Trendy v podnikání - Business Trends (2018), 8(1), 3-11.
https://doi.org/10.24132/jbt.2018.8.1.3_11

INVESTMENTS IN CRYPTOCURRENCIES: HOW RISKY ARE THEY?

Jiří Málek[1], Van Quang Tran[2]

[1] doc. Jiří Málek, Ph.D., Department of Banking and Insurance, University of Economics Prague, malek@vse.cz
[2] Ing. Van Quang Tran, CSc, Ph.D., Department of Monetary Theory and Policy, University of Economics Prague, tran@vse.cz

Abstract: The article analyzes the probability distribution of returns of the daily data of four cryptocurrencies (Bitcoin, Ethereum, Ripple,Litecoin). Alpha-stable distribution and normal inverse Gaussian distribution (NIG) are used as approximation of the empirical distribution of log-returns as they allow to capture the "power" tails. First basic information about all four cryptocurrencies are given, followed by definition of alpha-stable distribution and normal inverse Gaussian distributions which is special case of generalized hyperbolic distribution. These distributions are used to approximate empirical distributions of these cryptocurrencies. The difference between these two distributions is that the stable distribution can model heavier ends than the NIG (NIG has so called semi-heavy tails). The parameters are estimated using MLE (Maximum Likelihood Estimation) method, which has proved to be the most accurate one. First, we compare the empirical distribution of Bitcoin with NIG and alpha-stable distribution (the stable distribution appears to be much more accurate than the NIG). Then the only stable distribution is used and its parameters are searched for all four cryptocurrencies. α of all cryptocurrencies is close to one, which means that the probability distribution is similar to Cauchy one. The smallest α (and therefore the fattest tail) has Litecoin, followed by Ripple, Bitcoin, and the highest α of Ethereum. On the other hand, Ethereum has the highest sample volatility.

Keywords: Cryptocurrencies, alpha-stable distribution, NIG distribution, fat tails

JEL Classification: G10, C46

INTRODUCTION

Cryptocurrencies are very popular at present, and their growth (relative to the dollar and consequently other currencies) attracts investors' attention. In spite of the rapid growth trend, it is often forgotten about large declines whose character cannot be accurately estimated. At the moment there are hundreds, perhaps thousands, of cryptocurrencies, many of which arise and then quickly disappear. The main advantage of most of them is their decentralization, the impossibility of influencing by central authority. Whether it is their advantage or disadvantage is a question. Those who regard it as an advantage must take into account the high volatility of their exchange rate. It is also a question of what is the reason for investing in cryptocurrencies. Bitcoin and Litecoin seem to be more speculative, but Ethereum and Ripple offer much more. For example, their blockchain is very advanced, however, all operations on their platforms must go through settlement in an appropriate currency.

This contribution only deals with the risk of investing in the cryptocurrencies. The aim is to find the most appropriate probability distribution their log-returns that best approximates the empirical distribution. We have chosen the alpha-stable distribution and the normal inverse Gaussian distribution (NIG). NIG has a thinner tails than alpha-stable but stronger than the normal one (so-called semi-heavy)

First, we compare the empirical distribution of Bitcoin with NIG and alpha-stable distribution (the stable distribution appears to be much more accurate than the NIG). Then the only stable distribution is used and its parameters are estimated for all four cryptocurrencies (Bitcoin, Ethereum, Ripple, Litecoin). Estimates are obtained using the maximum likelihood method (MLE). For the NIG distribution the procedure is straightforward because there is a probability density in the explicit form. However, a stable distribution can be expressed only in the form of a characteristic function, so it is necessary to use the inverse Fourier transform to express the density, which leads to numerical problems. Part three briefly shows the method which seems to be the most appropriate in computational terms. Of course, there are other methods of estimating stable distribution parameters. For example very simple method of (Fama and Roll, 1971) which is only suitable for symmetrical stable distributions and (McCulloch 1980, 1997) which generalized the previous one. Koutrouvelis (1980, 1981) created an approach based on linear regression. Over time, however, the MLE method has gained dominance in most applications for its higher accuracy.

1. SHORTLY ABOUT SOME CRYPTOCURRENCIES

1.1 Bitcoin

Bitcoin (BTC), the most popular crypto, was proposed by an unknown person or persons, under the name Satoshi Nakamoto in October 2008, as a combination of a digital asset and a peer-to-peer payment system in his study: Bitcoin: A Peer-to-Peer Electronic Cash System. The first bitcoin was minted on January 4, 2009, and the first payment was January 11, 2009. The software was released as an open source on January 15, 2009, allowing anyone with sufficient technical skills and computer equipment to engage in development. For a long time, the bitcoin was of a little interest. From the second quarter of 2012, transaction volumes began to grow dramatically. The current daily average volume of bitcoin transactions during their lifetime (from January 4, 2010 to January 1, 2017) is 19,301,677 USD.

