Fractional-Order High-Pass Filter with Electronically Adjustable Parameters

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Abstract – **This paper presents possible solution of fractional-order high-pass filter (FHPF) with electronically adjustable order between 1 and 2 and also with electronically adjustable pole frequency. It is based on well-known Follow-the-Leader Feedback (FLF) topology adjusted for utilization with operational transconductance amplifiers (OTAs) and adjustable current amplifiers (ACAs). This 3rd-order topology is used in order to approximate FHPF response in particular frequency band of interest. Design is supported by Pspice simulations for three particular values of the filter's order (1.25, 1.5, 1.75) and for several values of pole frequency. Moreover, simulation results with two different approximations are compared.**

Keywords- adjustable current amplifier; approximation; ACA; FHPF; FLF; follow-the-leader feedback; fractional-order; operational transconductance amplifier

I. INTRODUCTION

Traditionally, integer-order analog filters are studied [1]. Transition from pass band to stop band of these filters is based on the slope of integer-order approximation of the filter. Therefore, there is a step change of the order of the filter and their attenuation is equal to $20n$ dB/decade theoretically, where n is non-zero unsigned integer representing the order of the filter. However, synthesis of fractional-order electronic systems, for instance passive components, frequency filters or oscillators is of great interest of many research groups [1]-[19].

In case of the fractional-order filters, the change of the order of the filter is fractional, represented by $20 \cdot (n + a)$ dB/decade slope of attenuation [1] in case of the low-pass (LP) or high-pass (HP) response and parameter a follows this inequality: $0 < a < 1$.

Two common ways are used to obtain fractional-order filter. The first is to use so-called fractional-order element (FOE) in the filtering structure, because of absence of its physical form, usually approximated by passive RC network, e.g. [4]-[14]. Some filters are designed with respect to the passive prototype [8], [10], [11], [14], some of them are based on the Kerwin-Huelsman-Newcomb (KHN) topology [9], [11]-[13] or Sallen-Key topology [9], [11] and usually including two FOEs [11]-[14]. Key disadvantage of FOE is that it is designed to behave as fractional element of particular order only in certain frequency band of interest, usually not exceeding two decades. For instance, it is band from 0.1 to 10 kHz [10] or 0.2 to 20 kHz [14].

There is a second way how to obtain fractional-order filter. Fractional-order transfer function can be approximated by integer-order function of higher order [15]-[18]. Actually, the term s^a in the transfer function is approximated by a ratio of two rational polynomials (at least of second order). Its issue is similar to FOEs, it behaves as fractional element of particular order only in certain frequency. When third-order function is used, transfer function having fractional order between one and two can be obtained. It can be then implemented by various topologies, e.g. Follow the Leader Feedback (FLF) topology [15]-[18].

Standard non-tunable Operational Amplifier (OA) or Current Feedback Operational Amplifier (CFOA) are considered as active elements frequently [9-13], [17], but also advanced active elements, e.g. Differential Difference Current Conveyor (DDCC) [16], or electronically controllable elements, e.g. Operational Transconductance Amplifier and Adjustable Current Amplifier [18] are implemented in fractional-order filters. Design of low-pass (LP) filter with fractional character is presented frequently [8], [9], [11]-[13], [15]-[18], but there are also papers presenting solutions of high pass (HP) response [8], [9], [12], [13], [15] or band pass (BP) [9], [10], [12]-[14].

Some of journal papers provide great details about whole design procedure [15]. Note that pole frequency of reported filters is usually non-controllable and in range from 1 kHz up to 10 kHz [8]-[17], sometimes higher bands are tested [18].

Our contribution includes designed fractional-order HP filter based on FLF with inverting integrators. Basic building blocks are Operational Transconductance Amplifiers (OTAs) [20] and single-input Adjustable Current Amplifiers (ACAs) [21] active elements. The fractional order and pole frequency can be controlled electronically by control of the active element transfer coefficients. Design is supported by PSpice simulations with three different

values of the fractional order (1.25, 1.5 and 1.75) and for three particular values of pole frequency. Moreover, two common approximations are mutually compared in order to highlight their significant differences.

