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IDENTIFICATION OF MULTI-COMPARTMENT DARCY FLOW MODEL MATERIAL PARAMETERS

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Abstract: This papper deals with modelling of blood perfusion in the liver tissue and focuses on the identification of the model parameters. For a description of perfusion a multi-compartment Darcy model is used. This model describes a hierarchical flow in the tree structure of the liver vasculature. The research is motivated by modelling of liver perfusion with the purpose to enable an improved analysis of CT scans. The identification problem is formulated as an optimization problem where the optimization parametrs are approximated by using a splinebox. For the numerical simulations the finite element method in the in-house developed FEM based software *SfePy* is used.

Keywords: Liver perfusion; multi-compartment model; porous media; Darcy flow; identification; optimization

1 Introduction

Modeling of blood perfusion in liver tissue called parenchyma is a significant problem of present biomechanical and biomedical research. Although the perfusion models can be also used for describing coronary perfusion of the heart [4], blood flow in the brain and more. This research is motivated by the needs of surgeons who would like to have an effective tool for easier and more accurate surgery planning as well as for predicting and simulating changes in hepatic perfusion. These changes in perfusion occur with changes in structure or volume of liver tissue due to various illnesses, such as cancer and liver cirrhosis, or due to subsequent treatment primarily such as resection, which is removing part of the tissue affected by the tumor. A significant disadvantage of liver perfusion models is difficulty to determine their parameters. Some of them correlate to the vascular trees geometry but others are defined only in the context of the model and can not be measured directly. Therefore a significant part of this paper deals with identification of parameters associated with perfusion models.

2 Hierarchical modeling of liver perfusion

The structure of liver tissue and liver vasculature is very specific and complex, see [1]. There are two bloodstreams and three vascular tree systems in the liver. A complex hierarchical vessels branching system of hepatic artery tree and portal vein *(vena portae)* tree with various diameters which diminish consecutively at each bifurcation, begining from largest vessels down to capillaries. The ascending system of hepatic vein *(vena hepatica)* tree is arranged in the reverse sense. All systems bifurcate to about 20 generations [3]. At the capillary level the blood is filtrated in the hepatic units called lobules which are typically considered as hexagonal prisms. The blood flow through liver is characterized at several scales for which different models are used. Flow in the larger vessels is described by 1D model based on Bernoulli equation. Flow in the liver tissue which is considered as a porous media is described by 3D multi-compartment Darcy flow model. Both models are connected through the sources and sinks, see Figure 1.

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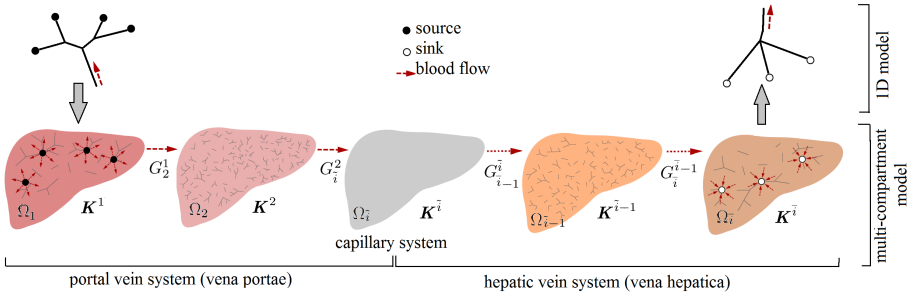

Figure 1: Structure of multi-compartment model and its connection to the 1D model. [5]

3 Multi-compartment Darcy flow model

Compartments are associated with the lower hierarchies of vascular trees. Each compartment occupies the continuum domain Ω and contains only blood vessels of a certain diameter, see Figure 1. The properties of i-th compartment are given especially by the permeability tensor $\boldsymbol{K}^i$ and perfusion parameters G^i_j between compartments i and j. The state problem of 3D perfusion multi-compartment model is defined in weak form (1) which is benefitial for the numerical implementation in in-house developed FEM software *SfePy (Simple Finite Elements in Python)*, [1], [5]:

$$\int_{\Omega_i \setminus \Sigma_i} \boldsymbol{K}^i \nabla p^i \cdot \nabla q^i + \int_{\Omega_i \setminus \Sigma_i} \sum_j \underbrace{G^i_j \left(p^i - p^j\right)}_{\mathcal{J}^i_j} q^i = \int_{\Omega_i \setminus \Sigma_i} f^i q^i, \quad \forall q^i \in Q^i, \tag{1}$$

where $i, j = 1, 2, ..., \bar{i}$ are indexes of compartments, p^i (p^j) is pressure in i-th (j-th) compartment, f^i is external source or sink of i-th compartment, q^i is the test function and $\mathcal{J}^i_j$ is intercompartmental flux which describes the amount of fluid transported from compartment i to j. The solution of the state problem equation (1) are pressure fileds in all compartments $\boldsymbol{p} = (p^1, p^2, ..., p^{\bar{i}})$.

