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Overview of absolute nodal coordinate formulation and usage of recently formulated finite elements

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This paper is dedicated to a description of finite elements defined using the absolute nodal coordinate formulation (ANCF). The ANCF is a modern finite element formulation which is suitable for modelling of flexible bodies that are parts of multibody systems. The main advantage of these elements is their ability to describe large displacements and rotations of flexible bodies. As it is mentioned in [3, 5], the ANCF elements can exactly describe an arbitrary rigid body motion of a discretized flexible body. This ability is achieved by using absolute position vectors of nodes and their gradients (slopes) with respect to local element coordinates as nodal coordinates. Unlike the ANCF method, classical finite element formulations of beams and shells use infinitesimal rotations instead of slopes as nodal coordinates which leads to a nonzero element elastic forces in case of an arbitrary rigid body motion [5] and thus these classical finite elements are not suitable for large motion problems.

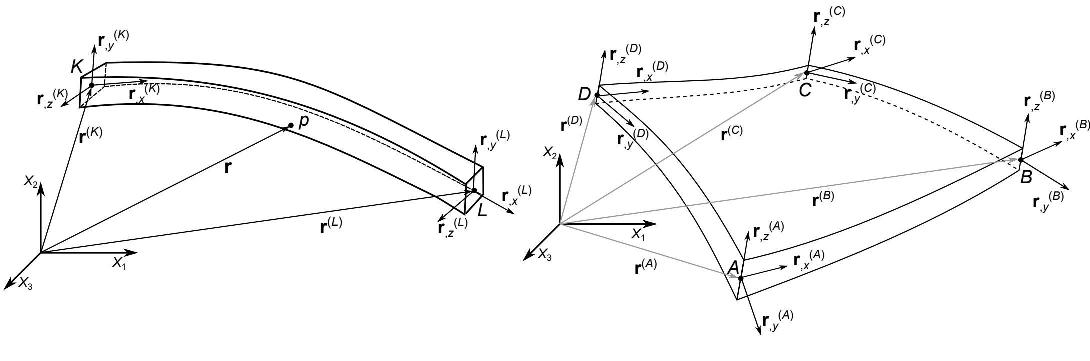

Fig. 1. Deformed fully parameterized ANCF beam element and ANCF plate element with vectors which defining the nodal coordinates (absolute positions of nodes and gradients in nodes)

In Fig. 1, a deformed fully parameterized original ANCF beam element and ANCF plate element are shown. In this picture, coordinates $X_1X_2X_3$ represent the global (absolute) coordinate system and xyz is a local element coordinate system. For the beam element, coordinate $x \in \langle 0,\ l_e\rangle$ is the axial parameter of the beam centerline and l_e is the beam element length. Coordinates y and z are cross-sectional beam coordinates. In case of the plate element, coordinates $x \in \langle 0,\ a_e\rangle$ and $y \in \langle 0,\ b_e\rangle$ are plate mid-surface coordinates and $z \in \langle -\frac{t_e}{2},\ \frac{t_e}{2}\rangle$ represents the coordinate perpendicular to plate mid-surface. Parameters a_e, b_e and t_e are the plate element length, width and thickness.

The absolute position vector of node i is denoted as $\mathbf{r}^{(i)}$, where $i = \{A,\ B,\ C,\ D,\ K,\ L\}$. In Fig. 1, and in further text of this paper, following symbolic notation for partial derivatives of node position vector is used

$$\mathbf{r}_{,j}^{(i)} = \frac{\partial \mathbf{r}^{(i)}}{\partial j}, \qquad j = \{x,\ y,\ z\}\,. \tag{1}$$

These partial derivatives represent slope vectors or gradients of node position vector with respect to local coordinates.