II. BUTTERWORTH APPROXIMATION OF THE FRACTIONAL-ORDER HIGH-PASS FILTER

Design procedure of the fractional-order high-pass filter (FHPF) having order of $(1 + a)$ was precisely described in [15]. Starting transfer function is in the following form:

$$K_{1+a}^{HP}(s) = \frac{K_1 s^{1+a}}{K_2 s^{1+a} + K_3 s + 1} \quad , \qquad (1)$$

where from [15]:

$$K_1 = 1 \; ,$$
$$K_2 = 0.2937a + 0.71216 \; , \qquad (2)$$
$$K_3 = 1.068a^2 + 0.161a + 0.3324 \; ,$$

coefficients K_X, where $X = 1, 2, 3$ influence directly the shape of the transfer characteristic. Note that the values of K_X are dependent on the parameter a, i.e. on the order of the filter. Second-order approximation of s^a from (1) is given by:

$$s^a = \frac{a_0 s^2 + a_1 s + a_2}{a_2 s^2 + a_1 s + a_0} \quad . \qquad (3)$$

There are several ways how to approximate coefficients a_0, a_1 and a_2 from (3) and two of them are going to be compared within this paper. First set of coefficients can be calculated as follows [17]:

$$a_0 = 2(1 + a) \; ,$$
$$a_1 = 5 - a^2 \; , \qquad (4)$$
$$a_2 = 2(1 - a) \; .$$

Second approximation [19] is defined by:

$$a_0 = a^2 + 3a + 2 \; ,$$
$$a_1 = 8 - 2a^2 \; , \qquad (5)$$
$$a_2 = a^2 - 3a + 2 \; .$$

The following 3rd-order transfer function is going to be used to emulate FHPF with Butterworth approximation of order $(1 + a)$ [15]:

$$K_{1+a}^{HP}(s) \cong \frac{K_1}{a_0} \frac{a_0 s^3 + a_1 s^2 + s a_2}{s^3 + \dfrac{b_1}{b_0} s^2 + \dfrac{b_2}{b_0} s + \dfrac{1}{b_0}} \quad . \qquad (6)$$

Coefficients b_X are calculated directly form a_X and K_X, where $X = 1, 2, 3$, as shown below [15]:

$$b_0 = \frac{a_0 K_2 + a_2 K_3}{a_0} \quad ,$$
$$b_1 = \frac{a_1(K_2 + K_3) + a_2}{a_0} \quad , \qquad (7)$$
$$b_2 = \frac{a_1 + a_2 K_3 + a_2 K_2}{a_0} \quad .$$

One of possible ways how to implement 3rd-order transfer function is to use FLF filtering topology. Fig. 1 shows the particular block diagram implementing the topology, consisting of inverting integrators, supplemented by three feedback loops and three forward signal paths. Transfer function of this general topology is

$$K(s) = \frac{I_{OUT}}{I_{IN}} = \frac{s^3 G_1 + s^2 \dfrac{G_2}{\tau_1} + s \dfrac{G_3}{\tau_1 \tau_2}}{s^3 + s^2 \dfrac{1}{\tau_1} + s \dfrac{1}{\tau_1 \tau_2} + \dfrac{1}{\tau_1 \tau_2 \tau_3}} \quad . \qquad (8)$$

Equations how to obtain τ_X and G_X from a_X and b_X coefficients are obvious when comparing (6) and (8), when $X = 1, 2, 3$.
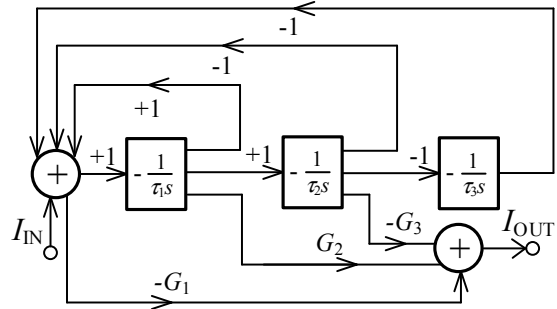

Figure 1. Block diagram of a 3rd-order FLF (follow-the-leader feedback) topology used for approximation of the FHPF (fractional-order high-pass filter) of $(1 + a)$ order

