Cylindrical structure with superconducting layer in a uniform electromagnetic field – analytical solution

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Abstract **An attempt at obtaining a model analytical solution of a nonlinear problem in the electromagnetic field is presented. The analysis is brought forth for a three-layer structure, with a nonlinear conducting region, which is placed in a uniform harmonic electromagnetic field. The solution is obtained by the method of small parameter. Integral error calculations have been performed in order to check the solution.**

Keywords **superconducting, analytical solution, nonlinear conductivity.**

I. Introduction

The aim of this paper is to present an analytical solution to a boundary value problem. A three-layer structure has been placed in a uniform harmonic electromagnetic field (Figure 1) of industrial frequency (thus, displacement currents can be omitted). Each layer consists of a material with different conductive properties. The interior core is a linear conductive layer (μ_c, γ_c properties of copper). It is surrounded by a dielectric insulator (assumed γ_d=0). The exterior layer consists of a superconducting material.

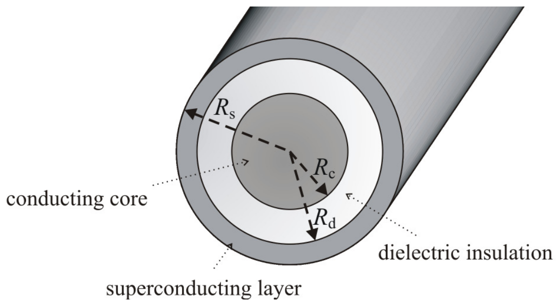

Fig. 1. Cylindrical structure with nonlinear region

The considerations are theoretical yet the material properties resemble real values. A characteristic of a real superconducting BSCCO [1] material has been chosen to introduce a nonlinear conducting material. Magnetic permeability on the other hand, has been assumed as a constant value expressed by relative magnetic permeability of $\mu_{rs}=10^{-3}$ following diamagnetic properties of a superconductor. The solution for this layer has been obtained using the method of small parameter [2]. Further on, an integral error [3] is defined to check the solution of the nonlinear region.

II. Approximation of superconductor *J-E* curve

The *J-E* curve of a superconductor, for a constant temperature, can be expressed by the odd power series:

$$J(E) = \sum_{k=1,3,5...}^{m} \gamma_{sk} E^k. \tag{1}$$

To reduce the order of the polynomial, a comparison between the real characteristic and its approximations has been made (Figure 2) for values below a certain threshold. The maximum value of electric field strength is 1.5μV/cm. It has been chosen following a commonly used value of 1μV/cm for which a superconductor obtains its critical field values [4].

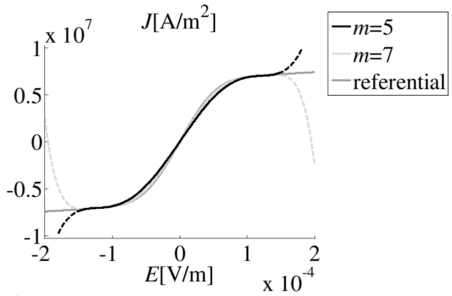

Fig. 2. *J-E* curve of superconductor. Comparison between real curve and approximations

For the chosen interval, *m*=5 and *m*=7 approximations do not differ strongly. Therefore, only a fifth order polynomial with odd terms has been chosen.

III. Analytical solution

The solution is presented using the magnetic vector potential. The differential equation describing the problem is:

$$\begin{aligned} L(t,r) &= \frac{\partial^2 A_s(t,r)}{\partial r^2} + \frac{1}{r}\frac{\partial A_s(t,r)}{\partial r} + \gamma_{s1}\mu_s \frac{\partial A_s(t,r)}{\partial t} = \\ &= \gamma_{s3}\mu_s \left(\frac{\partial A_s(t,r)}{\partial t}\right)^3 + \gamma_{s5}\mu_s \left(\frac{\partial A_s(t,r)}{\partial t}\right)^5 = R(t,r). \end{aligned} \tag{2}$$

The solution is obtained using the method of small parameter, therefore a solution of the form:

$$A_s(t,r) = \sum_{i=1}^{n} \gamma_{s5}{}^{i-1} A_i(t,r), \tag{3}$$

is obtained, consisting of *n* terms for which differential equations are formulated:

$$\frac{\partial^2 A_i(t,r)}{\partial r^2} + \frac{1}{r}\frac{\partial A_i(t,r)}{\partial r} + \gamma_{s1}\mu_s \frac{\partial A_i(t,r)}{\partial t} = W_i(t,r). \tag{4}$$

The solution is simplified to only one harmonic component, like in known literature where the method was used [5], [6]. Often only two or three terms of (3) are considered meaning one or two correction terms. This is because obtaining the right hand side terms of (4) is very time consuming (the higher the index of the equation, the more complicated the calculations). However, the authors have developed an algorithm, which aids in the construction of the W terms. In addition, the solution is obtained with respect to constant parameters c and θ [3]:

$$\begin{bmatrix} \underline{A}_{i,h}(r) \\ \dfrac{\mathrm{d}\underline{A}_{i,h}(r)}{\mathrm{d}r} \end{bmatrix} = \begin{bmatrix} e^{\mathrm{j}\theta} \sum\limits_{k=1,3,5...}^{p} c^k \underline{a}_k(r) \\ e^{\mathrm{j}\theta} \sum\limits_{k=1,3,5...}^{p} c^k \underline{b}_k(r) \end{bmatrix}. \quad (5)$$

where $\underline{a}$ and $\underline{b}$ represent complex coefficients which are dependent on the geometry and material properties.

