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Galerkin method for approximate modelling of finite-length journal bearings

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Galerkin method as a weighted residual method is one of many possible approaches to solve a partial differential equation whose solution is approximated by combination of trial functions. These functions are mutually linearly independent and satisfy boundary conditions.

Pressure field in a circular journal bearing is governed by Reynolds equation in the dimensionless form [2]

$$\frac{\partial}{\partial\varphi}\left(\overline{h}^3\frac{\partial\overline{p}}{\partial\varphi}\right)+\left(\frac{R}{L}\right)^2\frac{\partial}{\partial\overline{Z}}\left(\overline{h}^3\frac{\partial\overline{p}}{\partial\overline{Z}}\right)=\left(2\frac{\dot{\gamma}}{\omega}-1\right)\varepsilon\sin\varphi+2\frac{\dot{\varepsilon}}{\omega}\cos\varphi, \tag{1}$$

where $\overline{p}(\varphi,\overline{Z})$ is the dimensionless pressure field, φ and $\overline{Z}$ are the circumferential and axial coordinates, respectively, ω is the angular speed, R is the radius of a bearing shell, L is the length of a bearing, $\overline{h}$ is the dimensionless height of thin oil film, ε is the relative eccentricity and $\dot{\varepsilon}$ and $\dot{\gamma}$ are the velocities of relative eccentricity and attitude angle, respectively.

Unknown pressure distribution in the circular journal bearing is approximated by Fourier series in circumferential direction and by goniometric sinus function in axial direction. Final form of the pressure field approximation [1] is written as

$$\overline{p}\approx\hat{p}\left(\varphi,\overline{Z},t\right)=\overline{p}_{amb}+\sum_{i=0}^{N_O}\sum_{j=1}^{N_A}\left[a_{i,j}\sin\left(i\varphi\right)+b_{i,j}\cos\left(i\varphi\right)\right]\sin\left(j\pi\overline{Z}\right), \tag{2}$$

where $\overline{p}_{amb}$ is the dimensionless ambient pressure, $a_{i,j}, b_{i,j}$ are unknown coefficients of Fourier series and $N_O, N_A \in \mathbb{N}$ are the numbers of trial functions in the circumferential and axial direction. Aim of weighted residual methods is to minimize a residuum [1]

$$r=\frac{\partial}{\partial\varphi}\left(\overline{h}^3\frac{\partial\hat{p}}{\partial\varphi}\right)+\left(\frac{R}{L}\right)^2\frac{\partial}{\partial\overline{Z}}\left(\overline{h}^3\frac{\partial\hat{p}}{\partial\overline{Z}}\right)-\left[\left(2\frac{\dot{\gamma}}{\omega}-1\right)\varepsilon\sin\varphi+2\frac{\dot{\varepsilon}}{\omega}\cos\varphi\right]. \tag{3}$$

In the case of the Galerkin method, weight functions are the same as the trial functions and final generalized scalar multiplication is defined as follows

$$\int_0^1\int_0^{2\pi} r\cdot\left[\sin\left(k\varphi\right)+\cos\left(k\varphi\right)\right]\sin\left(l\pi\overline{Z}\right)\,\mathrm{d}\varphi\,\mathrm{d}\overline{Z}=0. \tag{4}$$

After consecutive generalized scalar multiplication by all weight functions, final system of algebraic equations can be rewritten into matrix form [1]

$$\begin{bmatrix}\mathbf{A} & \mathbf{0}\\ \mathbf{0} & \mathbf{B}\end{bmatrix}\begin{bmatrix}\mathbf{a}\\ \mathbf{b}\end{bmatrix}=\begin{bmatrix}\mathbf{f_1}\\ \mathbf{f_2}\end{bmatrix},\qquad \mathbf{A}^{(\mathrm{GAL})}\mathbf{x}^{(\mathrm{GAL})}=\mathbf{f}^{(\mathrm{GAL})}, \tag{5}$$