Bitcoin “coins” are created (mined) by a network of computers with specialized software programmed to release new coins at a steady but still declining pace. The number of coins in circulation should reach 21 million in year 2140 when the coinage should be terminated

Transactions take place between users directly, without an intermediary. These transactions are verified by network nodes and recorded in a blockchain. Since the system works without a central repository or single administrator, bitcoin is called the first decentralized digital currency.

Blockchain is a public list (or rather a database) where all transactions that have been made with bitcoin (or other cryptocurrency), are anonymously recorded. It is a chronological string of transaction blocks that is constantly growing and shared among all users - meaning that each computer connected to the network's bitcoin gets a copy of a blockchain that will automatically download at the next logon. At present, more than 90% of banks are considering using blockchain. Possibilities of applying this technology to payment systems, risk management, etc. are considered.

Bitcoin is almost anonymous, transactions are executed in minutes (even across the world), transaction costs are minimal (pennies, maximum crowns). The possibility of influencing or otherwise devaluing the currency is also very unlikely, basically impossible.

Bitcoin mining is the process by which transactions are verified and added to the block chain, and also the means through which new bitcoin are released. Anyone with access to the internet and suitable hardware can participate in mining. The mining process involves compiling recent transactions into blocks and trying to solve a computationally difficult puzzle. The participant who first solves the puzzle gets to place the next block on the block chain and claims the rewards and these rewards in the form of new Bitcoins are one of incentives for mining.

1.2 Ethereum

Ethereum (ETH) was founded in 2015. It is currently the second most popular virtual currency, benefiting mainly from somewhat different technology compared to bitcoin, which makes the currency more commercially interesting. This is reflected in the increased interest of commercial institutions, including large

banks and governments. They joined and formed the Enterprise Ethereum Alliance (EEA) in May 2017 around 86 members including companies such as JPMorchan Chase or Microsoft.
Ethereum (similar to bitcoin) is networks based on a public and decentralized blockchain database. But Bitcoin and Ethereum totally differ in their purpose and capabilities. While the Bitcoin blockchain is used to track bitcoin virtual currency transactions, Ethereum blockchain focuses on triggering the source code of any decentralized application (so called smart contracts). Another difference is the average time to extract one block - in the bitcoin database it is about 10 minutes, whereas for Ethereum it is only 12 seconds.

1.3 Ripple

Although Ripple (XRP) is often referred to as cryptocurrency, it is rather an open payment system with its own digital currency, which can be denominated in the usual "fiat" currencies. Like Bitcoin, Ripple uses a distributed database and has its own Ripple Transaction Protocol. It has some features of decentralization, albeit on a smaller scale than Bitcoin.
Ripple company has built a real-time digital payment system for real-time financial transactions, and is also the creator and owner of XRP. Ripple, unlike most cryptocurrencies, is not mined. At the beginning its creators, mined 100 billion gradually being put into circulation. It cannot be obtained by buying or donating. XRP is profiting from the growing interest in blockchain, but unlike cryptocurrencies available, Ripple is mostly owned by one single company. Ripple is developing as the increasingly popular digital payment standard of the financial sector and is gradually acquiring new financial institutions for its payment platform. Large multinational banks such as Bank of America, Royal Bank of Canada (RBC) and Union Bank of Switzerland (UBS) are already its clients.
As of December 21st 2017, the market capitalization of XRP is $51 billion, making it the 3rd largest cryptocurrency in circulation.

1.4 Litecoin

Litecoin (LTC) was designed as a copy of Bitcoin with several modifications. It involved adjusting some parameters, so the principle remained very similar. The result is a currency that is mined 4 times faster and the maximum number of coins is 4 times the limit specified by Bitcoin. At the same time, however, four times more data is stored in the network. Also, the hash algorithm used for mining is also different.
Compared to Bitcoin, Litecoin is cheaper. You pay for a transaction in the order of one thousandths or hundredths of LTC. Bitcoin, on the other hand, has recently faced the problem of rapidly rising fees. They originally also started with symbolic amounts, but now you might pay hundreds of crowns on a fee.

