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STOCK MARKET RISK MEASURED BY VAR AND CVAR: A COMPARISON STUDY

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Abstract: VaR and CVaR are effective quantitative measurement of market risk. These measures can quantify the risk of unexpected changes within a given period. In this paper, we examine the market risk of four stock indices: the Czech PX, the Austrian ATX, the London FTSE, and the American S&P 500. First, the returns of these indices are approximated using two distributions showing semi-heavy tails: a t-distribution and a normal inverse Gaussian distribution. For comparison, the normal and empirical distributions are also included since they often work as convenient alternatives. Subsequently, the VaR99 and CVaR97.5 values corresponding to four candidate distributions are calculated for each index. We also analyze the ability of theoretical distribution to approximate the left tail behavior of stock market indices returns. It turns out that the normal distribution is not suitable for this purpose. Furthermore, it appears that CVaR97.5 is higher (in absolute value) for all indices than the corresponding VaR 99, which may require higher need for economic capital, which banks should allocate.

Keywords: Stock market indices, t-distribution, normal inverse Gaussian distribution, VaR, CVaR

JEL Classification: C58, G14

INTRODUCTION

Volatility is an integral part of stock market dynamics. It provides opportunities to make a lot of money as well as to face huge losses. Therefore, any market participant has to take adequate risk management measure to counter its negative exposure. In order to do so, risk has first to be quantified. The same holds for regulatory purposes. Value at Risk (VaR) and Conditional Value at Risk (CVaR) are ones of the main measurement of market risk. These two measures are very simple and popular quantificators of market risk and they are widely used in practice. Moreover, they are applicable in measuring other types of risk as well.

Computing Var and CVaR heavily depends on the specification of distribution used for modeling price or return dynamics. It has been known for quite a long time that the distribution of financial asset returns in general as well as stock price returns has heavier tails and sharper peak than the corresponding normal distribution. Not only can the correct choice for their distribution help to find the answer to our problem, but it is also of great importance for VaR and CVaR evaluation as well as for asset. So far many efforts of researchers as well as practitioners have been devoted to this task. There are two ways how to deal with it.

The first one, which is less inconvenient but may not yield the needed accuracy, is to replace the normal distribution by an alternative distribution with the same number of parameters as the normal one which exhibits the leptokurtic property. In general, the probability distribution with heavy ends (alpha stable distribution) or the distribution with so-called semi-heavy tails (generalized hyperbolic distribution and its special cases) are considered. For the generalized hyperbolic distribution family, see Prause 1999, and Eberlein and Keller 1995, for the skewed generalized t-distribution family, see Theodossiou, 1998, Zhu and Galbraith 2012, Platen, E., and R. Rendek. 2008. and Guo 2017. The second way how to solve this problem is to use a candidate distribution with more than two parameters. In this case, the additional parameter(s) will capture the tail and peak behavior of the distribution of financial asset returns. However,

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additional parameters also make estimation procedure more complicated. In the literature, two distributions with semi-heavy tails are often chosen for this task: normal inverse Gaussian and Student-t distributions. Hence, VaR and CVaR as measures of stock market risk can be calculated with these two distributions. The objective of this research is to find how market risk can be adequately quantified by these two measures and which distribution is a good one for approximation of returns of stock market indices.

To find the answer to our research question raised above, we choose two candidate distributions potentially suitable for capturing heavy tails. We select four stock market indices: Prague Stock Exchange index PX (Czech Republic), Austrian index ATX, FTSE index (Great Britain) and American index S&P 500. The first two indices represent small stock markets and last two are the most liquid stock markets in the world. The data used for this research are daily series of these indices from 2000 to 2018. These series are converted into return series and then they are used to estimate the parameters of each chosen distribution which will be estimated by maximum likelihood technique. Then the suitability of each distribution for each series will be tested by χ^2 goodness of fit test. They will be tested for the left loss tail. Based on this result, the appropriate candidate distribution for returns will be recommended. Then, we calculate corresponding VaR99 and CVaR 97.5 for these two distributions together with normal and empirical distributions. The results will provide us information on the two measures as well as the risk pattern of the chosen stock markets.