The nodal coordinates of chosen node i can be written as

$$\mathbf{e}^{(i)} = \left[\mathbf{r}^{(i)T},\ \mathbf{r}_{,x}^{(i)T},\ \mathbf{r}_{,y}^{(i)T},\ \mathbf{r}_{,z}^{(i)T}\right]^T. \tag{2}$$

From Eq. (2), it is apparent, that each node has three positional degrees of freedom and nine degrees of freedom related to the components of three gradients. The ANCF beam element has 24 degrees of freedom because it uses two nodes – K and L. Its resultant vector of nodal coordinates can be expressed as

$$\mathbf{e} = \left[\mathbf{e}^{(K)T},\ \mathbf{e}^{(L)T}\right]^T. \tag{3}$$

The vector of nodal coordinates of the ANCF plate element can be expressed in the similar way with a difference in number of nodes – it uses four nodes (A, B, C and D). The resultant element has 48 degrees of freedom and its vector of nodal coordinates is

$$\mathbf{e} = \left[\mathbf{e}^{(A)T},\ \mathbf{e}^{(B)T},\ \mathbf{e}^{(C)T},\ \mathbf{e}^{(D)T}\right]^T. \tag{4}$$

It must be noted here that the vector $\mathbf{r}_{,z}^{(i)T}$ (plate element) or vectors $\mathbf{r}_{,z}^{(i)T}$ and $\mathbf{r}_{,y}^{(i)T}$ (beam element) can be omitted from the Eq. (2). The resultant element is called gradient-deficient ANCF element [2], its element elastic forces are derived in another manner than it is described later in this paper and these elements are suitable mainly for thin structures.

In case of the original ANCF beam element, the position vector $\mathbf{r}$ of arbitrary beam point p is approximated using cubic polynomials in x and linear polynomials in y and z, which is written in following form [2]

$$\mathbf{r} = \begin{bmatrix} a_0 + a_1x + a_2y + a_3z + a_4xy + a_5xz + a_6x^2 + a_7x^3 \\ b_0 + b_1x + b_2y + b_3z + b_4xy + b_5xz + b_6x^2 + b_7x^3 \\ c_0 + c_1x + c_2y + c_3z + c_4xy + c_5xz + c_6x^2 + c_7x^3 \end{bmatrix}, \tag{5}$$

where coefficients a, b and c can be expressed using known local coordinates of nodes. In case of plate elements it is similar with the difference that a cubic approximation is used in y. Then the interpolation uses following set of basis polynomials

$$\left[1,\ x,\ y,\ z,\ xy,\ xz,\ yz,\ x^2,\ y^2,\ x^3,\ y^3,\ x^2y,\ xy^2,\ xyz,\ x^3y,\ xy^3\right]. \tag{6}$$

After several operations, the global position vector of arbitrary beam/plate point p can be expressed as

$$\mathbf{r} = \mathbf{S}\left(x,\ y,\ z\right)\mathbf{e}, \tag{7}$$

where $\mathbf{S}$ is the matrix of shape functions of a chosen element (differs for beam and plate elements).

After the kinematic description of ANCF elements, the element kinetic energy is

$$E_{ke} = \frac{1}{2}\int_{V_e} \rho\dot{\mathbf{r}}^T\dot{\mathbf{r}}\mathrm{d}V_e, \tag{8}$$

where V_e is the element volume and ρ is material density. With the use of (7), the resultant constant mass matrix has a form

$$\mathbf{M}_e = \int_{V_e} \rho \mathbf{S}^T \mathbf{S} \mathrm{d}V_e. \tag{9}$$

There are several procedures for ANCF element elastic force derivation. The original ANCF elements use the *continuum mechanics approach*. In this formulation, the Saint Venant–Kirchhoff material model is used and the strain energy is

$$U_e = \frac{1}{2} \int_{V_e} \boldsymbol{\varepsilon}^T \mathbf{E} \boldsymbol{\varepsilon} \mathrm{d}V_e, \tag{10}$$

where $\mathbf{E}$ is the matrix of the elastic constants of the material and $\boldsymbol{\varepsilon}$ is the vector of components of Lagrange–Green strain tensor written as

$$\boldsymbol{\varepsilon} = \left[\varepsilon_x,\ \varepsilon_y,\ \varepsilon_z,\ 2\varepsilon_{xy},\ 2\varepsilon_{xz},\ 2\varepsilon_{yz}\right], \tag{11}$$

where

$$\begin{aligned} \varepsilon_x &= \tfrac{1}{2}(\mathbf{r}_{,x}^T \mathbf{r}_{,x} - 1), & \varepsilon_y &= \tfrac{1}{2}(\mathbf{r}_{,y}^T \mathbf{r}_{,y} - 1), & \varepsilon_z &= \tfrac{1}{2}(\mathbf{r}_{,z}^T \mathbf{r}_{,z} - 1), \\ \varepsilon_{xy} &= \tfrac{1}{2}\mathbf{r}_{,x}^T \mathbf{r}_{,y}, & \varepsilon_{xz} &= \tfrac{1}{2}\mathbf{r}_{,x}^T \mathbf{r}_{,z}, & \varepsilon_{yz} &= \tfrac{1}{2}\mathbf{r}_{,y}^T \mathbf{r}_{,z}. \end{aligned} \tag{12}$$

