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Report on the Ph.D. thesis by Ing. Lukáš Kotrla entitled "On Uniqueness and Existence of Parabolic PDEs Involving p-Laplacian and Generalized Trigonometric and Hyperbolic Functions"

(a) Evaluation of the significance of the doctoral thesis for the given field.

The thesis is devoted to the study of solutions of a parabolic problem of the form

$$\left.\begin{array}{l} \frac{\partial u}{\partial t} = \Delta_p u + h(t,x,u) \text{ for } (x,t) \in \Omega \times (0,T) \\ u(x,t) = 0 \text{ for } (x,t) \in \partial\Omega \times (0,T) \\ u(x,0) = u_0(x) \text{ for } x \in \Omega\,, \end{array}\right\} \qquad (P_t)$$

and corresponding steady state problem for special case of $h(x,u)$ in one-dimension. For (P_t), existence of a nontrivial nonnegative solution was obtained by clever construction of ordered pair of upper and lower solution, when $h(t,x,u) = q(x)u^\beta$ with zero initial data for which trivial solution is guaranteed.

For $p > 2$ again with zero initial data, a nonnegative solution with compact support as well as nonnegative solution with a prescribed number of bumps with pairwise disjoint support are obtained by constructing Barenblatt-type supersolution. The fact that such solutions cannot be expected for the semilinear case $p = 2$ makes this result very interesting. One generally expects, not wisely, that if a result holds for $p = 2$ then it must also hold for p-Laplacian. This result serves as cautionary tale for going from Laplacian to p-Laplacian.

Second topic considered in this thesis relates to the following nonlinear eigenvalue problem for any $p > 1$

$$\left.\begin{array}{l} -(|u'(x)|^{p-2}u'(x))' = \lambda |u'(x)|^{p-2}u'(x) \text{ for } x \in (0,\pi_p) \\ u(0) = 0 = u(\pi_p)\,. \end{array}\right\} \qquad (\text{EVP})$$

The focus is on the positive eigenfunction, termed $\sin_p$, corresponding to the principal eigenvalue of the eigenvalue problem (EVP) and its derivative, termed $\cos_p$. Regularity of $\sin_p$ function is established in detail. In particular, Table 3.1 gives the summary of order of differentiability of sin_p depending on the value $p > 1$. The $\sin_p$ function is extended to complex domain as well in this thesis which is nontrivial.

(b) Statement on the approach to solving the problem, methods used, and fulfillment of the given objective.

Comparison principle combined with intricate properties of Lebesgue and Sobolev spaces are employed in the study of (P_t). Some comparison principle, when not available in the literature, were established. These results are of independent interest as they are powerful tool in the study of problems of the form (P_t). Technical details carried out carefully in the study of generalized trigonometric and hyperbolic functions are definitely praiseworthy.

(c) **Opinion on the results of the thesis and specification of the student's original contribution to the given area of knowledge.**

Results obtained in this thesis are nontrivial which are already published in refereed journals. Papers are published in highly reputed journals such as, Journal of Differential Equations, Applied Mathematics Letters, Electronic Journal of Differential Equations etc. In my opinion, results concerning (P_l) have made substantial contribution to the area of parabolic PDEs. Results obtained for (EVP) will help other mathematicians to understand and use the generalized trigonometric and hyperbolic functions in their work.

Each paper is accompanied by statement of co-authors stating the specific contribution made by Mr. Kotrla. These statements validate the nontrivial original contribution of the student in published papers and therefore in the thesis to the field of differential equations.

(d) **Statement on the systemic approach, clarity, approximation of form and language.**

Introduction and statement of results are informative, clear and explains the context by making connection through historical perspective. Important results from the published papers are discussed with appropriate reference to the literature. Images and tables provided in the thesis is helpful for clarity. The thesis is written well.

(e) **Statement on the student's publication.**

Mr. Kotrla has authored one paper and co-authored seven papers in highly reputed refereed journals, see (c) above. Each of the co-authored papers are accompanied with statement from co-authors stating the specific nontrivial contribution made by Mr. Kotrla. The publication record of Mr. Kotrla is very impressive by any standard and is enviable even to researchers with Ph.D. degree.

(f) **Unambiguous statement saying that the reviewer recommends/ does not recommend the thesis for defense.**

The content of this thesis has several nontrivial results that have already been published in reputed refereed journals. **I strongly recommend this thesis for defense to complete Ph.D. degree.**

Sincerely,

Chhetri. (November 6, 2018)

Maya Chhetri

Reviewer's Report on PhD Dissertation Thesis

On uniqueness and existence of solutions of parabolic PDE involving p-Laplacian and generalized trigonometric and hyperbolic functions

submitted by Lukáš Kotrla
in the field of Applied Mathematics
on University of West Bohemia

The dissertation thesis of Lukáš Kotrla deals with quasilinear parabolic and elliptic problems involving p-Laplacian. The thesis contains important results in the field. In particular, the results published by the author and his coauthors deal with uniqueness problems, validity of strong maximum principle and properties of half-linear trigonometric and hyperbolic functions.

The methods used in the thesis are relevant and nontrivial. It should be mentioned that the author does not restrict his attention to the similarities with linear case, like other researchers in the field often do. The author performs deep research also when the equation exhibits behavior different to the well known linear case. This approach is way harder and often requires utilization of new ideas and approaches.

The results have been published in the well respected journals of high quality. Most of the results have been published with one or more coauthors and the contribution of Lukáš Kotrla have been confirmed by the corresponding co-author's statements. This contribution is sufficient to judge that Lukáš Kotrla is capable to practice the research in mathematics and deserves to be awarded by the PhD degree.

The formal look of the thesis also meets the high standards. The thesis is well written, I noticed only few minor problems. I especially like the idea to write the short informal sketch of the proof into the thesis and provide full proof as an appendix of the thesis.

Conclusion. Lukáš Kotrla has performed large amount of important research which meets the high scientific level required in the field. The results presented in the thesis are new, nontrivial, interesting and have been published in high quality journals. For this reason I strongly support the thesis for defense.

For the defense I suggest short discussion related to the possible half-linear generalization of some other well known functions, such as exponential function or Bessel function. What has been already done and what would be interesting to know?

Brno, 8.11.2018

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