1. NORMAL INVERSE GAUSSIAN AND STUDENT-T DISTRIBUTIONS AS SPECIAL CASES OF GENERALIZED HYPERBOLIC DISTRIBUTIONS

This generalized hyperbolical distributions was introduced by Barndorff-Nielsen (1977) and at first applied them to model grain size distributions of wind blown sands. Eberlein and Keller (1995) were the first to apply these distributions to finance. The probability density function is as follows:

$$f(x) = \frac{\left(\alpha^2-\beta^2\right)^{\lambda/2}}{\sqrt{2\pi}\alpha^{\left(\lambda-\frac{1}{2}\right)}\delta^\lambda K_\lambda\left(\delta\sqrt{(\alpha^2-\beta^2)}\right)}(\delta^2+(x-\mu)^2)^{(\lambda-1/2)/2}K_{\lambda-1/2}\left(\alpha\sqrt{\delta^2+(x-\mu)^2}\right)\exp\left(\beta(x-\mu)\right), \quad (1)$$

where $K_\lambda(x)$ is the modified Bessel function of the third (second) kind with index $\lambda \in \mathbb{R}$. It can be defined as

$$K_\lambda(x) = \frac{1}{2}\int_0^\infty s^{\lambda-1}\exp\frac{x\left(s+s^{-1}\right)}{2}ds \quad (2)$$

The following distributions are the special cases of the generalized hyperbolic distribution (GH). For λ=1, we get the hyperbolic distribution, for λ=1⁄2, we get the Normal Inverse Gaussian distribution (NIG). So the probability density function of NIG distribution is (using some properties of Bessel functions):

$$f(x) = \frac{\alpha\delta\, K_1\left(\alpha\sqrt{\delta^2+(x-\mu)^2}\right)}{\pi\sqrt{\delta^2+(x-\mu)^2}}\exp\left(\delta+\beta(x-\mu)\right) \quad (3)$$

For $\lambda = -\frac{\nu}{2}, \nu > 0, \alpha = \beta = 0$, we get Student t-distribution where v is the number of degrees of freedom, hence t-distribution is a special case. The PDF od Student t-distribution is

$$f(x) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sigma\sqrt{\nu\pi}\Gamma\left(\frac{\nu}{2}\right)}\left(1+\frac{(x-\nu)^2}{\nu\sigma^2}\right)^{-\frac{\nu+1}{2}} \quad (4)$$

where $\Gamma(.)$ is the so called gamma function.

The semi-fat tail property of distributions of generalized hyperbolic distribution family coming from the following asymptotic property of Bessel function:

$$P(X \leq x) \approx |x|^{\lambda-1}\exp[(\alpha+\beta)x] \ \text{ as } x \to -\infty \quad (5)$$

We see that t-distribution has thicker left tail than NIG for certain values of λ.

2. VALUE AT RISK AN CONDITIONAL VALUE AT RISK

Value at risk (VaR) at the level $\alpha \in (0,1)$ is defined by

$$VaR_\alpha(Y) = inf\{x \in \mathbb{R} | F_Y(x) \geq \alpha\}, \quad (6)$$

where *Y* is loss random variable (losses are positive, gains negative) with cumulative distribution function $F_Y(x)$.

VaR has become a standard risk measure in finance. But it has a disadvantage of lacking subadditivity which means that a (diversified) portfolio may have a higher risk (VaR) than the sum of its individual parts.

Conditional VaR (CVaR), sometimes called Expected shortfall (ES)[1] , is defined by

$$ES_\alpha(Y) = \frac{1}{1-\alpha} \int_\alpha^1 VaR_s(Y) ds. \quad (7)$$

CVaR can be interpreted as a conditional mean value of losses provided that the VaR has been exceeded. CVaR has been proposed as an alternative to VaR risk because its subaddition property. However, it is often criticized for computational difficulty, limited backtesting capabilities and high sensitivity to extreme data (lack of robustness).

