Numerical simulation of non-equilibrium wet steam flow in a turbine cascade

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The work deals with numerical simulation of non-equilibrium wet steam flows. The non-equilibrium steam is described by the system of Navier-Stokes equations. Thermophysical properties of the steam are calculated according to IAPWS IF97 formulation [3] and the non-equilibrium condensation is described by an additional two-equation model for liquid phase mass fraction and specific number of droplets (the so called mono-dispersion no-slip model, see e.g. [2] or [6]).

The system of Navier-Stokes equations describing the movement of the gas-liquid mixture is

$$\rho_t + \nabla \cdot (\rho \vec{v}) = 0, \tag{1}$$

$$(\rho \vec{v})_t + \nabla \cdot (\rho \vec{v} \otimes \vec{v}) + \nabla p = \nabla \cdot \vec{\vec{\tau}}, \tag{2}$$

$$(\rho E) + \nabla \cdot (\rho H \vec{v}) = \nabla \cdot (\vec{\vec{\tau}} \cdot \vec{v}) - \nabla \cdot \vec{q}. \tag{3}$$

Here ρ is the density, $\vec{v}$ is the common velocity, p is the pressure, $\vec{\vec{\tau}}$ is the effective deviatoric stress tensor, $E = e + v^2/2$ is the specific total energy where e is the specific internal energy, $H = h + v^2/2$ is the specific total enthalpy, and q is the effective heat flux. The system is coupled to standard two equation $k - \omega$ SST turbulence model.

The non-equilibrium condensation is described by the classical nucleation theory using the wetness w defined as the mass fraction of the liquid phase in the mixture and the specific number of droplets Q_0. These quantities are described by the following system of equations

$$(\rho w)_t + \nabla \cdot (\rho w \vec{v}) = \frac{4}{3} \pi r_c^3 \rho_l J + 4 \pi r_{30}^2 \rho_l \rho Q_0 \dot{r}, \tag{4}$$

$$(\rho Q_0)_t + \nabla \cdot (\rho Q_0 \vec{v}) = J, \tag{5}$$

where r_c is the critical droplet radius, $r_{30} = \sqrt[3]{3w/4Q_0 \pi \rho_l}$ is the average droplet radius, $\dot{r}$ is the droplet growth ratio, ρ_l is the liquid phase density, and J is the nucleation rate. For details see, e.g., [2].

Thermophysical properties of the mixture are calculated using IAPWS IF97 equation of state for gaseous phase (denoted by subscript g) and combined with polynomial properties of liquid phase (subscript l) at the saturation line in the following way

$$\frac{1}{\rho} = \frac{w}{\rho_l} + \frac{1-w}{\rho_g} \approx \frac{1-w}{\rho_g}, \tag{6}$$

$$h = wh_l + (1-w)h_g = h_g - wL, \tag{7}$$

$$p = p_g, \tag{8}$$

$$T = T_g = T_l, \tag{9}$$

where $L = h_g - h_l$ is the specific latent heat of evaporation.

The whole system of 9 partial differential equations is solved with an in-house finite volume solver based on the OpenFOAM framework [4]. The time dependent solution is of coupled system of equations is obtained using a modified version of sequential pressure correction method for compressible flows. The basic structure of the algorithm is described in the following listing for one time-step:

Algorithm 1: Pressure correction loop structure

    input : State at the time t^n
    output: State at the time t^{n+1}

1 update ρ using continuity eq. (1)
2 **for** *outer corrector = 1, ...* **do**
3   update $\vec{v}$ using momentum eq.(1) with actual pressure
4   update h using energy eq. (1) formulated in terms of enthalpy
5   correct condensation model by solving eq. (4) and (5)
6   calculate h_g using current L and w using eq. (7)
7   calculate gas phase properties from p and h_g using IF97
8   calculate mixture properties using eqns. (6-9)
9   **for** *piso corrector = 1, ...* **do**
10   correct the pressure in order to satisfy continuity
11   update velocity from the pressure correction; update turbulence model;
12   **end**
13 **end**

For most of calculations we use 2 PISO correctors (line 9) and 5-10 outer correctors (line 2), although the actual number of outer correctors depends on the setup of convergence criteria.

