COMPUTATIONAL MECHANICS 2018
34th conference with international participation
Srní
October 31 - November 2, 2018

Self-oscillation of the two-axis gyroscopic stabilizer

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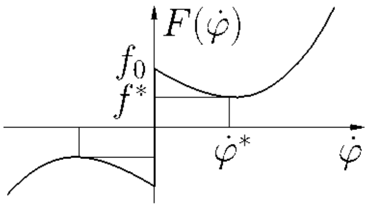
Van der Pol introduced a solution of the self-oscillations of spring suspended body siting upon the uniform velocity moving rough conveyor belt in 1934. Friction between body and conveyor belt was non-Coulomb, which characteristics has negative slope in the certain interval – see Fig. 1. This classic problem, which is introduced in lots of non-linear vibrations related textbooks, motivated several works (see [1, 2, 3, 5]) whose refer to the fact that solution leads to the same equation (such as pin rotating in hub). Chernikov in [3] demonstrates that transformation of one-axis gyrostabilizer self-oscillations lead to the same equation in certain case. Mentioned Chernikov's work motivated us to analyze self-oscillations of two-axis gyroscopic stabilizer with non-Coulomb friction in the axis of stabilizer outer gimbal caused by uniform rotation speed of the stabilizer base.



$$M_T(\dot{\varphi}) = f_0\left[\operatorname{sign}\dot{\varphi} - d_1\dot{\varphi} + d_3\dot{\varphi}^3\right] \qquad (1)$$

Fig. 1. Non-Coulomb friction characteristics

Similarly to Chernikov, we base our analysis on the system simplified as much as possible, using the following assumptions:

1. Mechanical system is in basic configuration – axes of gimbals are mutually perpendicular and horizontal, while flywheel's axes are vertical.
2. Correction and compensation feedbacks are deactivated – so that skew symmetric matrix of non-conservative forces is zero.
3. All gimbals and flywheels are static and dynamic balanced.
4. Flywheels angular momentum is sufficiently high.

Routh equations of motion are:

$$\begin{aligned} a_{11}\ddot{\varphi} + a_{13}\ddot{\varepsilon} - H\dot{\mathrm{E}} &= M_{T1}, \\ a_{22}\ddot{\psi} - H\dot{\varepsilon} + a_{24}\ddot{\mathrm{E}} &= M_{T2} = 0, \\ a_{31}\ddot{\varphi} + H\dot{\psi} + a_{33}\ddot{\varepsilon} &= 0, \\ H\dot{\varphi} + a_{42}\ddot{\psi} + a_{44}\ddot{\mathrm{E}} &= -H\dot{\beta}_0, \end{aligned} \qquad (2)$$

where a_{11} to a_{44} stand for algebraic functions of moments of inertia, H stands for angular momentum of the flywheels, M_{Ti} are for dry friction torques and $\dot{\beta}_0$ stands for a constant rotation speed of the stabilizer base. Second third and fourth equations have first integrals. According to Merkin [4], roots of characteristic polynomial are divided into two groups – nutation and precession, if angular momentum H is sufficiently high. But in spite of the absence of non-conservative positional forces (so called radial corrections), roots of the precession motion are identically zero. The only torque acting on the system is non-Coulomb friction in the first Eq. (2), which describes a motion of the outer gimbal. Thus the characteristic polynomial of the system Eq. (2) with no damping has two pairs of the pure imaginary roots whose imaginary parts represents the nutation frequencies.

Discontinuity at zero mutual velocity and a cubic part of the non-Coulomb friction torque characteristics Eq. (1) are the only nonlinearities in the system. We are tackling this nonlinearity using a harmonic linearization method, similarly to [3]. We are assuming that the frequency of the self-oscillations will be close to one of the natural frequencies. We can determine the amplitudes of the self-oscillations using the condition of the real part of characteristic polynomial root of the system with a linearized damping force to be identically zero at the natural frequency.

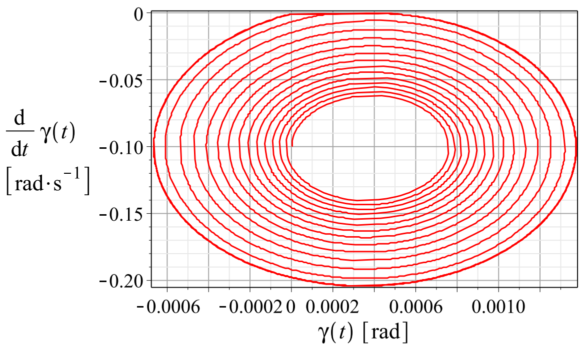


Fig. 2. Phase trajectory of outer gimbal with respect to inertial system – amplitudes reaching a limit cycle

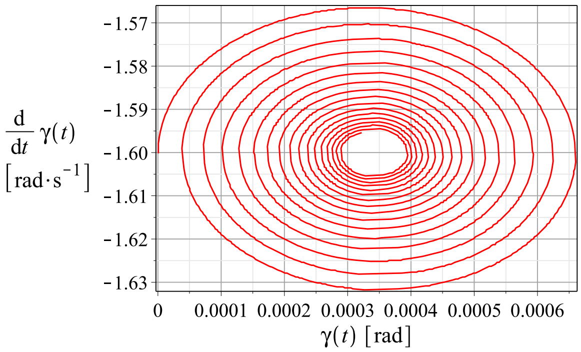


Fig. 3. Phase trajectory of outer gimbal with respect to inertial system – asymptotic stable

Limiting ourselves on the semi-trivial solution (assuming motion only with one frequency) and using table of the non-Coulomb friction characteristics according to Chernikov [1], we can show, by the numeric simulation of the system Eq. (2) , e.g. Fig. 2 and Fig. 3, that the phase trajectories of the motion for the certain setting of base rotation speed $\dot{\beta}_0$ are converging to the limit cycle and their amplitudes corresponds to the approximate solution of the linearized system. This can be considered as a proof of the self-oscillations existence.

Acknowledgements

This article was written at the Technical University of Liberec, Faculty of Mechanical Engineering with the support of the Institutional Endowment for the Long Term Conceptual Development of Research Institutes, as provided by the Ministry of Education, Youth and Sports of the Czech Republic in the year 2018.

References

[1] Andronov, V.V., Zhuravlev, V., Dry friction in problems of mechanics, Moscow, 2010.
[2] Blekhman, I.I., Vibracionnaya mekhanika, Moskva, 1994. (in Russian)
[3] Chernikov, S.A., Self-oscillation of a gyroscopic system with dry friction in gimbal axes during angular motion of the support, Herald of the Bauman Moscow State Technical University 95 (2) (2014) 28-39. (in Russian)
[4] Merkin, D.R., Giroskopicheskie systemy, Leningrad, 1974. (in Russian)
[5] Zhuravlev, V.V., Klimov, D., Prikladnie metody v teorii kolebanii, Moskva, 1998. (in Russian)