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TRANSIENTS ON NON-HOMOGENEOUS LINE WITH SPACE VARYING INDUCTANCE AND CAPACITANCE

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Abstract: *The paper deals with transient phenomena on non-homogenous transmission line. The problem is solved numerically in time domain, the transmission line with space varying capacitances and inductances is considered. Practical examples are evaluated to prove an efficiency of proposed algorithm.*

Key words: *non-homogenous transmission line, transient phenomena, space varying parameters of the line, time domain analysis*

INTRODUCTION

The models of transmission lines for very fast transient phenomena are usually created as a network with distributed parameters. However, there are many problems that the model of a homogenous line is not sufficient and it is necessary to consider a non-homogenous line whose electrical parameters are varying in space. For example if the line conductors are not parallel to the earth then the parameters of the line are changing in the dependence on the height of conductors above ground. Similarly, the connection between overhead line and underground cable can be modeled as non-homogenous (exponential) line or in the case of lightning surge response of the transmission line towers changing parameters have to be considered. There have been several approaches for searching transient phenomena on non-homogenous lines [1] – [4]. Mostly, an analysis in frequency domain is used. The solution in time domain is thereafter obtained via convolution integral or via numerical inverse Laplace transformation. In all these cases the lossless line is considered.

In this paper are searched transients on a non-homogenous lossy line in time domain. The system of partial differential equations with space varying coefficients is solved numerically. The algorithm comes out of our works [5], [7] where the homogenous transmission line was considered. The numerical process is based on Wendroff's implicit difference formula, as an

example is analyzed the voltage and current time-space distribution along the single-phase non-homogenous transmission line after lightning stroke attack. The space varying capacitances and inductances in dependence on the height conductor above ground are considered.

1 MATHEMATICAL MODEL

A general transmission system depicted on Fig. 1 consists of three parts: feeding and loading circuits and transmission line of length ℓ . The transmission line is modeled as a network with distributed parameters where voltage and current are time and space varying functions and the line parameters are depending on the distance x . In this way the transients on transmission line can be described by a system of partial differential equations of hyperbolic type with varying coefficients. The relation between voltage and current at the input and output of the line have to respect a configuration of a load circuit and source circuit and their mathematic formulation is used as the boundary conditions for solution of partial differential equations.

$$\begin{aligned} -\frac{\partial u(t,x)}{\partial x} &= R(x)i(t,x) + L(x)\frac{\partial i(t,x)}{\partial t} \\ -\frac{\partial i(t,x)}{\partial x} &= G(x)u(t,x) + C(x)\frac{\partial u(t,x)}{\partial t} \end{aligned} \quad (1)$$

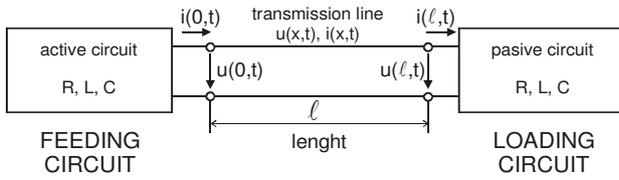


Fig. 1: Schema of the transmission system

2 ALGORITHM FOR NUMERICAL SOLUTION

For numerical solution of wave equations (1) the method of finite differences (FDTD) based on the implicit Wendroff's differential formula [5], [6] was used. The process of the way of discretization and the method of numerical solution is intimately described in previous works [5], [7]. The system of equations can be written in matrix form

$$\mathbf{A} \cdot \mathbf{v}^{(l)} = \mathbf{B} \cdot \mathbf{v}^{(l-1)} + \mathbf{D} \quad (2)$$

where $\mathbf{v}^{(l)} = [\{u_k\}, \{i_k\}]$ is a matrix of unknown discrete values of voltages and currents in an every space node $k=1,2,\dots,N+1$ of the grid at time l -level and can be evaluated from known values u_k, i_k at $l-1$ level. This time level corresponds to time $t = \Delta t (l-1)$, where Δt is a time step, the relationship between Δt and space step Δx should fill the Courant-Friedrich-Levy stability condition. The coefficients of matrixes \mathbf{A} and \mathbf{B} respect the space varying parameters of the line and they are changing point to point. Matrix \mathbf{D} is depending on the character of sources. This equation (2) can be solved in MATLAB.

