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Wave propagation in graded bars

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Nowadays, additive technology is modern and high technology for manufacturing of complex bodies with advanced properties and various and complicated shapes, where conventional technologies are not possible to use. Applications of 3D printing technologies of metals can be found in mechanical, biomechanical or aerospace engineering. For that reason, understanding to wave processes in heterogeneous, layered and functionally graded materials is important issue for this time [5]. In this contribution, an explicit local time stepping scheme for modelling of discontinuous wave propagation problems in heterogeneous bars by the finite element method is used based on work [7]. In this method, the local stepping algorithm with respect to local wave speed and local stability condition at each material point is employed. The accurate modelling of discontinuous wave propagation in elastic heterogeneous bodies is still an open problem in numerical methods. Wave propagation problems in graded elastic bars has been studied in [4]. Numerical methods currently used comprise the finite volume method [2], higher order discontinuous Galerkin formulation, the graded finite element method [6] and etc.

We study discontinuous wave propagation in a graded bar with the linear distribution of elastic modulus and constant mass density. The analytical solution of the problem can be found in [3]. The length of the bar is $L = 1$ m. In this test, the mass density is chosen as $\rho = 1$ kg/m^3. The elastic modulus on the left side of the bar is $E_1 = 1$ Pa and on the right side it is set as $E_2 = 2.25$ Pa. The bar is loaded on the left side by the Heaviside pulse with the stress amplitude as $\sigma_0 = 1$ Pa. Results of numerical solution of the elastic wave propagation problem in the graded elastic bar are presented in Fig. 1 for time $T = 0.75$ s obtained by the analytical solution [3], semi-analytical solution with the numerical inverse Laplace transform [1], the finite volume method [2], the finite element method with explicit time integration by the central difference method, the finite element method with/without the local time stepping. For FEM and FVM, the time step size was set as a minimum value of local stable time steps over all elements/cells. Based on the test, the nominated scheme is suitable for accurate modelling of wave propagation in graded media.

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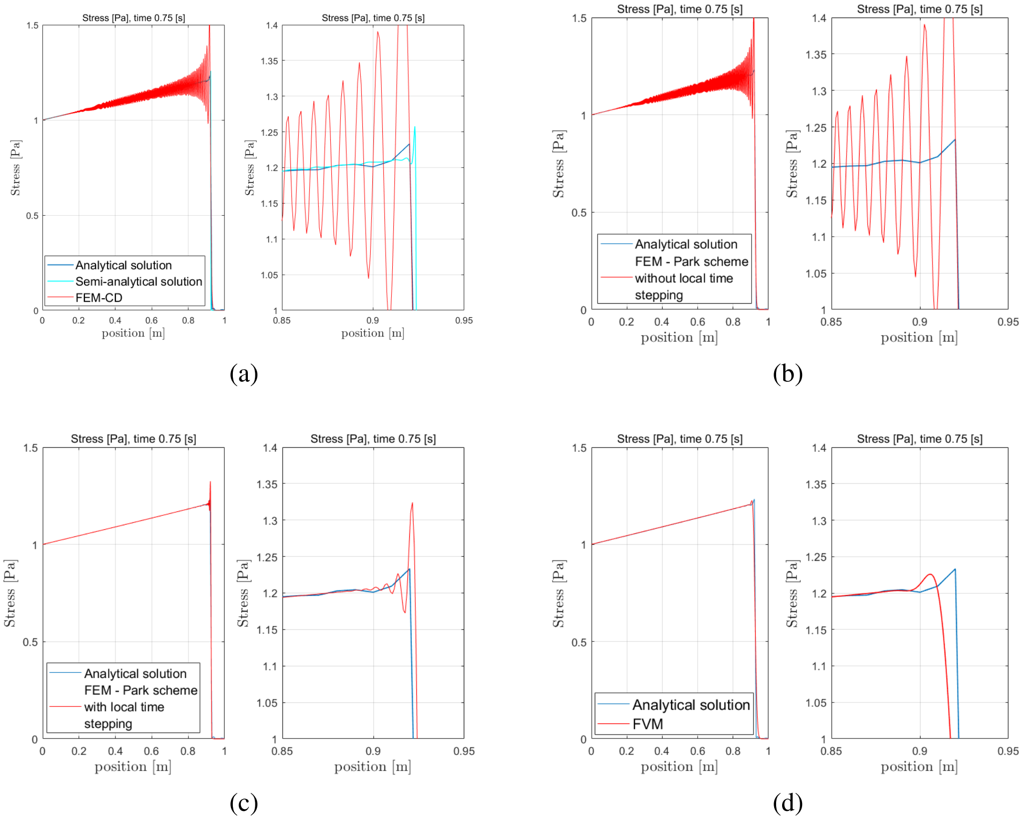

Fig. 1. Stress distributions in a graded bar obtained by a) FEM with the central difference method (CD), b) FEM with the Park method without local time stepping, c) FEM with the Park method with local time stepping, and d) FVM

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References

[1] Adámek, V., Valeš, F., Červ, J., Numerical Laplace inversion in problems of elastodynamics: Comparison of four algorithms, Advances in Engineering Software 113 (2017) 120-129.

[2] Berezovski, A., Engelbrecht, J., Maugin G.A., Numerical simulation of waves and fronts in inhomogeneous solids, World Scientific Pub Co Inc., 2008.

[3] Brepta, R., Huněk, I., Impulse propagation in a thin bar with variable Young modulus, Technical Report Z1032/87, Institute of Thermomechanics CAS, 1987.(in Czech)

[4] Chiu, T.-C., Erdogan, F., One-dimensional wave propagation in a functionally graded elastic medium, Journal of Sound and Vibration 222 (1999) 453-487.

[5] Ebrahimi, R., Advances in functionally graded materials and structures, InTech, 2016.

[6] Kim, J.-H., Paulino G.H., Isoparametric graded finite elements for nonhomogeneous isotropic and orthotropic materials, Journal of Applied Mechanics 69 (2002) 502-514.

[7] Park, K.C., Lim, S.J., Huh, H., A method for computation of discontinuous wave propagation in heterogeneous solids: Basic algorithm description and application to one-dimensional problems, International Journal of Numerical Methods and Engineering 91 (6) (2012) 622-643.