2 STABLE AND NORMAL INVERSE GAUSSIAN DISTRIBUTION

In this section, we give definitions of alpha-stable and normal inverse Gaussian distribution along with their basic characteristics. While NIG distribution can be defined by its density function, stable distribution is defined using its characteristic function.

2.1 Stable distribution

Let $X, X_1, X_2, X_3, \ldots, X_n$ be i.i.d. A random variable *X* is said to have the α-stable distributions if there is for any $n \geq 2$ a positive number c_n and *a* real number d_n such that

$$X_1 + X_2 + \cdots + X_n \overset{d}{=} c_n X + d_n$$

where $\overset{d}{=}$ denotes equality in distribution.

Thus, any sum of independent equally distributed random variables have the same distribution except for the “mean“ and “variance“. Unfortunately there is no general form of the probability density function (pdf), we know only the general form of the characteristic function:

$$\Phi(t) = \exp\left\{-\sigma^{\alpha}|t|^{\alpha}\left(1 - i\beta\,\mathrm{sgn}(t)\tan\frac{\pi\alpha}{2}\right) + i\mu t\right\} \quad \text{for } \alpha \neq 1$$

$$\Phi(t) = \exp\left\{-\sigma|t|\left(1 + i\beta\frac{2}{\pi}\mathrm{sgn}(t)\log|t|\right) + i\mu t\right\} \quad \text{for } \alpha = 1 \qquad (1)$$

where

α … tail power (tail index), as α decreases tail thickness increases

β … skewness parameter, determines asymmetry, a positive β indicates that the right tail is father than left one and vice versa, β=0 corresponding to a symmetric distribution

μ…location parameter, corresponding to mean value for α>0

σ … scale parameter, generalized standard deviation, for α=2 corresponding to a standard deviation of normal distribution

There is another (equivalent) parametrization of the characteristic function that differs from the previous parametrization only in location parameter.

2.2 Properties of stable distributions

The following properties holds for stable distributions:

1. Let X_1, X_2 are independent stable random variables, with $X_i \approx S(\alpha, \beta_i, \sigma_i, \mu_i),\ i = 1,2$ then $X_1 + X_2 \approx S(\alpha, \beta, \sigma, \mu)$ with

$$\sigma^{\alpha} = \sigma_1^{\alpha} + \sigma_2^{\alpha}$$

$$\beta = \frac{\beta_1\sigma_1^{\alpha} + \beta_2\sigma_2^{\alpha}}{\sigma_1^{\alpha} + \sigma_2^{\alpha}}$$

$$\mu = \mu_1 + \mu_2$$

2. If $X \approx S(\alpha, \beta, \sigma, \mu)$ and $a \in R$ then

$$X + a \approx S(\alpha, \beta, \sigma, \mu + a)$$

3. If $X \approx S(\alpha, \beta, \sigma, \mu)$ and $a \in R$ $a \in R$ and $a \neq 0$ then

$$aX \approx S(\alpha, \mathrm{sgn}(a)\beta, |a|\sigma, a\mu) \ \text{ for } \alpha \neq 1$$

$$aX \approx S\left(1, \mathrm{sign}(a)\beta, |a|\sigma, a\mu - \frac{2}{\pi}\beta\sigma a \log|a|\right) \text{ for } \alpha = 1$$

Fat tails of stable distribution

The power of the tail is the index α which approximately means that $P(X < x) \approx c_{\alpha}|x|^{-\alpha}$

as $x \to -\infty$. (The exact formula for c_{α}can be found in Nolan 2003.)

2.3 NIG distribution as special case of the generalized hyperbolic distribution

The generalized hyperbolic distributions (GHD) was introduced by Barndorff-Nielsen (1977) and at first applied them to model grain size distributions of wind-blown sands. Eberlein and Keller (1995) were the first to apply these distributions to finance. The probability density function of (GHD) is as follows:

$$f(x) = \frac{(\alpha^2-\beta^2)^{\lambda/2}}{\sqrt{2\pi}\alpha^{\left(\lambda-\frac{1}{2}\right)}\delta^{\lambda} K_{\lambda}\left(\delta\sqrt{(\alpha^2-\beta^2)}\right)}(\delta^2 + (x-\mu)^2)^{(\lambda-1/2)/2} K_{\lambda-1/2}\left(\alpha\sqrt{\delta^2+(x-\mu)^2}\right)\exp\left(\beta(x-\mu)\right)$$