where all unknown coefficients $a_{i,j}, b_{i,j}$ from Eq. (2) are associated into corresponding subvectors $\mathbf{a} \in \mathbb{R}^{N_A N_O,1}$ and $\mathbf{b} \in \mathbb{R}^{N_A(N_O+1),1}$. Coefficients $a_{0,j} = 0$ for $j = 1, 2, \ldots, N_A$. Submatrices $\mathbf{A} \in \mathbb{R}^{N_A N_O, N_A N_O}$ and $\mathbf{B} \in \mathbb{R}^{N_A(N_O+1), N_A(N_O+1)}$ are sparse, diagonal and regular. Each element of submatrices results from solution of each integral (4) and similarly for right-hand side subvectors $\mathbf{f}_1 \in \mathbb{R}^{N_A N_O,1}$ a $\mathbf{f}_2 \in \mathbb{R}^{N_A(N_O+1),1}$. Based on mutual orthogonality of goniometric functions, the elements of submatrices and subvectors are analytically derivable.

Hydrodynamic force is determined by the hydrodynamic pressure and the force components are obtained from pressure integration

$$\begin{bmatrix} F_{rad}^{hd} \\ F_{tan}^{hd} \end{bmatrix} \approx RL \frac{6\mu R^2 \omega}{c^2} \int_{-\frac{1}{2}}^{\frac{1}{2}} \int_{0}^{\pi} \hat{p}(\varphi, \overline{Z}, t) \begin{bmatrix} \cos\varphi \\ \sin\varphi \end{bmatrix} \mathrm{d}\varphi \, \mathrm{d}\overline{Z}, \qquad (6)$$

where μ is the dynamic viscosity, c is the radial clearance and pressure field $\hat{p}$ satisfies half-Sommerfeld boundary condition [2]. Components of the hydrodynamic force can be written in the closed form using previously calculated coefficients $a_{i,j}, b_{i,j}$ from Eq. (5). The hydrodynamic force can be expressed in Taylor series and linearized stiffness and damping coefficients of fluid film can be found. Stability of a rotor-bearing system is then determined based on Routh-Hourwitz stability criterion [2].

Presented Galerkin method (GAL) was applied on chosen bearing system with following parameters: $R = 50$ mm, $L = 100$ mm, $c = 0.8$ mm, $\mu = 0.04$ Pa.s, $p_{amb} = 0$ Pa and bearing load $F_0 = -147.15$ N. In-house software [3] with implemented solver based on finite difference method (FDM) was also used in order to perform comparison with obtained results. Comparison of journal trajectories and stability borderlines for both applied methods is depicted in Fig. 1. Intersection point of these two curves is a moment when system becomes unstable.

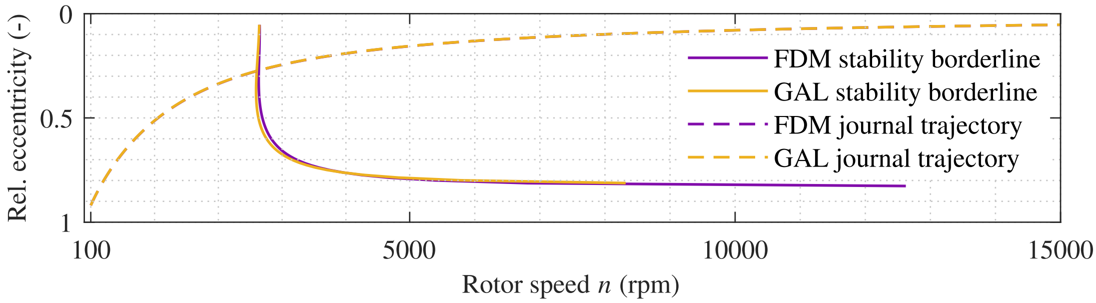

Fig. 1. Comparison of journal trajectories and stability borderlines

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