The resultant nonlinear vector of element elastic forces is

$$\mathbf{Q}_e = \frac{\partial U_e}{\partial \mathbf{e}}. \tag{13}$$

Based on the described formulation an in-house software in Matlab that uses ANCF beam and plate elements was created and was used in several applications. The typical benchmark simulation for ANCF beam elements is the simulation of a motion of a flexible pendulum. This benchmark problem is used to demonstrate the ability to describe large motions of the flexible bodies. In Fig. 2, the snapshots of the pendulum motion in discrete time steps are shown. The pendulum is 2 m long, has a square cross-section with the edge length of 0.01 m, material density is 4000 kg.m^{-3}, Young's modulus is 10^8 Pa, Poisson's ratio is 0.3 and 10 ANCF beam elements are used.

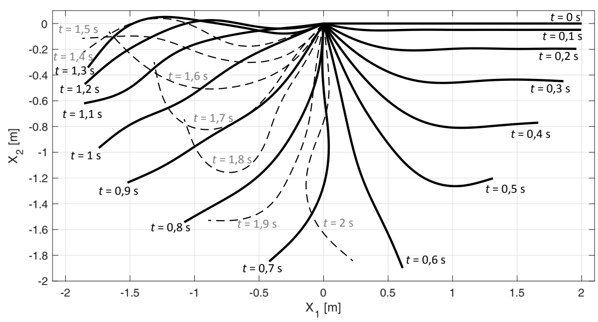

Fig. 2. Visualisation of the flexible pendulum in discrete time steps

Similar benchmark problem is typical for plate elements testing and demonstration – flexible plate pendulum. The tested flexible plate length is 1 m, width is 1 m and thickness is 0.02 m. The plate is connected to the ground by spherical joint in one plate corner. The chosen material density is 1000 kg.m^{-3}, Young's modulus is 10^5 Pa, Poisson's ratio is 0.3 and 8×8 plate elements are used. The snapshots of the flexible plate motion in discrete time steps are shown in Fig. 3.

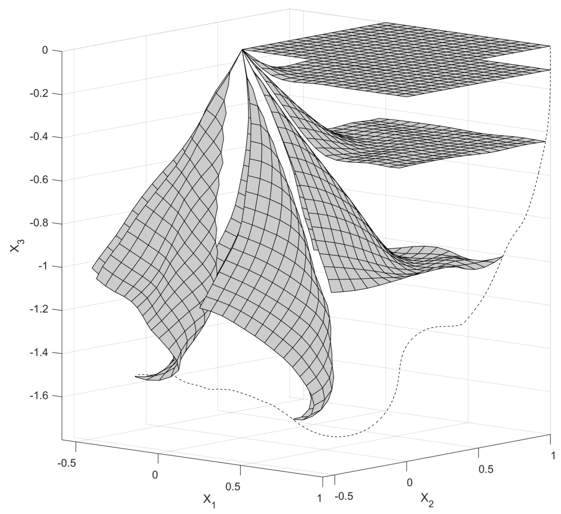

Fig. 3. Visualisation of the flexible plate in discrete time steps

There are a lot of practical use of various ANCF elements. In paper [1] ANCF beam elements were used for cable modelling and the cable–pulley interaction model is investigated and verified by experiment. The ANCF beam and plate elements were successfully used for nonlinear dynamic analysis of parabolic leaf springs in paper [6]. In fact, the ANCF method can be used anywhere, where the flexibility of bodies of multibody systems is not negligible, such as detailed tire modelling, dynamics of thin membrane structures used in aerospace or modelling of railway catenary. For completeness it is necessary to mention that also solid ANCF elements were developed recently [4] and they can be used, e.g., for rubber structures modelling. The main disadvantage of ANCF method is, that the resultant dynamic simulations are relatively time consuming which is caused by nonlinearity of the elastic forces and higher number of element degrees of freedom.

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