Where $K_{\lambda}(x)$is modified Bessel function of the third kind with index $\lambda \in \mathbb{R}$. It can be defined as

$$K_{\lambda}(x) = \frac{1}{2}\int_0^{\infty} s^{\lambda-1}\exp\frac{x(s+s^{-1})}{2}ds$$

The NIG distributions is the special cases of the generalized hyperbolic distribution for $\lambda = -1/2$. So the probability density function of NIG is (using some properties of Bessel functions):

$$f(x) = \frac{\alpha\delta\, K_1\left(\alpha\sqrt{\delta^2+(x-\mu)^2}\right)}{\pi\sqrt{\delta^2+(x-\mu)^2}}\exp\left(\delta+\beta(x-\mu)\right)$$

Fat tails of GH

The asymptotic probability of GH is the following

$$P(X \le x) \approx |x|^{\lambda-1}\exp[(\alpha+\beta)x] \text{ as } x \to -\infty$$

3 MAXIMUM LIKELIHOOD ESTIMATION FOR STABLE DISTRIBUTION

Because there is not the density function in the explicit form we must use the inverse Fourier transformation of the characteristic function. This leads to numerical problems. We used the algorithm of Borak, Hardle and Weron (2005) which is preferred in most applications.

After substitution $\zeta = -\beta\tan\frac{\pi\alpha}{2}$ the density of standard $\alpha -$ stable random variable (μ=0, σ=1) for $\alpha \neq 1$ can be expressed as:

for $x > \zeta$:

$$f(x;\alpha,\beta) = \frac{\alpha(x-\zeta)^{\frac{1}{\alpha-1}}}{\pi|\alpha-1|}\int_{-\xi}^{\frac{\pi}{2}} V(\theta;\alpha,\beta)\exp\left(-(x-\zeta)^{\alpha/\alpha-1}V(\theta;\alpha,\beta)\right)d\theta,$$

for $x = \zeta$:

$$f(x;\alpha,\beta) = \frac{\Gamma\left(1+\frac{1}{\alpha}\right)\cos\xi}{\pi(1+\zeta^2)^{\frac{1}{2\alpha}}}$$

and for $x < \zeta$:

$$f(x;\alpha,\beta) = f(-x;\alpha,-\beta)$$

where

$$V(\theta;\alpha,\beta) = (\cos\alpha\xi)^{\frac{1}{\alpha-1}}\left(\frac{\cos\theta}{\sin\alpha(\xi+\theta)}\right)^{\alpha/\alpha-1}\frac{\cos[\alpha\xi(\alpha-1)\theta]}{\cos\theta}$$

$$\xi = \frac{1}{\alpha}\arctan(-\zeta)$$

When estimating the parameters of alpha-stable distribution from data by MLE, we have to find such vector $\alpha, \beta, \delta, \mu$ that maximize the likelihood function $\sum_{i=1}^{n} \log f(z_i; \alpha, \beta, \delta, \mu)$ with respect to parameters $\alpha, \beta, \delta, \mu$, where $z_i = \frac{x_i - \mu}{\delta}$.

4 DATA AND RESULTS

The first part compares parameter estimates for stable and NIG distribution with empirical BTC log-return one (data in the period 13.9. 2011 to 16.6. 2017). In the next section only stable distribution is used (the considered daily data are up to 2017.11.10). Data was taken from www.kaggle.com. All four parameters for the BTC, ETH, XRP and LTC cryptocurrencies are estimated.
Tab. 1 shows the basic sample statistics and the table the following estimates of the relevant parameters for the stable and NIG distributions (it is not possible to compare the parameters of both distributions even though they are denoted by the same labels). From Table 2 it is clear that the SE (standard error) is significantly better for stable than the NIG distribution. Figure 1 shows the Cryptocurrencies / USD exchange rate, Figure 2 log-return.

Tab. 1: Descriptive statistics logarithmic returns series

	Bitcoin	Ethereum	Litecoin	Ripple
mean	0,003143	0,005671	0,0016	0,002299
median	0,002242	-0,00134	0	-0,00291
maximum	0,445543	0,412337	0,828968	1,027356
minimum	-0,66395	-1,30211	-0,51393	-0,61627
std. deviation	0,052287	0,086397	0,067627	0,076261
skewness	-1,48794	-3,80479	1,864021	2,060077
kurtosis	29,69565	67,98133	32,26929	35,10307
num. of obs	2229	823	1654	1556