The solver is validated using 2D flows through a transonic nozzle. The nozzle inflow is characterized by the value of total pressure $p_{tot} = 78.39\,\text{kPa}$ and total temperature $T_{tot} = 373.15\,\text{K}$. The outflow is supersonic. The simulation is performed using a structured mesh with 500×100 hexahedral cells with near-wall refinement corresponding $y^+ \approx 1$.
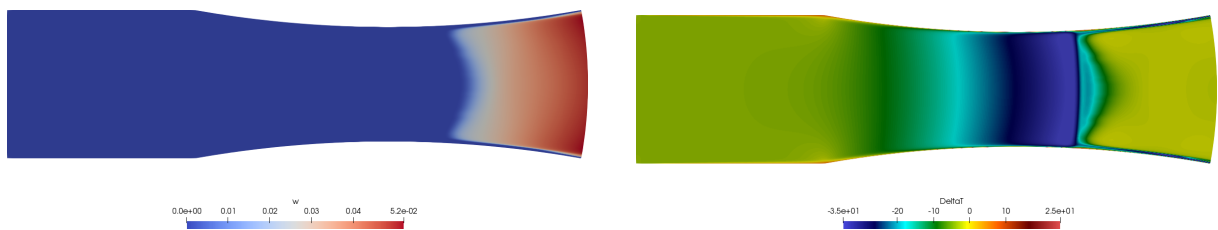

Fig. 1. Wet steam flow through a transonic nozzle: (*left*) wetness w, (*right*) sub-cooling $T - T_{sat}$

Fig. 1 shows the distribution of the wetness w (on the left) and the subcooling defined as the difference of T and the saturation temperature $T_{sat}(p)$. One can see that the condensation starts approximately at $T - T_{sat} \approx -35\,\text{K}$, whereas the equilibrium condensation would start at $T = T_{sat}$.

Fig. 2 shows the distribution of the pressure, wetness, and droplet radius along the axis of the nozzle. The pressure is compared to experimental data taken from [1]. One can see that the current calculation overpredicts the strength of the condensation shock, although the position of the shock corresponds very well to the experimental data. Similar behavior was found also
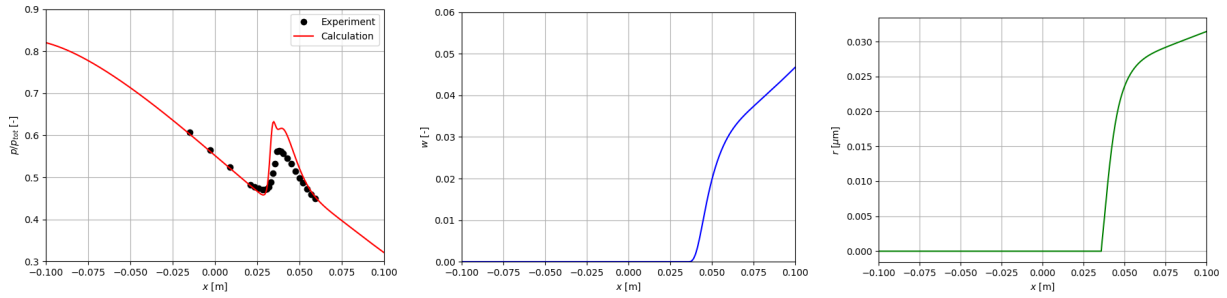
Fig. 2. Distribution of the pressure (*left*), wetness (*middle*), and droplet radius (*right*) along the nozzle axis

for completely different numerical method based on advanced Riemann solvers in [2] with very fine meshes. The wetness and the droplet radius correspond to results published in [2].

Fig. 3 shows the results of simulation of flows through a 2D model of turbine cascade. The simulation is done using an unstructured mesh with approximately 25 000 cells. The mesh is refined in the vicinity of the blade with $y^+ \approx 50$ and the near wall treatment with standard wall functions is used. The regime is characterized by the inlet total pressure $p_{tot} = 40.3$ kPa, total temperature $T_{tot} = 354$ K and axial flow direction. The outlet pressure is $p_2 = 16.3$ kPa. Fig. 3 shows the distribution of the pressure, wetness, and the entropy in the form if is-lines. One can recognize an entropy production in the zone of condensation indicating the energy losses due to phase transition. The distribution of the wetness documents that the model predicts also evaporation at the shock wave cased by the temperature jump across the shock. The distribution of the pressure along the blade shows quite good agreement with experimental data [5].

The presented results show that the solver is able to simulate wet steam flows with non-equilibrium condensation model. The method is based on a very simple two-equation model which usually does not predict very well droplet sizes. Nevertheless, the wetness as well as the flow field is predicted with reasonable accuracy. The use of real gas equation of state naturally slows down the execution speed of the solver, although this slowdown is not prohibitive for pressure correction family of schemes where on spends a large portion of time in the solution of the Helmholtz equation for the pressure.

Acknowledgements

References

[1] Barschdorf, D., Course of state variables and gas-dynamic correlations in the spontaneous condensation of pure water vapor in Laval nozzles, Forschung im Ingenieurwesen 37 (5) (1971) 146-157. (in German)

[2] Halama, J., Hric, V., Numerical solution of steam flow in a nozzle using different non-equilibrium condensation models, Applied Mathematics and Computation 272 (2016) 657-669.

