

### External Referee's report on PhD thesis

Author: Tomáš Sobotka

Title: *FRACTIONAL STOCHASTIC VOLATILITY MODELS OF FINANCIAL MARKETS*

Supervisor: prof. B. Maslowski

Technical supervisor: J. Pospíšil

Submitted at: Department of Mathematics, University of West Bohemia

The thesis submitted in order to complete PhD studies of the author at University of West Bohemia is, according to the title, focused on *Fractional stochastic volatility models* of financial markets. The author studies contemporary topic which is attractive for both, researchers as well as practitioners of mathematical finance and financial engineering and falls well into given specialization (*Applied mathematics*).

The content of the thesis is in line with the topic – the author briefly reviews theoretical foundations of option pricing with specific focus on stochastic volatility case and provides several original extensions, either already published in high quality journals or still in the manuscript form. However, the aim (or goal) of the research is not specifically defined as an implication of the knowledge gap within the introduction, but given only indirectly. In fact, the “aim” is mentioned in the Conclusion as threefold, though a purpose is rather shown (the principal reason for writing any thesis should not be to provide comprehensive introduction to the topic).

Formally, the thesis consists of five standard chapters and two appendices. The first chapter is introductory and helps to specify the knowledge gap and motivation for the research of the author. The second and third chapters provide rather brief review of stochastic volatility models and option pricing models, including calibration. Author's original contribution is provided especially in Chapter 4. It consists mainly of analysing the variance swaps under the assumption of rough volatility. The author also shows significant computational improvements. These improvements might show to be crucial for practice as well as theoretical understanding of derivatives pricing.

Finally, the author provides chapter entitled Conclusion. This chapter, however, does not show any formal conclusion to the thesis (instead, new definitions, formulas, and statements are provided) and thus, the thesis/author's contribution can be deduced only informally. The same applies for Appendix B, which provides 5 papers already published in internationally well recognized journals, co-authored by the thesis author – in neither of the papers the thesis author is the first or corresponding author (in most cases, it is the technical supervisor).

Because of that I suggest to discuss the contribution of the thesis, and especially the author, to the state of the art of stochastic volatility modelling with respect to financial derivatives pricing in more details during the defence.

The thesis does not contain any severe formal or language errors and the form of presentation is at high level overall.

I acknowledge that the dissertation of T. Sobotka is of very high quality – when writing the thesis, the author proved the qualification and skills obtained during the studies and provided high portion of original independently achieved results that add to the knowledge in the field of applied mathematics and which have already been published in internationally well recognized journals. In my opinion, the thesis quality is above the average as well as the author's publications. Therefore, without any doubt, I recommend the thesis to be defended and, subsequently, giving the author the degree Doctor of Philosophy – PhD at University of West Bohemia.

Ostrava, February 10, 2020

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CHARLES UNIVERSITY  
Faculty of mathematics  
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**Tomáš Sobotka, Ph.D. Thesis**  
**Fractional stochastic volatility models of financial markets**

**Referee Report**


February 12, 2020

This Ph.D. thesis focuses on contingent claim pricing under the presence of stochastic volatility. Specifically, it studies plain vanilla option prices and variance swaps under various stochastic volatility models. The thesis includes about 40 page overview of the theory followed by references, data plots and 5 published papers listed in the appendix.

Following the requested template for the referee report, let me address the mandatory points.

- (a) **Evaluation of the importance of the Ph.D. thesis for the field.** The thesis topic of option pricing is one of the central themes of the field of mathematical finance. Stochastic volatility is also one of the major research areas. The research presented in the thesis brings new results in the area of using specific stochastic volatility models. The author has produced 5 publications in indexed journals. The results presented in the thesis are appropriate for the doctoral dissertation.
- (b) **Statement regarding the solution methods used and fulfillment of the research goals.** The methods used in the thesis are based on a general no-arbitrage pricing theory. This is based on the fact that properly discounted prices are martingales. This leads to a dual representation of the prices by either stochastic representation in terms of conditional expectations or by partial differential equations. The contribution of the author is to find the relevant pricing representations in the stochastic volatility setup, which typically leads to mathematically non-trivial problems that also requires sophisticated numerical implementations.

- (c) **Statement regarding the results of the Ph.D. thesis and to the original contribution of the author.** The presented work is original. Moreover, it resulted in several journal publications, meaning that it already went through an extensive peer review process. The only part that requires clarification is the exact contribution of the author of the thesis in the presented coauthored papers. In fact, all presented papers have at least one coauthor, and the exact contribution of the author of the thesis is just not clear. A statement about the contribution from the coauthors would be helpful.
- (d) **Statement regarding clarity, formal presentation and language quality of the thesis.** The thesis is clearly organized, the presented papers already meet the publication standards required.
- (e) **Statement regarding the publications.** The publications are in decent journals, five papers is probably more than one would require for the thesis. However, given the relatively large number of coauthors of the publications, the coauthors should issue a clarifying statement regarding what exact parts of the presented research can be solely attributed to the author of the thesis.
- (f) **Explicit recommendation of the reviewer to recommend or not to recommend for defense.** The reviewer recommends to accept the presented thesis for defense. The clarification of the exact contribution from the author and the coauthors should be produced in the meantime.



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