Source: Own processing, 2017

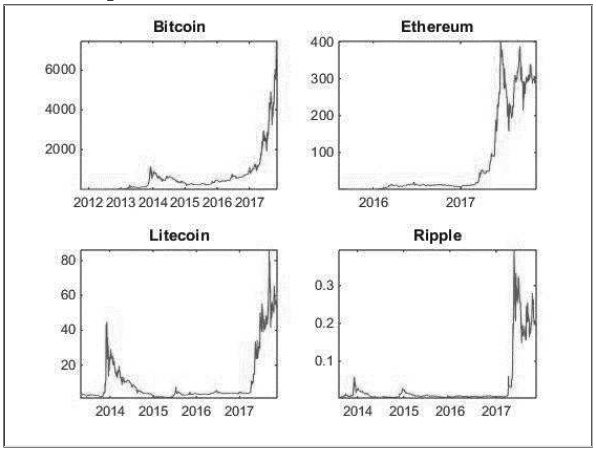
Tab. 2: Parameter estimation result for bitcoin returns series, period 2011 – 2017

	NIG distribution		Alpha-stable Distribution	
	value	S.E.	value	S.E.
α	5,8080	0,0308	1,1866	0,0193
β	0,1556	0,0661	0,0977	0,0155
σ	0,0166	0,0005	0,0152	0,0004
μ	0,0024	0,0003	0,0074	5,31E-06

Source: Own processing, 2017

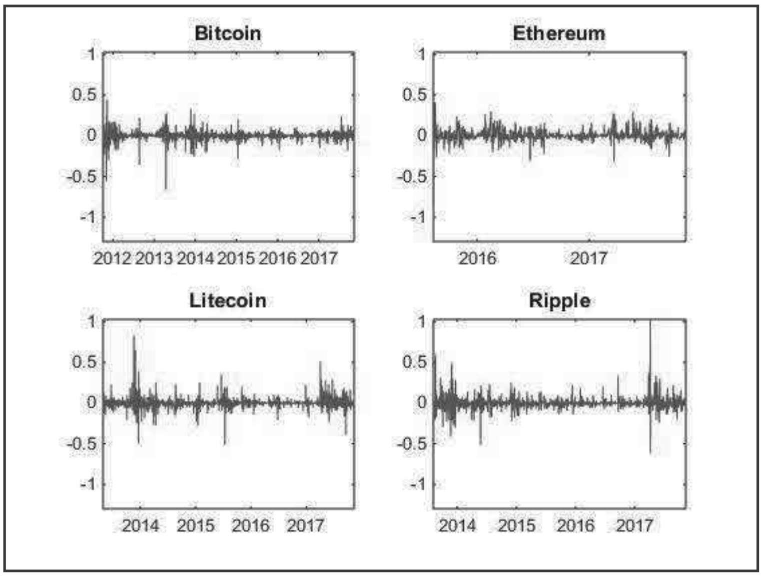
https://doi.org/10.24132/jbt.2018.8.1.3_11

Fig. 1 Cryptocurrencies USD exchange rate



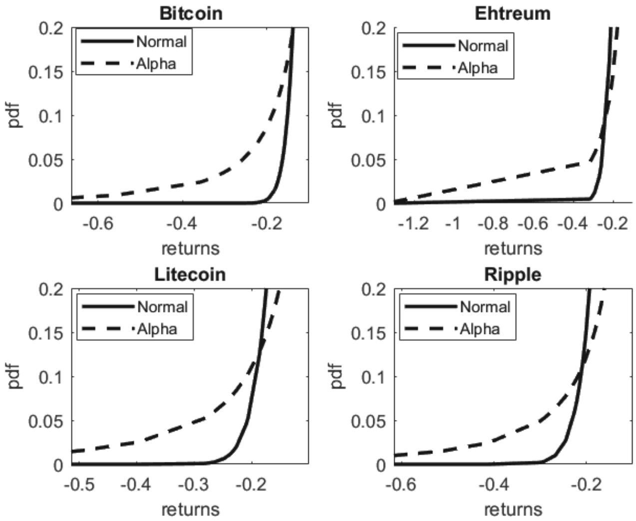
Source: Kaggle - The Home for Data Science (www.kaggle.com)

Fig. 2 Log-return



Source: Kaggle - The Home for Data Science (www.kaggle.com) + Authors

Fig. 3 Comparing the left tail of alpha-stable and normal distributions of individual cryptocurrencies



