Modelling of Metallic Cables at G.fast Frequencies

Pavel Lafata
Dept. of Telecommunication Engineering
Czech Technical University in Prague, FEE
Prague, Czech Republic
pavel.lafata@fel.cvut.cz

Marek Nevosad
Dept. of Telecommunication Engineering
Czech Technical University in Prague, FEE
Prague, Czech Republic
marek.nevosad@fel.cvut.cz

***Abstract* – Metallic cables are still frequently used in access telecommunications networks, especially in the last-mile network segments, together with digital subscriber line (DSL) technologies. Recently, the G.fast system for reaching gigabit transmission speed over short metallic lines has been introduced. In order to achieve such performance, the frequency band of G.fast was extended up to 106 MHz or 212 MHz and several innovative transmission concepts were adopted as well. However, these innovations require accurate approximations of a G.fast transmission channel. Due to that it is necessary to provide accurate approximation of metallic cable transmission characteristics, the attenuation constant $\alpha(f)$ as well as the phase constant $\beta(f)$. Therefore, an innovative LN model was proposed and is presented in this paper. Its main motivation is to outperform the existing models with equal number of necessary parameters. The accuracy of a proposed LN model was tested for various metallic cables and compared with two typical existing models.**

Keywords-metallic cables; transmission lines; G.fast; modeling; propagation constant.

I. INTRODUCTION

Today, the metallic cables and metallic lines in access networks are still often used for digital subscriber lines and systems (DSL) [1]. The previous DSL generation, VDSL2, can occupy a frequency band up to 30 MHz, however, a newly developed G.fast lines exploit much wider frequency bands [2], up to 106 MHz (G.fast 106a version) or up to 212 MHz (G.fast 212a version) [3]. Thanks to that the G.fast line can achieve transmission speed up to 1 Gbps for short local loops [4]. Moreover, several new transmission concepts were implemented into G.fast as well [3], such as the Vectored Discrete Multi-tone Modulation (VDMT) for far-end crosstalk elimination, the reverse power-feeding, the time-division duplex (TDD) transmission technique, etc. However, the performance of all these enhancements strongly depends on the accuracy of a transmission channel modeling [5], especially on the modeling of its propagation constant $\gamma(f)$.

The propagation constant $\gamma(f)$ is one of the secondary line coefficients [6] and since it is a complex number, it can be decomposed into a real part, the attenuation constant $\alpha(f)$, and a complex part, the phase constant $\beta(f)$. Both of these constants are required for modeling of transmission characteristics of metallic cables [7], for signal propagation modeling, for digital transmission system performance modeling, etc. Therefore, the accuracy of a propagation constant $\gamma(f)$ approximation can positively influence further estimations and modeling.

On the other hand, most of the existing models of metallic cables and lines were originally designed for VDSL/VDSL2 frequency spectrum up to 30 MHz [7], therefore their accuracy for G.fast frequencies is limited. Basically, there are two main approaches for modeling of frequency characteristics of metallic cables [8]. The first type of models represented mainly by the British Telecom models (BT) and newly presented G.fast model (in ITU-T G.9701) approximates the primary line coefficients, $R(f)$, $L(f)$, $C(f)$ and $G(f)$, or their combination, a longitudinal impedance $Z_S(f)$ and a shunt admittance $Y_P(f)$. The secondary line coefficients, including the propagation constant $\gamma(f)$, can be then calculated using well known telegraph equations [6]. The main disadvantage of these models is that they usually require a high number of parameters and these models are also quite complex. The second group of models directly approximates the secondary line coefficients, especially the propagation constant $\gamma(f)$, using parametric *k*-models. These models are either based on mathematical approximation of the propagation constant $\gamma(f)$ equation by using Taylor series expansion and Hilbert transform [8], or a best character-fitting method. This group of models is mainly represented by Chen's (KM1) model, KM2 and KM3 models and their derivatives and modifications.