III. ACTIVE ELEMENTS USED IN FILTERING TOPOLOGY

The designed filter, presented in the following section is based on two well-known types of active elements. First one is Operational Transconductance Amplifier (OTA) [20] with suitable number of current outputs. Schematic symbol of OTA in case of two current outputs is shown in Fig. 2. Note that this element is also frequently referred to as Balanced-Output OTA (BOTA).
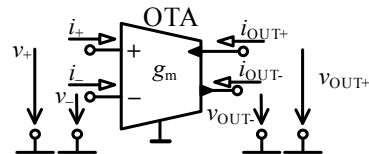

Figure 2. Operational Transconductance Amplifier (OTA) schematic symbol (dual-output version)

Standard operation of OTA is described by these equations: $i_{OUT+} = -i_{OUT-} = g_m \cdot (v_+ - v_-)$, while $i_+ = i_- = 0$. Note that transconductance parameter (g_m) of OTA is usually controlled by external DC current.

Second type of active element that is implemented in the proposed FLF topology of the filter is the single-input Adjustable Current Amplifier (ACA) [21] with adjustable current gain. This element is depicted in Fig. 3. Number of current outputs also dependents on particular requirements.

Standard operation of ACA is described by simple equations: $i_{OUT+} = -i_{OUT-} = B \cdot i_{IN}$. Current gain ($B$) is also usually controlled externally, in many cases by DC voltage [22].
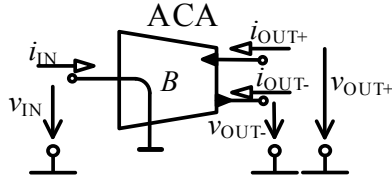

Figure 3. Adjustable Current Amplifier (ACA) schematic symbol (dual-output version)

Note that for current gain fixed to 1 or −1, this element is frequently referred to as Current Follower (CF) in case of having single output. If there are two current outputs, element is known as Dual-Output CF (DO-CF) [23]. Description of DO-CF element is because of its simplicity omitted in this paper.

Utilization of OTA and ACA elements is beneficial because of the possibility of electronical control of parameters of the designed filter by electronic control of internal parameters of these active elements.

IV. Designed Fractional-Order HP Filter

Designed FHPF with active elements introduced in previous section and following the concept of FLF block topology (Fig. 1) is shown in Fig. 4.
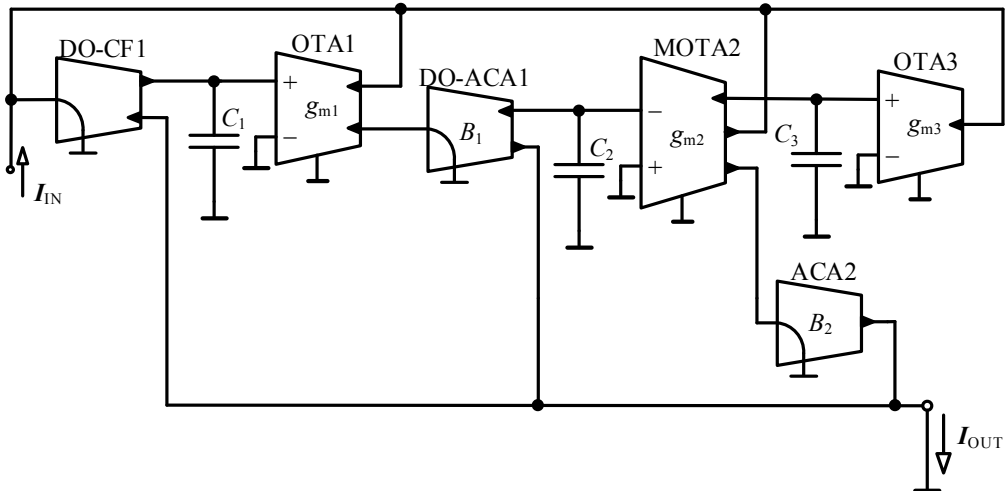
Particular transfer function of the filter from Fig. 4 that has inverting character is as follows:

$$K(s) = \frac{I_{OUT}}{I_{IN}} = \frac{N(s)}{D(s)} \ , \qquad (8)$$

where

$$N(s) = -s^3 - s^2 \frac{g_{m1}B_1}{C_1} - s \frac{g_{m1}g_{m2}B_1B_2}{C_1C_2} \ , (9)$$

and

$$D(s) = s^3 + s^2 \frac{g_{m1}}{C_1} + s \frac{g_{m1}g_{m2}B_1}{C_1C_2} + $$
$$+ \frac{g_{m1}g_{m2}g_{m3}B_1}{C_1C_2C_3} \ . \qquad (10)$$

Fig. 4 contains all variables that are used in (9) and (10). Note that except values of C_1, C_2 and C_3, all these parameters can be controlled electronically, which is beneficial feature not only for control of the pole frequency of the filter but also for electronic change of the order of the HP filter as will be demonstrated in the following section.