From a statistical point of view, CVaR should not be preferred to VaR, but CVaR has one advantage because it is far more difficult to manipulate. Banks can manipulate risk measures by selecting a specific estimation method. However, there is no certainty that the estimation method that gives positive results for the bank today will do the same tomorrow.

3. EMPIRICAL RESULTS

For our empirical analysis four stock market indices are chosen. They are the Czech stock market index PX, the Austrian stock market index ATX, the London stock market index FTSE, and the American stock market index S&P 500. Data are daily series of close index values from 2000 to 2018. Indices are obtained from Bloomberg database. The original series then are transformed into the return series of four stock market indices. We display their basic descriptive statistics in two tables 1a and 1b.

Tab. 1a: Descriptive statistics

Descriptive statistics of original series				
	PX	ATX	FTSE	SP500
mean	988,73	2438,56	5737,76	1446,34
median	986	2394,44	5861,92	1318,32
mode	1016,8	1127,75	4489,7	1092,54
minimum	320,1	1003,72	3287	676,53
maximum	1936,1	4981,87	7778,64	2872,87
std	354,30	970,22	949,87	443,91
skewness	0,30	0,57	-0,28	0,96
kurtosis	2,87	2,77	2,35	3,13
Num of ob	4773	4770	4735	4735

Source: Authors' own research

[1] These two terms (ES and CVaR) are not exactly the same, but they are identical for continuous distributions.

Tab. 1b: Descriptive statistics

Descriptive statistics of log-return series				
	PX	ATX	FTSE	SP500
mean	1,47E-04	2,20E-04	9,30E-06	1,31E-04
median	4,81E-04	1,76E-04	0,00E+00	2,13E-04
mode	0	0	0	0
minimum	-0,162	-0,103	-0,093	-0,095
maximum	0,124	0,120	0,094	0,110
std	0,013	0,014	0,012	0,012
skewness	-0,477	-0,340	-0,161	-0,222
kurtosis	16,496	10,602	9,573	12,052
Num of ob	4772	4769	4734	4734

Source: Authors' own research

From Table 1b we can see that returns of index PX display the widest range, the Austrian ATX index has the highest risk measured by the standard deviation. The London FTXE index has the lowest risk. In terms of average returns, they are similar except the case of index FTSE. The skewness is negative for all four indices, the highest skewness (in absolute value) is the Czech index PX, the lowest value is the FTXE index. The PX index also has the highest kurtosis, the lowest kurtosis index FTXE. All these statistics suggest that the probability distribution of log-returns is not normal.

3.1 Estimation parameters of distributions

The return series are first used to estimate parameters of hypothesized distributions described in the previous section by maximum likelihood estimation technique. The normal distribution is also included for comparison as well as it is a special case of all multi-parameter distribution classes. All computation is implemented in Matlab. The estimation results are displayed in Tables 2a, 2b. Besides the values of the parameters of each distribution, the asymptotic standard errors (SE) of the estimates are also computed and they are displayed under the estimated values of the parameters in the tables. The estimation results show that all estimated parameters of t- distribution of all four returns series are statistically different from 0. The estimation results for NIG distribution are similar to the ones of t-distribution except for the expected valued of return of index ATX.

Tab. 2a: Parameter Estimation results for t-distribution

	parameter	estimate	SE	z-stat	p-value
PX	mu	0,00048	0,00016	3,04581	0,00232
	sigma	0,00861	0,00013	68,28416	0
	nu	3,29627	0,02276	144,81449	0
ATX	mu	0,00076	0,00038	1,98808	0,04680
	sigma	0,00838	0,00050	16,70400	0
	nu	2,81838	0,03813	73,92200	0
FTSE	mu	0,00028	0,00013	2,18216	0,02910
	sigma	0,00723	0,00014	51,66309	0
	nu	2,86471	0,15189	18,86088	0
SP500	mu	0,00052	0,00014	3,60532	0,00031
	sigma	0,00650	0,00011	57,92318	0
	nu	2,35565	0,00519	453,69690	0