The Chen's (KM1) model is one of the most often used parametric models today for approximating both attenuation constant $\alpha(f)$ as well as phase constant $\beta(f)$ of metallic lines. It uses 3 *k*-parameters individually specified for each metallic cable (line). The model, which was first proposed by Chen in [6] and its $\beta(f)$ approximation was slightly modified in [8], is given as:

$$\alpha(f) = k_1\sqrt{f} + k_2 f \,, \tag{1}$$

$$\beta(f) = k_1\sqrt{f} - k_2\frac{2}{\pi} f \ln f + k_3 f \,. \tag{2}$$

ISBN 978-80-261-0386-8,

Where, $\alpha(f)$ is an attenuation constant in dB/km, $\beta(f)$ is a phase constant in rad/km, f is a frequency and k_1, k_2 and k_3 stand for the parameters of a model. Although this model was originally designed for VDSL2 frequencies, it can be applied for G.fast estimations, however, with limited accuracy only. On the other hand, it provides a quick approximation of the propagation constant $\gamma(f)$, therefore it is often used in practice as well. In [8], Acatauassu et al. provided some mathematical derivation and validation of Chen's model and they also presented two completely new parametric models, KM2 and KM3. Both models are based on Taylor series expansions and approximations of the propagation constant $\gamma(f)$ and they are also suitable for G.fast frequencies up to 200 MHz. These models can also provide more accurate estimations of the propagation constant $\gamma(f)$ compared to the Chen's model (KM1), however, they require a higher number of k-parameters, the KM2 model needs 4 k-parameters and KM3 model requires 5 k-parameters. The KM3 model can be, according to [8], expressed as:

$$\alpha(f) = k_1\sqrt{f} + k_2 f + k_5\frac{1}{\sqrt{f}} + k_4\,, \quad (3)$$

$$\beta(f) = k_1\sqrt{f} - k_2\frac{2}{\pi}f\ln f + k_3 f - k_5\frac{1}{\sqrt{f}}\,. \quad (4)$$

It is evident that the extra accuracy provided by the KM3 model negatively influences its overall complexity and the number of required parameters, on the other hand. Therefore, in order to increase the accuracy of attenuation and phase constants $\alpha(f)$, $\beta(f)$ modeling, a completely different approach should be adopted. Due to that the main motivation of an innovative LN parametric model presented within this paper is to outperform the accuracy of existing Chen's (KM1) model with the same number of necessary parameters. The proposed LN model is based on the application of an inverse hyperbolic sine function, which could be successfully fitted into the typical frequency character of the attenuation and phase constants. To verify the accuracy of a proposed LN model, measurements of real metallic cables up to 250 MHz were performed and the presented LN model was compared with the Chen's (KM1) model as well as with the KM3 model to illustrate its potential.

II. Proposed LN Model

In this section, an innovative LN model for modeling of a propagation constant $\gamma(f)$ at G.fast frequencies up to 250 MHz is presented. Based on numerous experience obtained during measurements of various metallic cables and lines with different constructional parameters, the application of an inverse hyperbolic sine function was proposed. The initial idea of adopting hyperbolic functions for propagation constant $\gamma(f)$ modeling was presented in [9], in which two attenuation constant $\alpha(f)$ models based on an inverse hyperbolic cotangent function were proposed. However, the application of an inverse hyperbolic sine function helps to achieve better accuracy of modeling, since its exponential (logarithm) character should better fit the typical characters of attenuation and phase constants. Moreover, using a Taylor expansion of a propagation constant $\gamma(f)$ expression, the resulting model LN with arsinh function and optimum frequency fractions could be obtained. The resulting LN model could be therefore expressed as:

$$\alpha(f) = k_1 \operatorname{arsinh}\left(k_2\left(f^{0.3} + \sqrt{f}\right) + \frac{k_2}{k_1^{2.5}} f^{1.1}\right), \quad (5)$$

$$\beta(f) = k_3 f - \operatorname{arsinh}\left(k_3 f\right). \quad (6)$$

The LN model requires 3 k-parameters, the same number of parameters as Chen's (KM1) model, therefore their complexity is generally equal. The verification of presented LN model through comparisons with real measured characteristics as well as other models is presented in the following section.