V. Simulation Results

Filter from Fig. 4 was simulated in OrCAD PSpice under two scenarios in order to provide comparison of two possible approximations of a_0, a_1 and a_2 calculated from (4) and (5). All active elements were emulated by behavioral models as described below in order to provide realistic simulation results besides theoretical expectations. OTAs were substituted by 3rd-level model of Universal Current Conveyor (UCC) [23] and in case of ACAs, simulation model of EL2082 chip from Elantec [22] was used as a replacement. Unfortunately, EL2082 has only one current output, therefore, in case of DO-ACA1, additional copy of current is required. EL2082 was in this particular case supplemented by one UCC connected as DO-CF [24]. Note that DO-CF1 element within the structure shown in Fig. 4 was also emulated by UCC.

Figure 4.   Designed 3rd-order HP filter approximating $(1 + a)$-order FHPF with controllable active elements

Starting pole frequency was selected as $f_p = 100$ kHz, capacitor values are $C_1 = 560$ pF, $C_2 = 910$ pF and $C_3 = 3.9$ nF for both approximations. Other calculated parameters of the filter are summarized for the first approximation in Table I and for the second approximation in Table II.

The simulation results in the case of the first values of coefficients (referred to as 1st set) and the second values coefficients (designated by 2nd set) respectively are compared to theoretical presumptions and also mutually in the following figures.

Fig. 5 provides comparison of theoretical expectations vs. theory for both approximations and for three values of a parameter. From both graphs it is obvious that theory and simulation results match very well in case of first and also second approximation. Theoretically, attenuation in stop-band should be 25, 30 and 35 dB per decade for chosen values of a parameter. In case of first approximation, the obtained values are 24.1, 29.3 and 34.2 dB, in case of second approximation obtained values are 25.5, 30.5 and 35.4 dB.

In both cases particular band, where we can observe the validity of approximation is limited by the second-order approximation of s^a (3). Comparison of approximations in more detail is provided by Fig. 6 for both the magnitude and phase responses. As obvious from these graphs, second approximation has intended fractional character from f_p down to frequency around 4 kHz (vs. 10 kHz in the case of 1st approximation). However the second approximation does not follow expected behavior of Butterworth approximation ideally in the pass band - there is visible attenuation around the pole frequency. Note that these findings are valid for all the tested values of a parameter.

Based on these two key differences, proper type of approximation should be selected for particular application in order to obtain required character of response. Note that order of the FHPF is controlled electronically in both cases by simultaneous change of five parameters: g_{m1}, g_{m2}, g_{m3}, B_1, and B_2 as summarized in Table I and II. Moreover, if only g_{m1}, g_{m2}, g_{m3} parameters are tuned, pole frequency can be controlled without disturbing the order of the filter as is shown in Fig. 7 for both approximations. Three f_p-s were selected as an example: 33, 100 and 300 kHz. Change of g_{m1}, g_{m2}, g_{m3} parameters required for setting of these pole frequencies is obvious from Table III, where values for both approximations are summarized. Note that comparison with theoretical expectations was omitted in this particular case.

TABLE I. PASSIVE COMPONENTS AND CURRENT GAINS FOR A = 0.25, 0.5, 0.75 WHEN FIRST SET OF COEFFICIENTS WAS USED (4)

order (*a*)	0.25	0.5	0.75
$1/g_{m1}$ [Ω]	988	1114	1157
$1/g_{m2}$ [Ω]	1256	1179	978
$1/g_{m3}$ [Ω]	1178	1040	1002
B_1 [-]	0.686	0.621	0.516
B_2 [-]	0.218	0.142	0.063