Source: Authors' own research

Tab. 2b: Parameter Estimation results for NIG-distribution

	parameter	estimate	SE	z-stat	p-value
PX	alpha	60,05061	2,67299	22,46570	0
	beta	-5,40087	0,67187	-8,03858	8,8818E-16
	delta	0,01014	0,00284	3,57479	0,00035
	mu	0,00106	0,00031	3,42196	0,00062
ATX	alpha	50,19242	11,45128	4,38313	1,1699E-05
	beta	-6,07214	1,14486	-5,30382	1,1341E-07
	delta	0,00933	0,00090	10,41036	0
	mu	0,00136	0,00140	0,96618	0,33396
FTSE	alpha	58,95939	3,54231	16,64432	0
	beta	-4,75282	1,86182	-2,55279	0,01069
	delta	0,00803	0,00028	28,62191	0
	mu	0,00066	0,00019	3,53512	0,00041
SP500	alpha	46,44095	2,72325	17,05351	0
	beta	-4,79900	1,88373	-2,54761	0,01085
	delta	0,00666	0,00019	34,72542	0
	mu	0,00082	0,00015	5,38086	7,4129E-08

Source: Authors' own research

3.2 Pearson´s Chi squared goodnes of fit test

In this section the Pearson chi squared goodness of fit test is used to verify the suitability of each distribution. The advantage of this choice is that it can be applied globally as well as on individual segments of a distribution. It also takes into account the number of parameters of a distribution. The essence of the test is as follows. This test tests the null hypothesis whether data comes from a certain distribution. The measure of goodness of fit which is also test statistic compares the observed frequencies with the expected ones by summing up their differences as follows

$$\chi^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$$

where

O_i is observed frequency for the *i*-th bin

E_i is expected frequency for this bin

and E_i is computed as folow

$$E_i = N\big(F(ub)_i - F(lb_i)\big)$$

Where N is total number of observations, F is the CDF of the hypothesized distribution, lbi and ubi are the lower bound and upper bound of the i-th bin respectively. The test statistic under the null hypothesis has χ^2 with n –k -1 degrees of freedom, where n is the number of bins and k is the number of parameters of the tested distribution (see Snedecor and Cochrane).

The testing using chi squared goodness of fit test is proceeded as follows. First we test for the whole distribution. The whole interval [0,1] is divided into forty subintervals of length 0.025. If the inverse CDF of a distribution exists, then the boundary points for each bin are computed with the inverse CDF, otherwise, it is determined numerically. Then the boundary points for each bin are computed by interpolation. After that the number of observed frequencies is counted and so is the test statistic.

We also use the chi squared goodness of fit test to investigate the validity of the null hypothesis in several segments, namely the left tail. We test for the validity in segment (0, 0.025). This interval is divided into ten bins of equidistant length 0.0025 and the testing is proceeded as described above. The results are summarized in Table 3.

Table 3: Pearson's Goodness of fit test results for left tail returns of indices

Distribution	Test results	PX	FTSE	ATX	SP500
Normal	Test stat	155.18	113.63	215,0514	965,12
	p-value	0	0	0	0
Student-t	Test stat	11.07	17.92	27,5328	23,5634
	p-value	0.135	1.24E-2	2,67E-04	0,0014
NIG	Test stat	15.93	8.220	4,2768	6,2807
	p-value	1.41e-2	0.222	0,6393	0,3925

Source: Authors' own research

The results of the goodness of fit test show that for the left tail of return distribution, normal distribution is an unsuitable candidate which is expected. For the other two distributions, t-distribution is more appropriate for returns of index PX while for the remaining three indices, NIG distribution is a better alternative. It fits better for their left tail. This implies that returns of index PX is heavier than returns of the other indices. This result is in accordance with what we observed with descriptive statistics. Among the remaining three indices, using equation (5) for heavy tail property and the estimation results in Table 2b, one can infer that the left tail distribution of returns of indices ATX and FTSE is similar while the left tail of distribution of returns of index SP 500 is a bit tender than those of the previous two. However, it needs more thorough analysis to quantify the exact differences.

