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Application of the entropy based criteria on the ECG analysis

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This contribution follows up the paper [3] dealing with the finding of the best deterministic approximation of the Brownian motion using the known thermostatic oscillator. The purpose was to use this approximation in the model of the molecular motor dynein. We used different criteria like entropies, complexity, Lyapunov exponents etc. The developed program seems to be suitable also for the analysis of the ECG signal to predict the heart arrhythmia before they occur. It is an actual problem investigated on experimentally obtained rabbits' ECGs.

Some criteria exist, like index of cardiac electrophysiological balance (iCEB), or others which are more complicate. They need more leads and manual treating. Therefore, we have tried to use some of the criteria based on entropy mentioned above. As a basis seems to be useful the permutation entropy [2].

From all criteria, we chose after analysis two of them – new empirical amplitude aware permutation entropy [1] and information exergy index combining singular value decomposition (SVD) and information exergy [5]. The first criterion corresponds with the properties of the ECG signal in the one given time interval and the second one takes into account the changes during a longer time.

In the following, we try to explain both criteria very shortly:

Entropy (information entropy). Entropy is the measure of the disorder of a closed system. We suppose the rabbits' ECG signal with arrhythmia in comparison of the healthy rabbits has some irregularities. The criteria based on entropy seems to be the best suitable. A lot of possibilities to define the entropy of the time series exist, like the ECG signal. As a very simple and effective, we have chosen the permutation entropy, introduced in [2].

Permutation entropy. Let us have a one-dimensional time series $S = \{x_t;\ t = 1, 2, \ldots, N\}$. In the arbitrary time s, we introduce a D-dimensional vector $(s) \to (x_{s-(D-1)\tau}, x_{s-(D-2)\tau}, \ldots, x_{s-1\tau}, x_s)$, where τ is the delay (usually equal 1). To this vector, we assign the **ordinal pattern** which is permutation $\pi = (r_0, r_1, \ldots, r_{D-1})$ from the order number of the elements of the vector elements. It is defined with the inequality

$$x_{s-r_{D-1}} \leq x_{s-r_{D-2}} \leq \ldots \leq x_{s-r_1} \leq x_{s-r_0}. \tag{1}$$

The number of these permutations – patterns – is $D!$. For $N \gg D!$ we can write for the probability of π

$$p(\pi) = \frac{\text{number of patterns } \pi}{N - D + 1}\,. \tag{2}$$

The probability distribution is then $P = \{p(\pi)\}$. Putting these probabilities into the known formula of **Shannon entropy**

$$S(P) = -\sum_{j-1}^{N} p_j \ln p_j \,, \tag{3}$$

we obtain the permutation entropy.

Amplitude-aware permutation entropy (AAPE). Original permutation entropy has two weaknesses: First, it does not take into account the different absolute values and differences of vectors elements creating the same patterns (for example, (2,1,4) creates the same pattern as (20,1,40)). The second problem is that the vector has the same elements with the same values. Their in order of emergence is ranked (e.g., (2,4,4) can create the same pattern as (2,3,4)). Azami [1] tried to solve these problems in the following approach: To the probability, he added the probabilities

$$\frac{A}{D}\sum_{k=1}^{D}|x_{s-(k-1)\tau}| + \frac{1-A}{D-1}\sum_{k=2}^{D}|x_{s-(k-1)\tau} - x_{s-(k-2)\tau}|, \tag{4}$$

where A is chosen from the range [0,1]. In case of two or more vector elements having the same value, into account is taken only the corresponding part – for two same elements the half, for three the third etc.

Information exergy. Information exergy is equal the change (or transfer) of the Kullback information between the supposed state (the ECG of the healthy rabbits) and the actual state. Generally, it is given with formula

$$K(p_0, p) = \int p \log \frac{p}{p_0} \mathrm{d}t \,. \tag{5}$$

The actual state can be the function of time τ this can evaluate the change of state during the time

$$K(p_0, p(\tau)) = \int p(\tau) \log \frac{p(\tau)}{p_0} \mathrm{d}t \,. \tag{6}$$

The problem is how to express this change using only one variable. Zhang [5] calls this variable again information exergy and mentions that it is a "time cumulating function of information entropy"

$$\mathbf{Y}(\tau_2) = \int_{\tau_1}^{\tau_2} S(\tau) \mathrm{d}\tau, \tag{7}$$

where $S(\tau)$ is information entropy obtained in the time moment τ from the neighbouring part of the time series s.c. window WS. The discretised formula is

$$\mathbf{Y}(\mathbf{m-1}) = \sum_{i=1}^{m-1} \frac{S(i) + S(i+1)}{2} \,, \tag{8}$$

where $m = 1, 2, \ldots, M$ is a moment in the solved time interval. All these $\mathbf{Y}(\mathbf{m-1})$ can be arranged into a column vector Y. If we choose $\mathbf{K}$ different time intervals (τ_1, τ_2) from the time series (ECG), it is possible to create the information exergy matrix $\mathbf{F}((\mathbf{M-1}) \times \mathbf{K})$ from the corresponding vectors $\mathbf{Y}$:

$$\mathbf{F} = \begin{pmatrix} \frac{S(1,1)+S(2,1)}{2} & \frac{S(1,2)+S(2,2)}{2} & \cdots & \frac{S(1,K)+S(2,K)}{2} \\ \sum_{i=1}^{2} \frac{S(i,1)+S(i+1,1)}{2} & \sum_{i=1}^{2} \frac{S(i,2)+S(i+1,2)}{2} & \cdots & \sum_{i=1}^{2} \frac{S(i,K)+S(i+1,K)}{2} \\ \vdots & \vdots & \ddots & \vdots \\ \sum_{i=1}^{M-1} \frac{S(i,1)+S(i+1,1)}{2} & \sum_{i=1}^{M-1} \frac{S(i,2)+S(i+1,2)}{2} & \cdots & \sum_{i=1}^{M-1} \frac{S(i,K)+S(i+1,K)}{2} \end{pmatrix}. \tag{9}$$