3 LINE PARAMETERS DEPENDING ON CONDUCTOR SAG

It is obvious to consider parameters of the line as constant and only an average (or maximal or minimal) height of conductors above earth is supposed. In order to respect the real layout of the line the sag of conductor has been taking into account. The sag can be described with the catenaries equation [8],

$$y = \frac{\alpha}{2} (e^{x/\alpha} + e^{-x/\alpha}) \quad (3)$$

where y is the height of conductor above earth and x is the distance from the tower. The value of space varying parameters $L(x)$ and $C(x)$ are evaluated with respect to equation (3) and to the basic relationships for calculation of the capacitance and inductance (see for instance [5]). The line has been divided into small elements with varying height above the earth and the parameters have been recalculated for each element. In Fig. 2 is depicted the dependence of the value of both, capacitances and inductances on the distance x from input of the transmission line. These results are determined for 1 km long line with distance of 250 m between the towers.

The results confirm the predicted opposite change of these two parameters. The variation of inductance and capacitance with respect to the conductors sag is about 2.8%.

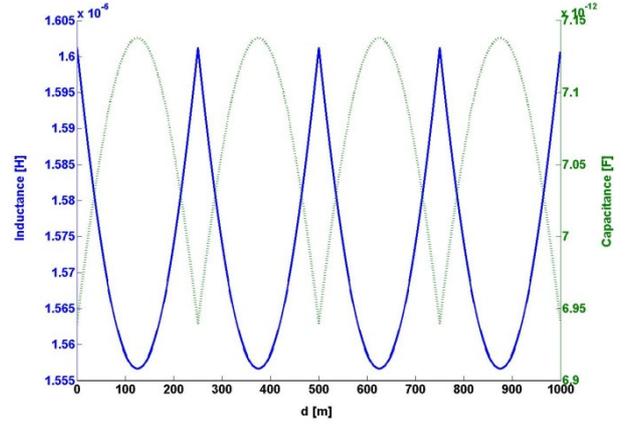


Fig. 2: Parameters L, C depending on conductor sag

3.1 Illustrative example

As an illustrative example was chosen previously mentioned 1 km long transmission line with 250 meters distance between the towers. A one phase line was considered because there is followed the influence of the conductor sag and it can be better observed on one phase line. Moreover, this example was solved in previous works and so it is possible to compare the obtained results.

The conductor have radius of 5 mm and is placed at the height of 15 m above the ground. The conductor sag is 3 m. The line is modeled like the matched line so there are no reflections on the ends. The lightning stroke impact is modeled in the middle of the line. The stroke is simulated like an injected current wave which is taken from previous works [5], [7].

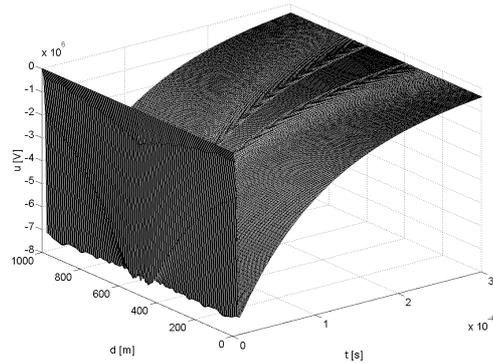


Fig. 3: Voltage distribution along the non-homogeneous line

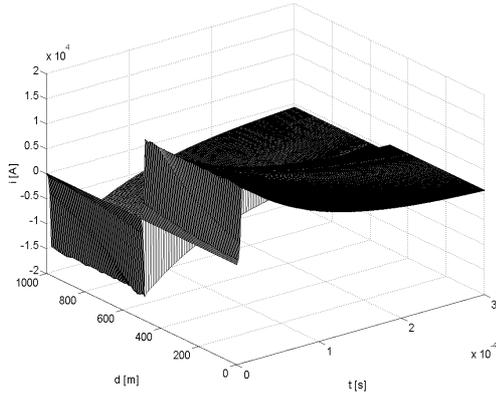


Fig. 4: Current distribution along the non-homogeneous line

3.2 Discussion

In this chapter are compared the obtained results for homogenous (Fig. 5 and Fig. 6) and non-homogenous (Fig. 3 and Fig. 4) transmission lines.

