically no influence to the current maximal value. Increasing the fiber length causes the shift up of the voltage corresponding to the current maximum, see Fig. 6.

The cutoff temperature changes the V-A characteristics in the following way. Setting this temperature higher makes fibers to overburn at higher currents reached at higher voltages. The form of the characteristic stays, however, unchanged.

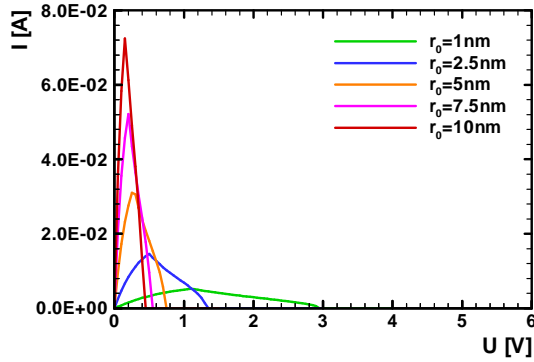


Fig. 5: V-A characteristics of the system of 1000 fibers in dependence on fiber radius r_0 . The maximum temperature is 300°C .

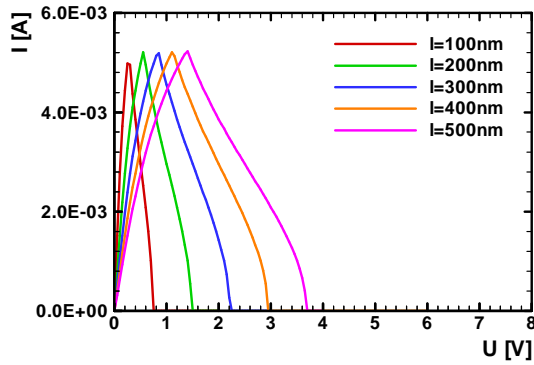


Fig. 6: V-A characteristics of the system of 1000 fibers in dependence on fiber length l . The maximum temperature is 300°C .

3 COMPARISON WITH EXPERIMENT

There is a number of parameters influencing the shape of the V-A characteristic of the system of metal fibers, that represent the behavior of a MIM structure. In the case of a fabricated MIM structure [7] we do not know exactly precise values of these parameters. Some of them can be only estimated, or can be taken from literature [1,2]. On the other hand there is the great spread of properties of fabricated MIM structures [7]. The metallic layers about 30 nm in thickness were prepared by sputtering over a mask to get the structure consisting of six MIM junctions located at crossings of gold strips shown in Fig. 7. The spin coating technique was used to prepare polymer insulating layers about 250 nm in thickness. Structures are placed on glass substrates.

The example of the measured V-A characteristic of one MIM junction from Fig. 7 is plotted in Fig. 8. This characteristic was measured on the MIM sample placed in vacuum of the order 10^{-5} Torr. A logarithmic current-voltage convertor was used to measure low currents passing through the MIM structure. The fabricated MIM structures are not stable, so the measured V-A characteristic is heavily distorted by an existing noise. The range of a negative differential resistance is, however, well apparent. The calculated V-A characteristics shown in Figs. 4, 5, and 6 have a very similar form to the measured characteristic.

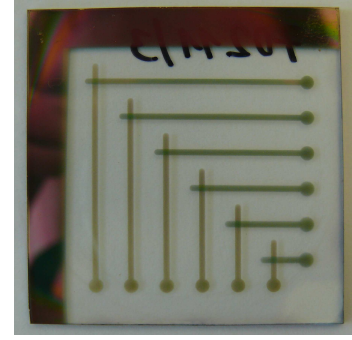


Fig. 7: Glass sheet with six MIM junctions. These junctions are represented by an insulating material at crossings of conducting strips deposited on both sides of the insulating material.

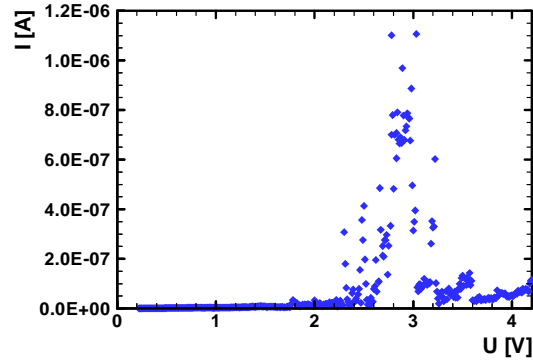


Fig. 8: Measured V-A characteristics of a MIM specimen.

4 CONCLUSION

The precise calculation of the V-A characteristics of a very thin metal fiber is presented. The nonlinear model of this characteristic accounts for both thermal losses inside the fiber and the thermal energy leaked from the fiber to a surrounding insulating material, i.e., cooling the fiber by this material. The aim was to determine the characteristic of a MIM structure, where the way of passing current can be explained by a filament model. Experiment showed that the V-A characteristic of the MIM junction exhibits an area of a negative differential resistance. In the case of only one metal fiber, this negative differential resistance can be found taking into account the nonlinear dependence of the metal resistance on temperature, i.e., not only linear term α_1 , but quadratic term α_2 as well. Under this condition the negative differential resistance appears in the V-A characteristic, but however, at the unrealistically high temperature.

The real MIM junction has a finite cross-section and there is a great number of conducting fibers created by the metal diffusion from conducting layers into the insulating material. The randomness of parameters of these fibers was taken into account by assuming their different resistances. The spread of the resistances was described by a normal distribution. The V-A characteristic of this system of fibers was calculated by increasing the voltage and calculating particular fiber temperatures. At a cutoff temperature the fiber is overburnt and the passing current due to this decreases causing the negative differential resistance. At a real MIM structure this process is, however, reversible. The

calculated V-A characteristics resemble the measured characteristics.

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