III. Measured Results and Comparisons

It is necessary to calculate the optimum k-parameters for each model used in the following comparisons. This can be performed by using least-square method, in which the summary squared error [10], E_S, of an attenuation constant, $E_{S\alpha}$, as well as of a phase constant, $E_{S\beta}$, is minimized. The summary squared error can be calculated as [8]:

$$E_S = \sum_f \left(X_M(f) - X(f)\right)^2\,. \quad (7)$$

Where, E_S is the resulting summary squared error between modeled values $X_M(f)$ (either a model of an attenuation constant $\alpha(f)$ or a phase constant $\beta(f)$) and real measured values $X(f)$ ($\alpha(f)$, $\beta(f)$) over the entire frequency band f. The summary squared errors of an attenuation constant, $E_{S\alpha}$, as well as a phase constant, $E_{S\beta}$, for the proposed LN model can be calculated as:

$$E_{S\alpha} = \sum_{i=1}^{N}\left(k_1 \operatorname{arsinh}\left(k_2\left(f_i^{0.3} + \sqrt{f_i}\right) + \frac{k_2}{k_1^{2.5}} f_i^{1.1}\right) - \alpha(f_i)\right)^2 \quad (8)$$

$$E_{S\beta} = \sum_{i=1}^{N}\left(k_3 f_i - \operatorname{arsinh}\left(k_3 f_i\right) - \beta(f_i)\right)^2\,. \quad (9)$$

The measurements were performed using calibrated Rohde&Schwarz ZVRE vector network analyzer together with proper NorthHills balun transformers in order to balance the impedance between the analyzer and measured metallic cables. To examine the accuracy of a proposed LN model, different metallic cables were selected and their propagation constants $\gamma(f)$ measured. Therefore the measurements were performed for these typical cables: standard UTP cat. 5e cable (50 meters) in a frequency band from 2 to 250 MHz, standard UTP cat. 6 cable (50 meters) in a frequency band 2-250 MHz, STP cat. 7 cable (97 meters) again in a frequency band between 2 and 250 MHz, TCEPKPFLE 75x4x0.4 cable (100 meters) in a frequency band from 2 to 150 MHz and

SYKFY 4x2x0.5 cable (25 meters) in a frequency band 2-150 MHz as well. Since the lengths of all cables are different, all values of $\alpha(f)$ and $\beta(f)$ were recalculated for a standard length of 100 meters.

A. Experimental Results

First, Tab. 1 contains the values of a summary squared error of an attenuation constant approximation, $E_{S\alpha}$, for all models (Chen's (KM1), KM3 and LN) for UTP cat. 5e, cat. 6 and STP cat. 7, while the squared error of a phase constant estimation, $E_{S\beta}$, for the same cables is illustrated in Tab. 2.

TABLE I. Summary squared error $E_{S\alpha}$ for the attenuation const. for UTP cat. 5e, 6 and STP cat. 7.

	cable		
model	***UTP cat. 5e***	***UTP cat. 6***	***STP cat. 7***
KM1	59.154	49.636	18.419
KM3	56.210	47.802	17.997
LN	57.607	49.106	18.176

TABLE II. Summary squared error $E_{S\beta}$ for the phase const. for UTP cat. 5e, 6 and STP cat. 7.

	cable		
model	***UTP cat. 5e***	***UTP cat. 6***	***STP cat. 7***
KM1	84780.485	80711.344	69958.572
KM3	83876.521	79880.916	69536.179
LN	545.793	565.568	663.037

The values of k-parameters calculated for each model for UTP cat. 5e are presented in Tab. 3, while Fig. 1 illustrates the attenuation constant modeling for UTP cat. 5e cable with an absolute error calculated and presented in Fig. 2. The same comparison for a phase constant is provided in Fig. 3 and Fig. 4.

TABLE III. k-parameters of each model calculated for UTP cat. 5e cable.