TABLE II. PASSIVE COMPONENTS AND CURRENT GAINS FOR A = 0.25, 0.5, 0.75 WITH SECOND SET OF COEFFICIENTS (5)

| order (*a*)	0.25	0.5	0.75
$1/g_{m1}$ [Ω]	722	863	980
$1/g_{m2}$ [Ω]	1345	1220	983
$1/g_{m3}$ [Ω]	1472	1164	1038
B_1 [-]	0.712	0.607	0.493
B_2 [-]	0.128	0.070	0.026
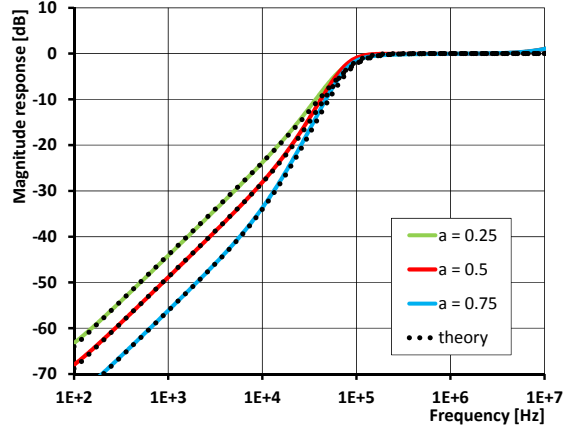
a)
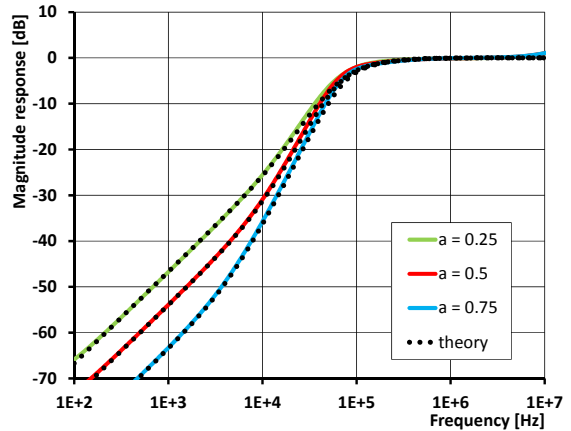
b)

Figure 5. Simulation results (colored solid lines) vs. theory (dotted lines) for three values of a parameter (magnitude responses) in case of a) 1st set of coefficients defined in (4), b) 2nd set of coefficients defined by (5). Both for fixed value of $f_p = 100$ kHz.

TABLE III. VALUES OF TRANSCONDUCTANCES WHEN TUNING THE POLE FREQUENCY IN BOTH THE DISCUSSED CASES AND FOR ORDER OF THE FILTER DEFINED BY A = 0.75
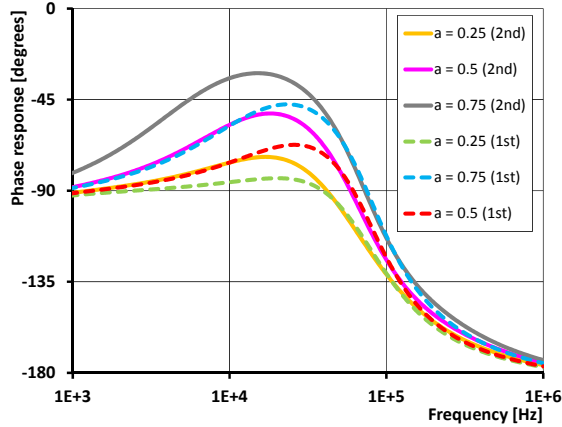
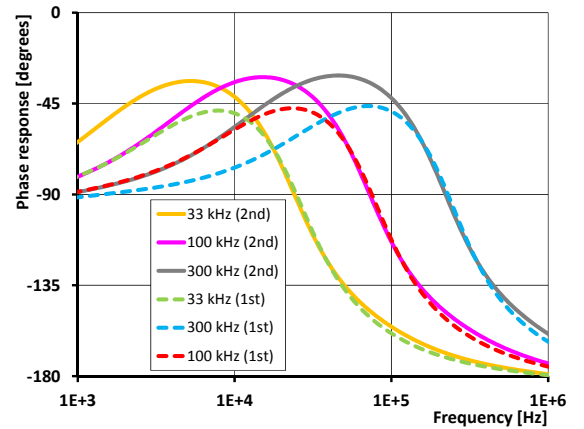| Pole frequency f_p [kHz]	33	100	300
$1/g_{m1}$ (1st) / $1/g_{m1}$ (2nd) [Ω]	3471 / 2940	1157 / 980	386 / 327
$1/g_{m2}$ (1st) / $1/g_{m2}$ (2nd) [Ω]	2934 / 2949	978 / 983	326 / 328
$1/g_{m3}$ (1st) / $1/g_{m3}$ (2nd) [Ω]	3006 / 3114	1002 / 1038	334 / 346