4 Identification of material parameters

As was mentioned above, the disadvantage of liver perfusion models is the difficulty to determine perfusion parameters G^i_j, which can not be measured directly. Therefore an identification problem was formulated as optimization problem, see [6]. The optimization parameters $\boldsymbol{\alpha} = (\alpha^i_j)$ for which $G^i_j = \bar{G} \cdot \alpha^i_j$, where $\bar{G}$ is a constant such that $\bar{G} > 0$ are identified. However for uncoupled compartments $G^i_j \equiv 0$ and $\alpha^i_j = 0$. We assumed that the correct intercompartmental fluxes $\bar{\mathcal{J}}^i_j$ can be measured, so that parameters G^i_j is computed by minimizing the objective function φ expressed as

$$\varphi(\boldsymbol{\alpha}, \boldsymbol{p}) = \sum_i \int_{\Omega_i} \sum_{j>i} \left| \underbrace{G^i_j(\alpha^i_j)(p^i - p^j)}_{\mathcal{J}^i_j} - \bar{\mathcal{J}}^i_j \right|^2, \tag{2}$$

where pressures p^i and p^j satisfies equation (1). The optimization parameters $\boldsymbol{\alpha} = (\alpha^i_j)$ are approximated by using the splinebox defined by the control polyhedron (3).

$$\boldsymbol{\alpha}(\boldsymbol{t}) = \sum_k \mathbf{c}_k B_{k,d}(\mathbf{t}), \qquad \mathbf{t} \in \Omega, \tag{3}$$

where $\mathbf{c}_k$ are splinebox control points and B_k are B-spline base function with degree d. For numerical simulations the cubic splinebox ($d = 3$) with B-spline base functions implemented in *SfePy*, see [2] was used. Besides solving the state problem is as part of the optimization process calculating the sensitivity analysis and solving the adjoint problem, see [1].

5 Validation of multi-compartment model

For a validation of the multi-compartment Darcy flow model, the 1D flow model decribing the Poiseuille flow on the branching network was considered as the reference model, see [5]. Where for validation the spatial pressure distributions from both models were compared. The vasculature tree was generated by the GCO *(Global Constructive Optimization)* method.

6 Numerical results

For numerical simulations a block goemetry of 144 (12x12x1) elements was used, see Figure 2. In this block geometry the perfusion tree with different vessels diameter was generated by the GCO method, see Figure 2. The multi-compartment Darcy model involved two compartments. At the top and bottom geometry face a periodic boudary conditions were prescribed. As the input data for 3D multi-compartment Darcy model the data computed by the 1D Poiseuille model were used. These are permeability tensors $\boldsymbol{K}^i$, initial values of perfusion parameters $\bar{G}$ and reference intercompartmental fluxes $\bar{\mathcal{J}}^i_j$. For approximation of optimization parameters $\boldsymbol{\alpha}$ the cubic splinebox with B-spline base functions and 20 control points was used, see Figure 2. All initial values of optimization parameters $\boldsymbol{\alpha}$ were set to 1. Box constrains conditions for $\boldsymbol{\alpha}$ were set as 0 and 10. For minimization of the objective function (2) the method *minimize* with *SLSQP* solver imlemented in *SciPy.optimize* was used.