***k*-parameters**	**Chen's (KM1)**	**KM3**	**LN**
k_1	0.0020	0.0020	269.9046
k_2	1.02139e-8	1.2382e-8	7.2476e-6
k_3	3.0357e-6	3.0655e-6	3.1055e-6
k_4	-	0.2194	-
k_5	-	6.8867e-6	-

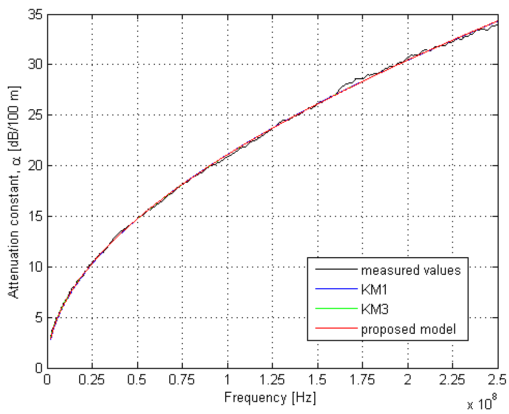

Figure 1. The attenuation constant of UTP cat. 5e cable.

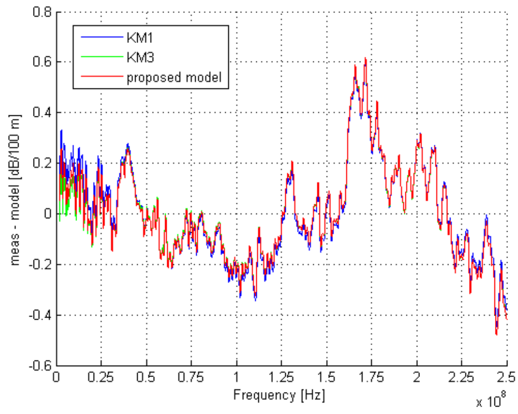

Figure 2. Absolute error in dB/100m between measured and modeled values of attenuation const. for UTP cat. 5e cable.

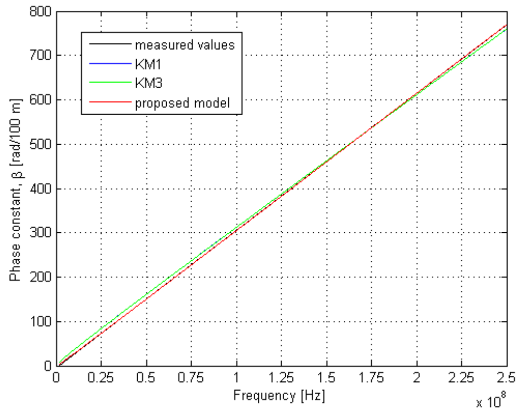

Figure 3. The phase constant of UTP cat. 5e cable.

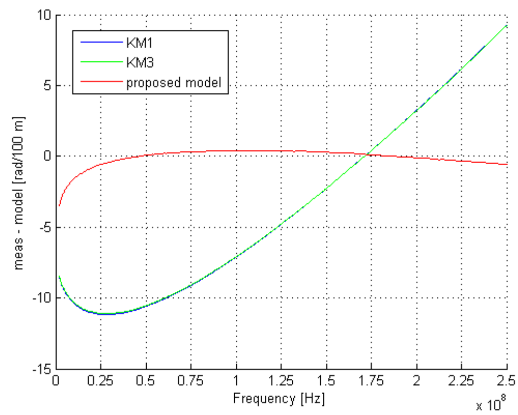

Figure 4. Absolute error in rad/100m between measured and modeled values of phase const. for UTP cat. 5e cable.

It is evident that the proposed LN model outperforms the accuracy of existing Chen's (KM1) model attenuation constant and phase constant approximation for UTP cat. 5e, cat. 6 and also for STP cat. 7 cable. Moreover, the approximation of a phase constant for all presented cables is even better than the KM3 model, although it uses more k-parameters. Next, the same comparison was performed for SYKFY and TCEPKPFLE cables in a frequency band from 2 to 150 MHz. Therefore, Tab. 4 contains the values of squared error of $\alpha(f)$ approximation, while

Tab. 5 illustrates the summary squared error of *β(f)* estimations.