From figures 3 to 6 can be seen that for both cases (homogeneous and non-homogeneous model) are the results similar. But in detail view on these graphs there are some differences. For example it was followed the space and time position of the current and voltage maximum which changed for these cases (see Tab. 1).

	Non-homogeneous line	Homogeneous line
U_{\max} [V]	$7,642 \cdot 10^6$	$7,725 \cdot 10^6$
time [s]	$5,5 \cdot 10^{-6}$	$5,5 \cdot 10^{-6}$
distance [m]	82	83
I_{\max} [A]	$1,599 \cdot 10^4$	$1,605 \cdot 10^4$
time [s]	$5,0 \cdot 10^{-6}$	$5,5 \cdot 10^{-6}$
distance [m]	222	918

Tab. 1: The maximum voltage and current time and space position comparison

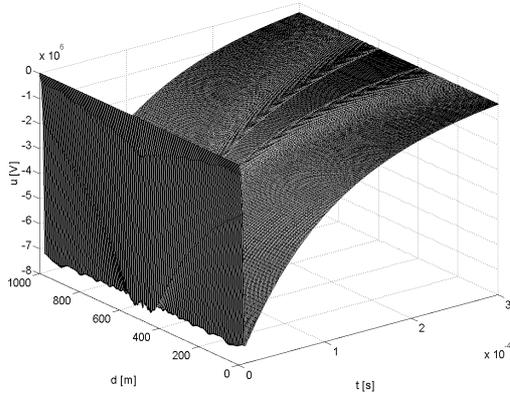


Fig. 5: Voltage distribution along the homogeneous line

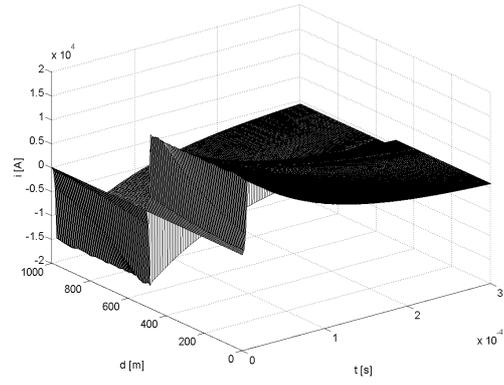


Fig. 6: Current distribution along the homogeneous line

The similar results for homogeneous and non-homogeneous transmission line model are caused by the small variation of capacity and inductance. But this example showed us that the proposed algorithm can solve the non-homogeneous problems as well.

4 EXPONENTIAL TRANSMISSION LINE

The second example which was solved is the connection between two transmission lines with various surge impedances. This connection is carrying out as an exponential line. The change of inductance and capacitance are increasing and decreasing exponential functions

$$\begin{aligned} L(x) &= L_0 e^{kx} \\ C(x) &= C_0 e^{-kx} \end{aligned} \quad (4)$$

Then it is possible to express the impedance $Z(x)$ as

$$Z(x) = \sqrt{\frac{L_0 e^{kx}}{C_0 e^{-kx}}} = e^{kx} \sqrt{\frac{L_0}{C_0}} \quad (5)$$

where the constant k is for the line length ℓ given from

$$k = \frac{1}{\ell} \ln \frac{Z_1}{Z_0} \quad (6)$$

4.1 Illustrative example

As the example of exponential transmission line was solved the interconnection between overhead line and cable line replaced with the exponential line. The length of the interconnection is 10m and the line is matched on both ends. In Fig. 7 a)-c) are shown three cases of this example. The variants a) and b) are for two boundary cases, where the line parameters are constant and are equal to $L(0)$, $C(0)$ or to $L(\ell)$, $C(\ell)$. The option c) is for varying parameters $L(x)$, $C(x)$ (see Fig. 8).

