

Application of the harmonic balance method for investigation of dynamic parameters of the rotors mounted on linear/nonlinear coupling elements

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In modelling of rotating systems, the connection of the rotor to the stator part is often performed by means of coupling elements that behave nonlinearly, such as hydrodynamic and magnetic bearings, squeeze film dampers [2], etc. The Harmonic Balance Method (HBM) represents a tool for investigating the steady-state response of nonlinear rotating systems, which is periodic or even quasiperiodic in time [3].

Computational models are designed for rotating systems [1] such as rotors and assume that individual parts are flexible bodies such as the shaft and the discs. The motion equation of the undamped rotating system including the stator part, expressed in the fixed space is

$$\mathbf{M}\ddot{\mathbf{q}}(t) + \omega_0 \mathbf{G}\dot{\mathbf{q}}(t) + \mathbf{K}\mathbf{q}(t) = \mathbf{f}(\mathbf{q}, \dot{\mathbf{q}}, t), \quad (1)$$

where \mathbf{M} , \mathbf{G} , and \mathbf{K} are global matrices of the mass, the gyroscopic effects, and the stiffness of the system, respectively, \mathbf{f} is generally a nonlinear vector which includes the loading and coupling forces, \mathbf{q} is the global vector of the nodal displacements, ω is the angular velocity of rotation of the rotor and t is the time. The symbol (\cdot) denotes a derivative with respect to time.

The computational model created this way can have many degrees of freedom n because the rotating system is discretized by solid finite elements (FE). HBM assumes that the steady-state response can be approximated by a finite number of members N of the Fourier series. When using HBM, the total number of primary unknowns (which are elements of vectors of the absolute, cosine and sine coefficients of the Fourier series) is equal to $(2N+1) \cdot n$, which can dramatically increase the demands on computing power. Therefore, a software library is being developed at the IT4Innovations National Supercomputing Center, which will enable problems of this type to be solved efficiently using high performance computing (HPC) resources.

A library for computational modelling of rotating systems is being developed in the MATLAB programming environment. Within this library, program packages are being created for: (i) stationary analysis, (ii) modal analysis, and (iii) calculation of transient and steady-state response of forced vibration. In this library, selected coupling elements between rotating and stationary parts can be realized with linear or nonlinear models of coupling.

The computational algorithms from the newly developed program library are tested on rotor systems. The first test rotor system (Fig. 1) consists of an elastic shaft and one elastic disc. The second test rotor system (Fig. 2) is comprised of an elastic shaft and two elastic discs. The first disc is located at the bearing span and the second one is located on an overhanging end of the shaft. Both test rotor systems are mounted on two radial hydrodynamic bearings and one axial roller bearing. The coupling areas are shown in Fig. 1

and Fig. 2 marked in green colour. The stator part is assumed to be represented by an absolutely rigid body. The rotor material was considered homogeneous, isotropic, and linearly elastic. The rotors are excited by centrifugal forces due to unbalance of the discs.

In the computational models, the shaft and the rotor discs are discretized by solid hexahedral isoparametric FEs with a linear basis function. Bearings are included in the computational models by means of force coupling and by stiffness and damping coefficients.

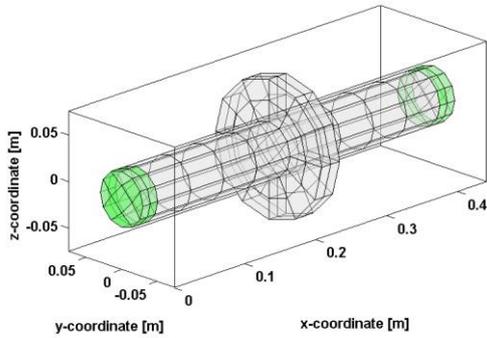


Fig. 1. Geometry of the first test problem

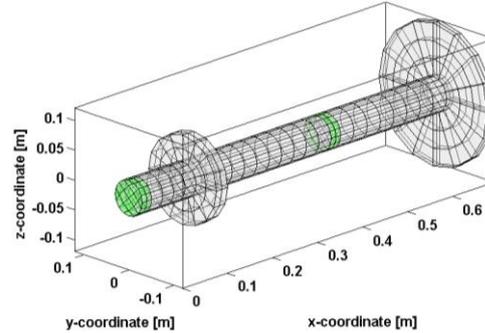


Fig. 2. Geometry of the second test problem

In Figs. 3-5, the results from the modal analysis software package are presented. Computations were performed for the first test problem, the rotor rotating with an angular velocity of 95 rad/s and with no coupling elements present. In Fig. 3, the first non-zero mode shape is plotted, and it corresponds to the bending deformation of the shaft (with two nodal lines) which lies in the xz plane. The third and fifth non-zero mode shapes are shown in Fig. 4 and Fig. 5, respectively. In the third shape, there is a bending deformation of the shaft (now with three nodal lines) and the fifth shape is a torsional deformation of the shaft.

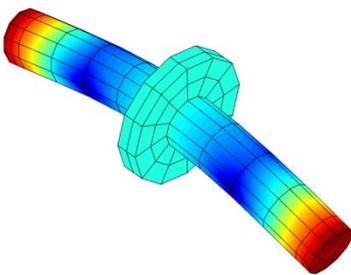


Fig. 3. First mode shape - 1415 Hz

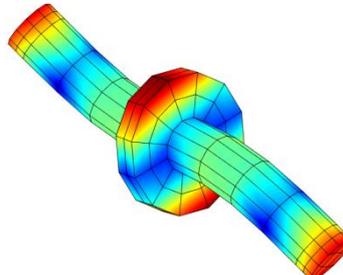


Fig. 4. Third mode shape - 3615 Hz

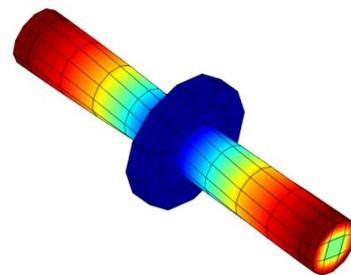


Fig. 5. Fifth mode shape - 3796 Hz

The newly developed library is intended for both stationary and nonstationary analyses. Special attention is given to the effective use of HBM using the IT4Innovations HPC resources. The simulation results obtained are verified with commercially available FE program systems.

Acknowledgements

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