3.3 VaR and CVaR computation

We use the estimation results in the previous part to compute VaR 99 and CVaR 97.5 for return series of all four indices. For the sake of completeness, in addition to the Student's t-distribution and NIG, the results for the normal and empirical distributions (HS) are also calculated. The computation results of four series are shown in Tables 4, 5, and 6. In Tables 4 and 5 the adequate values of VaR99 and CVaR975 are displayed in bold.

Tab. 4: VaR 99 computation results of four series

VAR99	Gauss	T	NIG	HS
PX	-0,0312	**-0,0360**	-0,0386	-0,0393
ATX	-0,0321	-0,0393	**-0,0422**	-0,0442
FTSE	-0,0272	-0,0338	**-0,0353**	-0,0334
SP500	-0,0276	-0,0365	**-0,0379**	-0,0348

Source: Authors' own research

Tab. 5: CVaR 97.5 computation results of four series

CVAR975	Gauss	T	NIG	HS
PX	-0,0314	**-0,0393**	-0,0400	-0,0416
ATX	-0,0322	-0,0443	**-0,0440**	-0,0444
FTSE	-0,0274	-0,0379	**-0,0367**	-0,0357
SP500	-0,0277	-0,0433	**-0,0398**	-0,0370

Source: Authors' own research

Tab 6: Differences of VaR 99 and CVaR 97.5

VAR99	Gauss	T	NIG	HS
PX	0,0002	0,0034	0,0014	0,0023
ATX	0,0002	0,0050	0,0018	0,0002
FTSE	0,0001	0,0041	0,0014	0,0023
SP500	0,0001	0,0067	0,0019	0,0022

Source: Authors' own research

The computation results of VaR99 and CVaR97.5 in the three previous tables show that using normal distribution to model returns can lead to underestimation the market risk. As we know from the previous subsection, t-distributions is the best option for returns of index PX, then the corresponding market risk level of index PX measured by VaR99 and CVaR975 is 0,0360 and 0,0393, respectively. As NIG distribution is best option for returns of indices for ATX, FTXE and SP 500, the corresponding market risk level measured by VaR99 and CVaR97.5 for Austrian stock market is 0,0422 and 0,0440, for London stock market is 0,0353 and 0,0367 and for American stock market with index SP500 is 0,0379 and 0,0398, respectively. Sorting by these two measures, the market risk exposure order is the following: London, Prague, America and Austria. This order implies that risk exposure may not originate from the market liquidity, but from the nature of their volatility. The other inference we obtained from our analysis is VaR99 and CVaR97.5 measures derived from historical simulation tend to be overestimated due to the missing values outsides of the observed range. For the same reason, the computed values of VaR and CVaR can be underestimated with historical simulation if the value of α is too small. Finally, while under the assumption of normal distribution, the value of VaR99 and CVaR97.5 are similar and their differences are negligible, for t-distribution and NIG distribution, the differences of the two measures are substantial and they are not interchangeable anymore. The results are consistent with what we observe.

CONCLUSION

We have measured the stock market risk of four markets: two very advanced and two less liquid with VaR and CVaR measures. Besides the traditional normal distribution and empirical distribution, we have used two heavy tail distributions: Student-t and NIG distributions. We have used daily data to estimate parameters of distributions we use and test their ability to capture left tail behavior of the distribution. Our results indicate that there is no distribution that can be universally used for modeling distribution of returns of all four indices. Further, the use of normal distribution or empirical distribution usually leads to incorrect evaluation of this two measures. We have also found that in general VaR99 and CVaR975 are not the same for distributions used in our analysis. The selection of two different type of markets in our analysis in terms of their respective liquidity provides us an interesting inference of the role of liquidity. Liquidity may not be the main driving factor of market risk. And using the appropriate distribution for determining VaR and CVaR leads to more correct evaluation of the risk level one may expose and consequently it helps to determine an adequate level of capital requirement for both risk management and regulatory purposes.

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