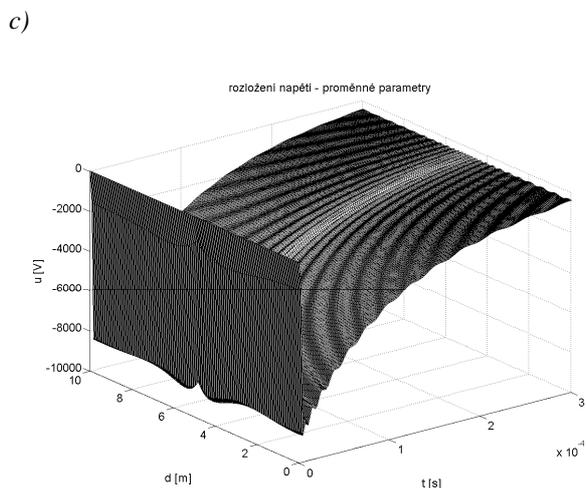
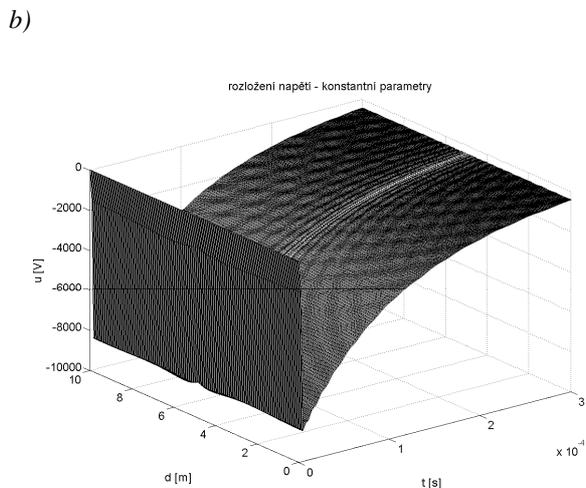
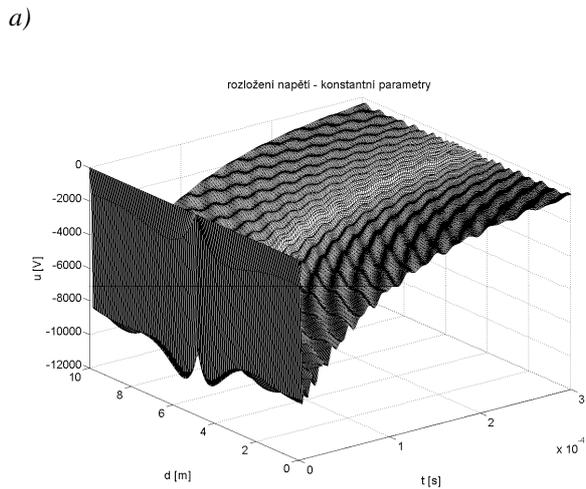


Fig. 7: Voltage distribution along the transmission line: a) for parameters $L(0)$, $C(0)$; b) for parameters $L(l)$, $C(l)$; c) for varying parameters $L(x)$, $C(x)$

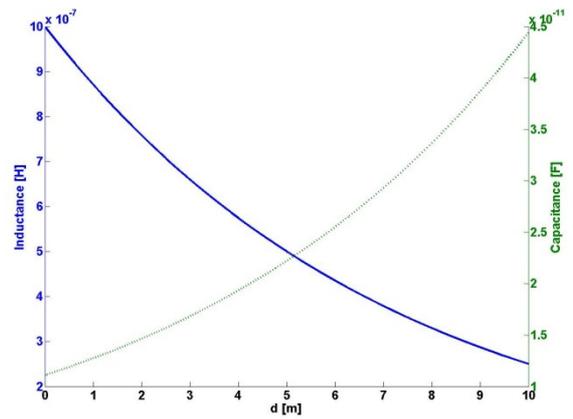


Fig. 8: Parameters L , C for exponential transmission line

5 CONCLUSIONS

The algorithm for numerical solution of wave equation for non-homogenous transmission line was introduced and it was applied on two practical examples. One of them was chosen for comparison of obtained results with the case of homogenous line which was solved in earlier works. However, in the second example was solved exponential line with very strong change of parameters, the proposed algorithm was stable and has provided results in agreement with known physical laws. So it was proved that the algorithm based on implicit Wendroff's difference formula is stable and suitable not only for homogeneous but for non-homogeneous line, as well.

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