a)
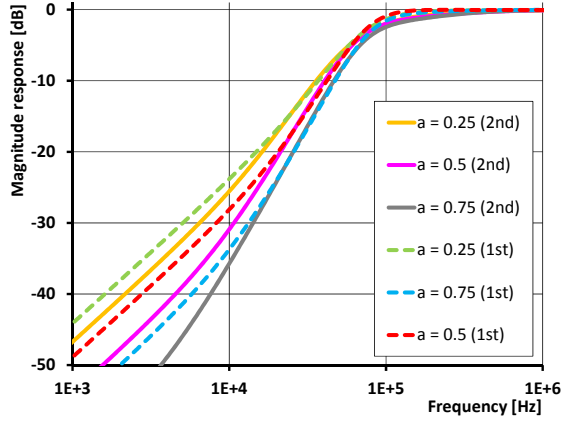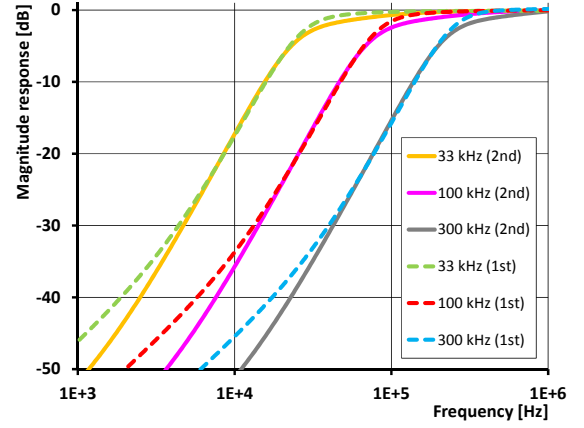
b)

Figure 6. Mutual comparison of both approximations in detail for three values of *a* parameter and fixed value of f_p = 100 kHz. a) magnitude responses b) phase responses.

a)

b)

Figure 7. Mutual comparison of both approximations in detail in case of tuning f_p and for *a* = 0.75 a) magnitude responses b) phase responses.

From both magnitude and phase response graphs in Fig. 7 it is obvious that order of the filter is not disturbed while tuning the pole frequency in case of the simulation results. Moreover, these graphs confirm the previously observed differences between two studied approximations, i.e. that first approximation has more limited validity of fractional character in stop band, however, its features are very good in the pass band, because there is no undesired attenuation. Of course, these differences are given by particular positions of zeros and poles of transfer functions that are dependent on the selected type of approximation of transfer function.

Note that design calculation were aimed to allow omitting of ACAs, if a number of active elements should be decreased. Because for any value of *a* parameter current gain is less than one (B_X < 1). Therefore, if electronical control of order is not required, number of active elements can be decreased to only 4 active elements.

In case of this FHPF response, sufficiently high values of output resistances of used active elements are very important. In this particular case, it influences results significantly first below 30 Hz frequency and therefore it is not an issue in band where fractional-order character is provided and it can be neglected.

VI. CONCLUSION

This paper presented design procedure and particular solution of fractional-order HP filter. The simulation results were obtained with help of behavioral models of OTAs and ACAs and the design is prepared in order to allow also future verification of features by laboratory measurements. Order of the filter and also the pole frequency of the filter can be controlled electronically. Moreover, two possible approximations were mutually compared in order to highlight their differences. As discussed in introductory part, most of the published solutions of fractional-order filters are designed to operate in low-frequency range below 10 kHz because of active elements used. The designed FHPF topology presented in this paper confirmed possibility to use concept of fractional-order filters also at higher frequencies. Of course, obtained features are always limited by features of active elements. Really precise values of transconductances, i.e. resistors, and also of current gains are required. Of course, in case of real implementation, the values of resistors have to be selected from at least E48 row, while precise values of current gains are usually not that problematic. This step was omitted in order to suppress impact of inaccurate resistors and to focus more on features of both approximations.

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