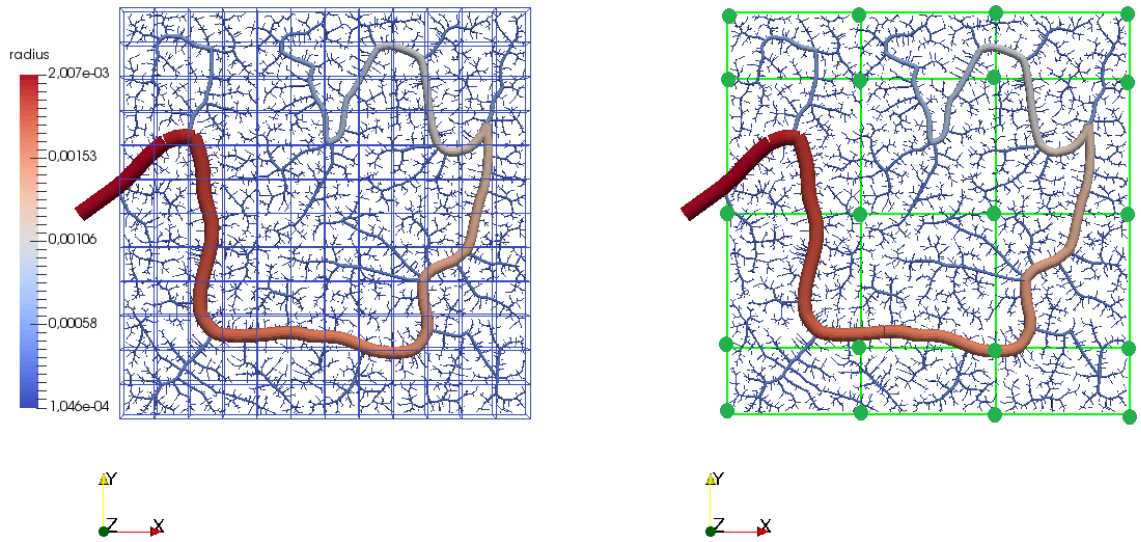

Figure 2: Left: Vacsular tree generated by GCO method in the block geometry used for numerical simulations. Right: Splinebox polyhedron with 20 control points used for approximation of optimization parameters displayed over the generated vascular tree.

The process of optimization is shown in the graph of objective function values depending on the number of computed iterations, see Figure 3. For finding the objective function minimum just 341 iterations were needed. During the identification the value of the objective function decreased by 32.5%.

In the Figure 4 is shown initial values of intercompartmental flows difference $f\_\mathrm{int} = \mathcal{J}^1_2 - \bar{\mathcal{J}}^1_2$ and values gained after optimization $f\_\mathrm{opt} = \mathcal{J}^1_2 - \bar{\mathcal{J}}^1_2$. Values of optimization parameters $\boldsymbol{\alpha}$ after optimization are also shown in Figure 4.

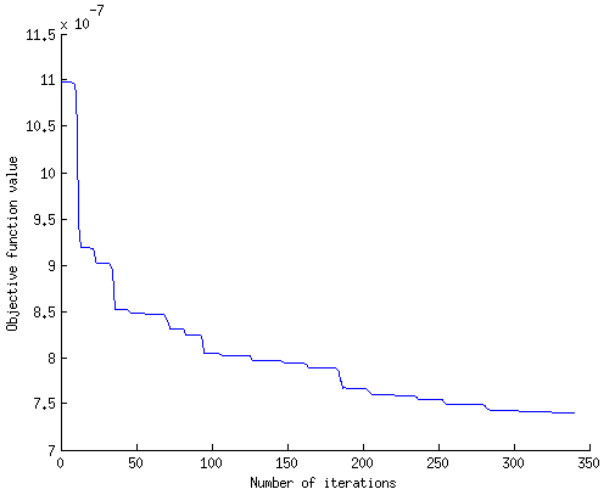

Figure 3: The objective function values depending on the number of iterations.

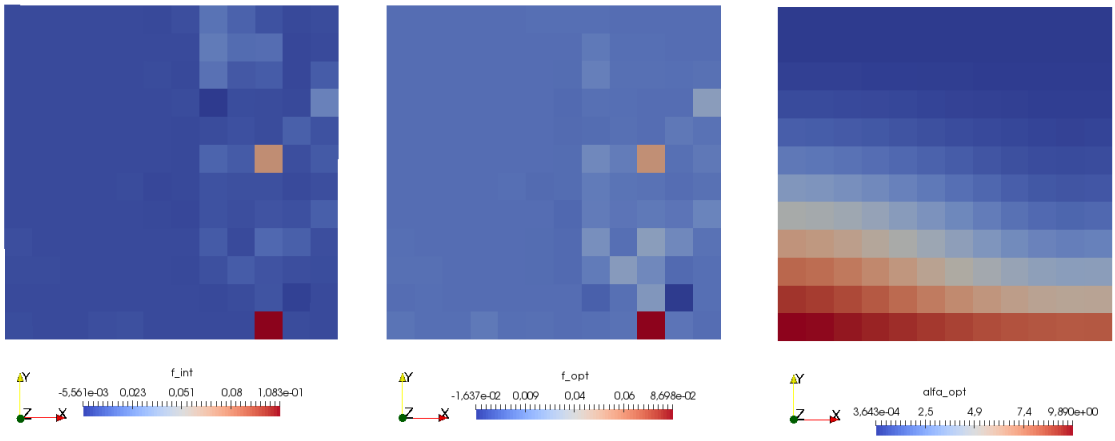

Figure 4: Left: Difference of initial values of intercompartmental flow and measured intercompartmental flow f_int. Middle: Difference of optimized values of intercompartmental flow and measured intercompartmental flow f_opt. Right: Values of parameters $\boldsymbol{\alpha}$ after optimization.

7 Conclusion

In the paper the multi-compartment model of the liver tissue blood perfusion which can be used for developing tools for improvement analysis of CT scans was proposed. A significant disadvantage of this model is the difficulty to determine their parameters which can not be measured directly. Therefore is the identification of perfusion model parameters a crucial part of future research. The identification problem was formulated as an optimization problem in which the splinebox was used for parameters approximation and initial values were computed by 1D Poiseuille model. Numerical simulations were performed in in-house developed FEM software *SfePy*.

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