[3] Wagner, W., Cooper, J. R., Dittmann, A., Kijima, J., Kretzschmar, H.-J., Kruse, A., Mareš, R. Oguchi, K., Sato, H., Stöcker, I. Šifner, O., Takaishi, Y., Tanishita, I., Trübenbach, J., Willkommen, T., The IAPWS industrial formulation 1997 for the thermodynamic properties of water and steam, Journal of Engineering for Gas Turbines and Power 122 (1) (2000) 150-184.

[4] Weller, H. G., Tabor, G., Jasak, H., Fureby, C., A tensorial approach to computational continuum mechanics using object-oriented techniques, Computers in Physics 12 (6) (1998) 620-631.
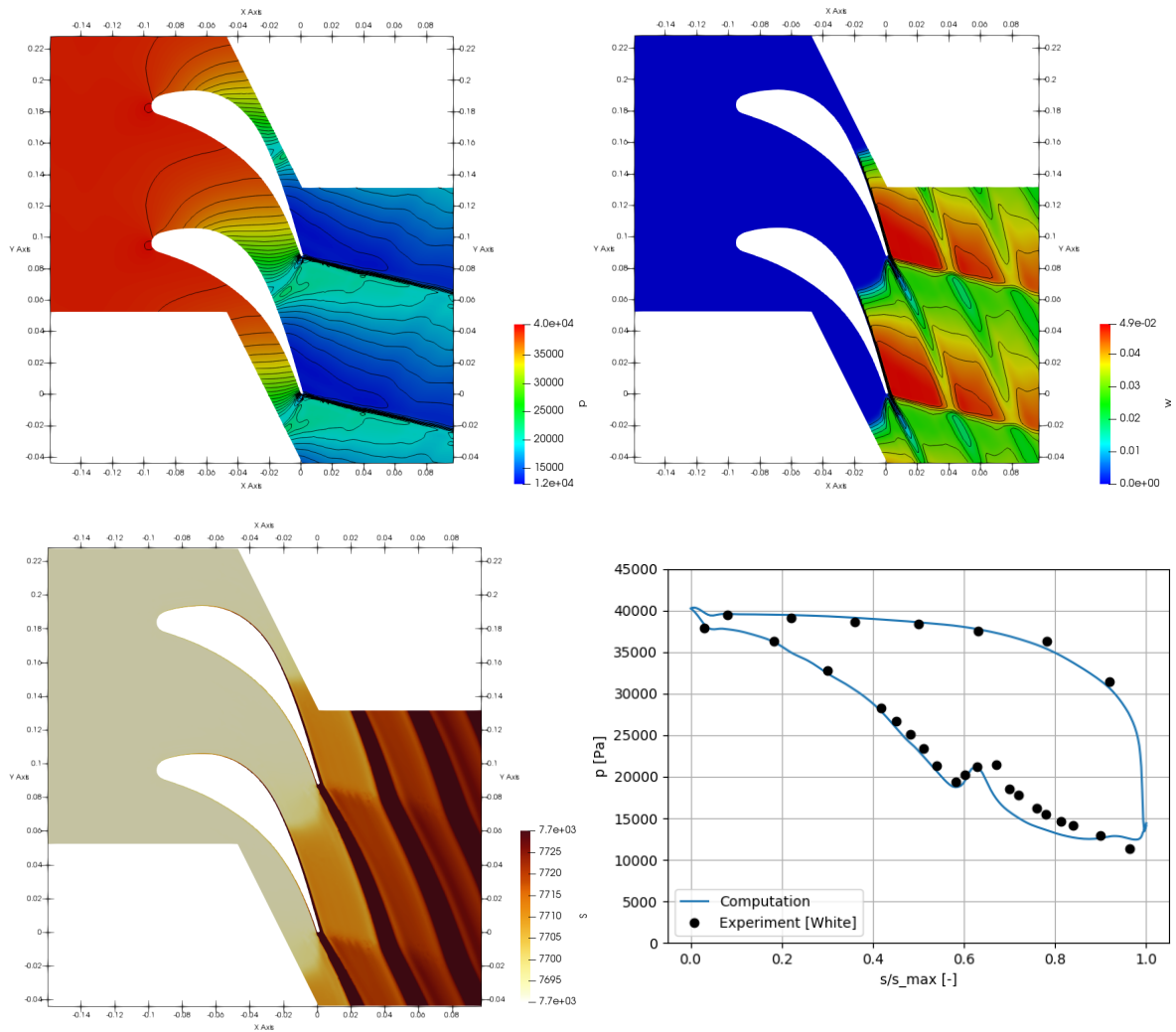
Fig. 3. Wet steam flow through a turbine cascade, the isolines of the pressure, wetness, and entropy, and the distripution of the pressure along the blade

[5] White, A. J., Young, J. B., Walters, P. T., Experimental validation of condensing flow theory for a stationary cascade of steam turbine blades, Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences 354 (1704) (1996) 59-88.

[6] Wróblewski, W., Dykas, S., Chmielniak, T., Models for water steam condensing flows, Archives of Thermodynamics 33 (1) (2012) 67-86.