Source: Own processing, 2017

Tab. 3: Stable parameters estimation

parameter	Bitcoin		Ethereum		Litecoin		Ripple	
	pam	SE	pam	SE	pam	SE	pam	SE
α	1,2110	0,0312	1,3087	0,0434	1,1253	0,0418	1,1769	0,0324
β	0,1022	0,0164	0,3227	0,0458	0,0327	0,0850	0,1085	0,0064
σ	0,0160	0,0005	0,0305	0,0014	0,0176	0,0014	0,0215	0,0006
μ	0,0073	0,0000	0,0154	0,0008	0,0017	0,2926	0,0051	0,0013

Source: Own processing, 2017

INTERPRETATION AND CONCLUSION

It can be seen from the figures (Fig. 3) that normal distribution is not appropriate to approximate the empirical ones. (It has not been further investigated). The stable distribution approximates the empirical BTC / USD exchange rate distribution more precisely than the NIG one.

The α of the stable distribution is close to 1 for all cryptocurrencies, which means that this distribution is close to Cauchy distribution.

The smallest α (and therefore the fattest tail) has Litecoin, followed by Ripple, Bitcoin, and the highest α of Ethereum. On the other hand, Ethereum has the highest sample volatility, as well as the σ parameter, which corresponds to the standard deviation. This is not inconsistent as the α of fat tails, the standard deviation or the sigma parameter are different measures of risk. As $\beta \to 0$ it can be said that the empirical distribution is very little different from the symmetrical one.

It can be seen that investing in cryptocurrencies is very risky. Despite the growth trend, there are large declines, and an investor never knows whether the bubble bursts or it is a momentary decline when investors are picking profits. It is clear that at present the growth of the Bitcoin exchange rate is led by speculators and resembles a pyramid game. For this reason, investing in (if you really want to invest in cryptocurrencies), Ethereum or Ripple seem more appropriate, because they offer much more than Bitcoin. If, however, the main factor that cryptocurrencies offer is decentralization, then it must be said that the main requirement of ordinary

https://doi.org/10.24132/jbt.2018.8.1.3_11

consumers of companies is stability. And it is not yet clear how without a central subject it can be achieved. Perhaps if there was one cryptocurrency around the world. But that's just a foolish dream.

Acknowledgments: Author Jiří Málek acknowledges the financial support of Czech Science Foundation with grant GAČR 18-05244S "Innovative Approaches to Credit Risk Management" and Institutional support IP 100040/1020. Author Tran van Quang is grateful for the financial support of grant GAČR 18-05244S "Innovative Approaches to Credit Risk Management" of Czech Science Foundation.

REFERENCES

Barndorff-Nielsen O. E. (1977). Exponentially decreasing distributions for the logarithm of particle size". Proc. Roy. Soc. London, A (353):401–419.

Borak, S., Härdle, W. and Weron, R. (2005). Stable Distributions, SFB 649 Discussion Papers SFB649DP2005-008, Sonderforschungsbereich 649, Humboldt University, Berlin, Germany

Chambers J. M., Mallows C. L., and Stuck B.W. (1976). A method for simulating stable random variables, J. Amer. Statist. Assoc. 71, 340-344

Eberlein E., Keller U. (1995). Hyperbolic distributions in fnance, Bernoulli 1, pp. 281-299

Fama, E. and Roll, R. (1968). Some Properties of Symmetric Stable Distributions. J. Amer. Statist. Assoc. 63, 817–836.

Fama, E.,and Roll R. (1971). Parameter Estimates for Symmetric Stable Distributions. J. Amer. Statist. Assoc. 66, 331–338.

Koutrouvelis, I. A. (1980). Regression–Type Estimation of the Parameters of Stable Laws. J. Amer. Statist. Assoc. 75, 918–928.

Koutrouvelis, I. A. (1981). An Iterative Procedure for the Estimation of the Parameters of the Stable Law. Commun. Statist.–Simul. 10 17–28.

McCulloch, J. H. (1980). Simple Consistent Estimators of Stable Distribution Parameters. Commun. Statist.–Simul. 15 (4), 1109–1136.

McCulloch, J. H. (1997). Measuring Tail Thickness to Estimate the Stable Index α: A Critique. Journal of Business and Economic Statistics 15, No 1

Nolan, J. P (2003): Modeling Financial Data with Stable Distribution. In Handbook of Heavy Tailed Distributions in Finance, Edit by Rachev S.T., Elsevier science.

The Home for Data Science & Machine learning (www.kaggle.com)

Weron, R. (1995). Performance of the Estimators of Stable Law Parameters, Wroclav University of Technology.

www.kaggle.com