TABLE IV. Summary squared error $E_{S\alpha}$ for the attenuation const. for TCEPKPFLE and SYKFY cables.

	cable	
model	***TCEPKPFLE***	***SYKFY***
KM1	407.903	436.369
KM3	225.260	321.319
LN	374.547	438.081

TABLE V. Summary squared error $E_{S\beta}$ for the phase const. for TCEPKPFLE and SYKFY cables.

	cable	
model	***TCEPKPFLE***	***SYKFY***
KM1	21028.464	71426.536
KM3	16612.899	64833.915
LN	416.867	10088.192

Again, the proposed LN model provided significantly better approximation of a phase constant compared to KM1 and even KM3 model. Its accuracy of an attenuation constant modeling is better (TCEPKPFLE cable) or equal (SYKFY cable) to KM1 model as well, generally, the LN model outperformed the KM1 model in case of both typical telecommunication cables too.

IV. CONCLUSIONS

This paper was focused on modeling of metallic cables' transmission characteristics at high frequencies. Since the newly proposed G.fast system can occupy frequencies up to 212 MHz, the necessity of providing accurate approximations up to 250 MHz is evident. The main motivation presented within this article was to propose a completely new parametric propagation constant *γ(f)* model, which can outperform the existing models with the same number of necessary parameters. The presented LN model is based on an inverse hyperbolic sine function and thanks to that its accuracy of *α(f)* and *β(f)* estimations is better compared with existing Chen's (KM1) model, moreover, the proposed LN model even outperformed the KM3 model in several situations. The comparisons performed for various metallic cables with different internal construction, conductor diameters and used materials proved the validity of a proposed LN model and illustrated its potential and accuracy. Thanks to that the presented LN model could be successfully adopted in practice to provide accurate propagation constant *γ(f)* approximations for various situations and scenarios and for modern transmission systems at high frequencies.

ACKNOWLEDGMENT

The work presented in this paper was supported by the Grant Agency of the Czech Technical University in Prague, grant No. SGS14/142/OHK3/2T/13.

REFERENCES

[1] O. Krajsa, R. Sifta, and J. Sedy, "A comparative study of the modern access networks," *36th International Conference on Telecommunications and Signal Processing,* pp. 123-126, July 2013.

[2] J. Vodrazka, "Potential use of gigabit digital subscriber lines in hybrid access networks," *International Conference on Digital Technologies*, pp. 71-74, May 2013.

[3] M. Timmers, M. Guenach, C. Nuzman and J. Maes, "G.fast: evolving the copper access network," *IEEE Communications Magazine*, vol. 51, no. 8, pp. 74-79, August 2013.

[4] I. Almeida, A. Klautau and Lu Chenguang, "Capacity analysis of G.fast systems via time-domain simulations," *IEEE International Conference on Communications*, pp. 4008-4013, June 2013.

[5] R. Strobel, R. Stolle and W. Utschick, "Wideband modeling of twisted-pair cables for MIMO applications," *IEEE Global Communications Conference*, pp. 2828-2833, December 2013.

[6] H. Hughes, *Telecommunications Cables: Design, Manufacture and Installation*. John Wiley&Sons Ltd., Chichester, England, 1997, pp. 32-86.

[7] W. Y. Chen, *DSL: Simulation Techniques and Standards Development for Digital Subscriber Line System*. Macmillan Technology Series, Indianopolis, USA, 1998, pp. 123-168.

[8] D. Acatauassu, S. Host, L. Chenguang, M. Berg, A. Klautau and P. O. Borjesson, "Simple and causal twisted-pair channel models for G.fast Systems," *IEEE Global Communications Conference*, pp. 2834-2839, December 2013.

[9] P. Lafata, "Simple attenuation models of metallic cables suitable for G.fast frequencies," *Advances in Electrical and Electronic Engineering*, vol. 13, no. 2, 2015.

[10] P. Jares and J. Vodrazka, "Modelling of Data Stream Time Characteristics for Use of Inverse Multiplexer," *Advances in Electrical and Electronic Engineering*, vol. 12, no. 4, pp. 260-264, 2014.