The exterior electromagnetic field is expressed by a constant E or H value on the boundary of the problem. The field on the side surface $r=R_s$ can be defined by imposing a Dirichlet boundary condition:

$$\underline{E}(R_p) = -\mathrm{j}\omega_0 \underline{A}(R_p) = \underline{\Xi}_D, \quad (6)$$

or if need be a Neumann boundary condition:

$$\underline{H}(R_p) = -\frac{1}{\mu_s}\frac{\mathrm{d}\underline{A}(R_p)}{\mathrm{d}r} = \underline{\Xi}_N. \quad (7)$$

The $\underline{\Xi}$ coefficients represent electric or magnetic field strength boundary values.

Using the (5) relation a polynomial of degree $p=4n+1$ must be solved. Thus, the full solution is evaluated.

IV. Integral Error and Exemplary Results

An integral error is defined in two forms – in reference to amplitude:

$$e_{\int \mathrm{amp.}} = \left(\frac{1}{R_s - R_d} \int_{R_d}^{R_s} \left| 1 - \left| \frac{\underline{R}(r)}{\underline{L}(r)} \right| \right| \mathrm{d}r \right) \cdot 100\%, \quad (8)$$

and phase:

$$e_{\int \mathrm{ph.}} = \left(\frac{1}{R_s - R_d} \int_{R_d}^{R_s} \left| \frac{1}{\pi} \arg\left(\frac{\underline{R}(r)}{\underline{L}(r)} \right) \right| \mathrm{d}r \right) \cdot 100\%. \quad (9)$$

The maximum value of electric field strength is applied in a Dirichlet boundary condition to check the accuracy of the solution in case of the strongest nonlinearity. Results are presented for one to six terms of (3) on Figure 3.

To ensure accurate results, six terms have been taken into account in the next calculations. Electric and magnetic field distributions in the nonlinear conductor, for three different values of applied Dirichlet and Neumann boundary conditions respectively, have been presented on Figure 4.

A heterogeneous distribution of electric field strength can be noticed which is a common occurrence in superconductors [7]. The exterior layer also expresses very good shielding of the magnetic field.

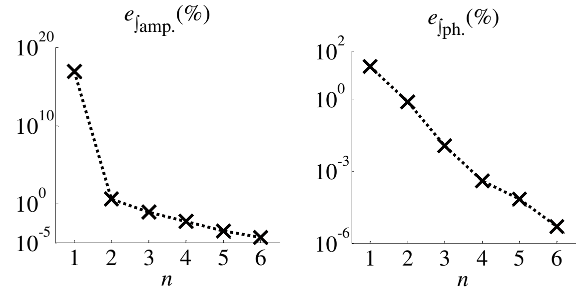

Fig. 3. Amplitude and phase integral error calculation results (in logarithmic scale)

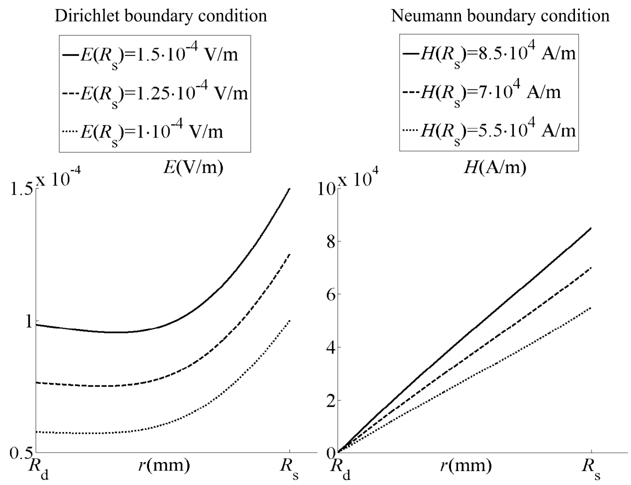

Fig. 4. Electric and magnetic field strength distribution for applied boundary conditions

V. Conclusions

Model analytical solutions for both Dirichlet (6) and Neumann (7) boundary value problems have been obtained. The analysis focused on the nonlinear region of the three-layer cylindrical structure. The method of small parameter has been used, therefore the solution was assumed in the form (3). Errors according to chosen criteria have been calculated. The error values have been presented for values of n from one to six. It could be noticed that for a higher number of terms the result becomes more accurate. For six terms used, exemplary E and H distributions have been calculated.

VI. References

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