It is necessary to mention that the different time intervals can be used as the different leads of ECG. Articles [4] and [5] suggest how to obtain from the information exergy matrix one parameter called information exergy index. For this purpose, it is necessary to use singular value decomposition. The decomposition of the matrix $\mathbf{F}$ in the product of three matrices

$$\mathbf{F} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^T, \tag{10}$$

where $\mathbf{\Sigma}$ is the diagonal matrix. The diagonal entries are the singular values. They are for the square matrices equal the square root of the eigenvalues of the matrix $\mathbf{F}\mathbf{F}^T$. The greatest singular value σ can be used as an indicator of entropy changes of the time series during the time.

As a most effective has been shown to use both parameters AAPE and σ. Both these criteria should distinguish two groups of ECG – ECG of the healthy rabbits and of the rabbits with both types of arrhythmia (Torsades de Pointes (TdP) and non TpD).

The first one characterises the basic ordering of the ECG signal and the second one its change during time. A time series is chosen the series the peaks of the ECG signal, which is a special type of the Poincare section (Fig. 1). Our code is called Rabbits_ExergyMES1.m.

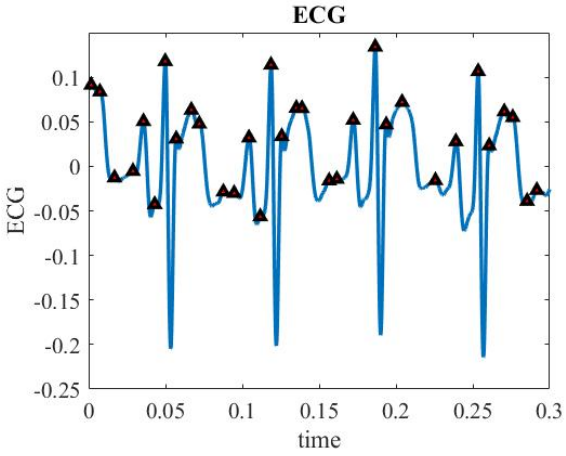

Fig. 1. Poincare section of the ECG signal

To verify the efficiency of this method, we have analysed 39 rabbits' one-lead ECG. They were divided into three groups healthy rabbits, rabbits with TdP arrhythmias and rabbits with non-TdP arrhythmias. The experiment is the result of the in vivo study realised in the Biomedicine Centre of the Faculty of Medicine in Pilsen. The result is shown in Fig. 2.

Conclusions. From the result, we can conclude:

1. AAPE allows distinguishing rabbits without arrhytmias and rabbits with (TdP) arrhythmias with good sensitivity and specificity.
2. The parameter σ cannot distinguish between healthy and non-healthy rabbits, but its value is in some boundary smaller for healthy rabbits.
3. The method does not allow to distinguish between TdP and non-TdP arrhythmias. Although, it can be seen the growing number of TdP arrhythmias with the growing σ. The tendency is opposite for the non-TdP. It can be a task for further research.

The introduced methods are very simple and offer the possibility to select the ECG of healthy rabbits. Further analysis and comparison with other published methods are needed. Also, it is necessary to apply to human ECG.

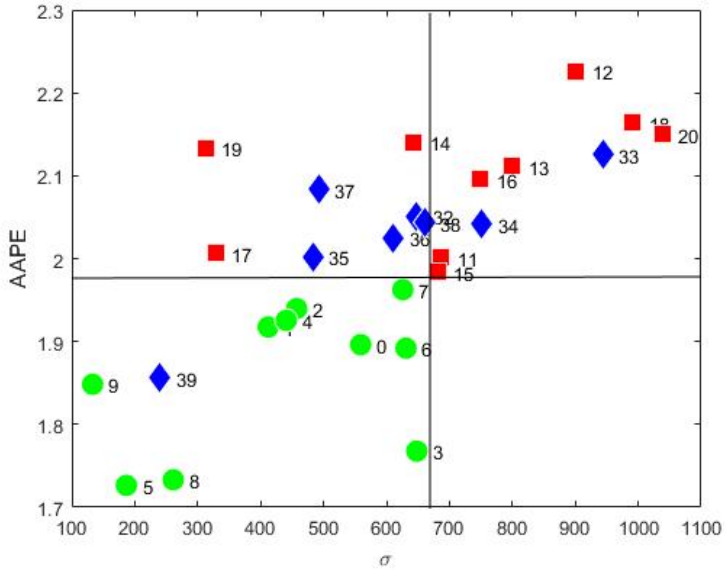

Fig. 2. Green-healthy rabbits, red-rabbits with TdP arrythmias and blue-rabbits with non-TdP arrythmias

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