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Use of advanced kinematic hardening rules for prediction of multiaxial ratcheting

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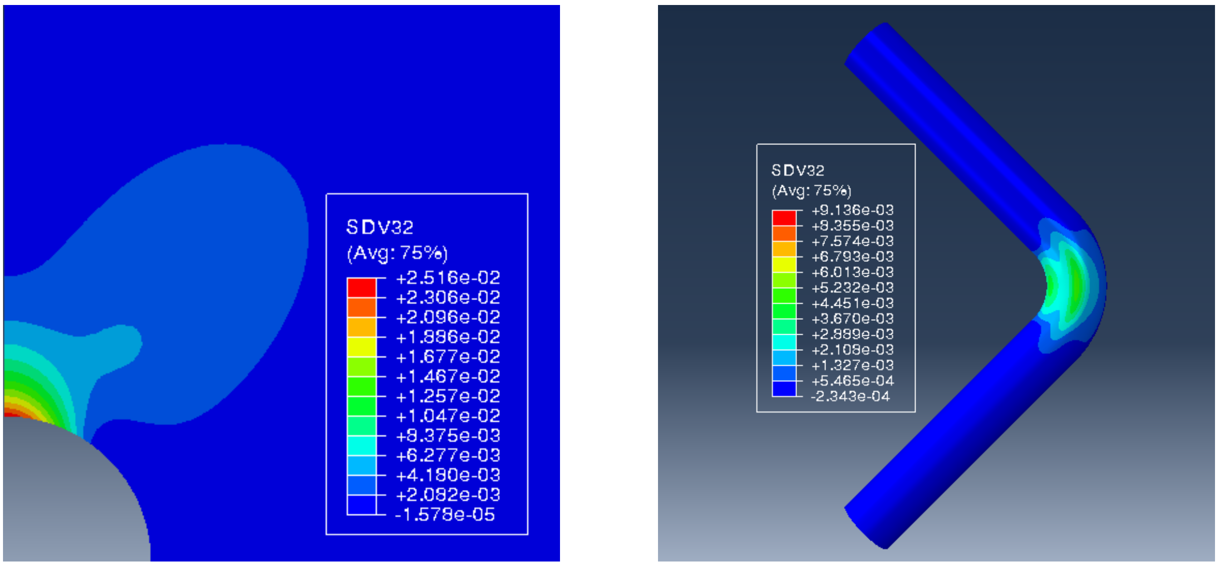
Cumulation of plastic strain due to cyclic loading, so called ratcheting, experimentaly observed as Baushinger effect, is possible cause of problems in range of engineering applications. This phenomenon occurs during unsymmetric cyclic loading in structures such as facilities pipes, offshore structures, etc., whenever occasional overloads occur. This can cause problems of structures service, or even their failure. To capture physical process of multiaxial ratcheting (MR) evolution, wide range of advanced plasticity models was developed. In this work we are focused on phenomenon of non-linear kinematic hardening (KH). Models for capturing of KH are based on concept of multi-component backstress [1]. There are many of these models published. In this work we present a model of *Multicomponent Armstrong–Frederick with Threshold with r modification* (MAFTr) published in [2]. For reason of MR prediction on complex structures, the model is implemented into FE code Abaqus through UMAT subroutines coded in FORTRAN programming language.



(a) (b)

Fig. 1. Numerical examples used to test performance of UMAT subroutine with implemented *MAFTr* model into Abaqus FE code

Table 1. Material parameters of MAFTr model [2]

a_{11} [−]	a_{12} [−]	a_{13} [−]	a_{14} [−]	a_{21} [MPa^{-1}]	a_{22} [MPa^{-1}]	a_{23} [MPa^{-1}]	a_{24} [MPa^{-1}]
225 353.7	12 123.0	3 004.0	56 338.0	0.05917	0.02199	0.004437	0.0591

Within the *MAFTr* model, an additive decomposition of small, rate independent strains is assumed as in $\boldsymbol{\varepsilon}_{\mathrm{tot}} = \boldsymbol{\varepsilon}_{\mathrm{pl}} + \boldsymbol{\varepsilon}_{\mathrm{el}}$. Elastic behavior of material follows Hooke's law in the form $\boldsymbol{\sigma} = \lambda \boldsymbol{I} \mathrm{tr} \boldsymbol{\varepsilon}_{\mathrm{el}} + 2\mu \boldsymbol{\varepsilon}_{\mathrm{el}}$, where $\boldsymbol{\sigma}$ is a stress tensor, λ and μ are Lamé elastic parameters and $\boldsymbol{I}$ is a unit second-order tensor. Hereafter, however, to keep the notion simple, we use Hooke's law in general form $\boldsymbol{\sigma} = \boldsymbol{C}\boldsymbol{\varepsilon}_{\mathrm{el}}$. Hence, further in this work, the term λ denotes exclusively the loading index. In this work we adpot classical von Mises yield criterion is used as

$$f = \sqrt{\frac{3}{2}\left(\boldsymbol{s} - \boldsymbol{\alpha}\right) : \left(\boldsymbol{s} - \boldsymbol{\alpha}\right)} - k^2. \tag{1}$$

Further we omit change in the isotropic hardening by keeping $\dot{k} = 0$. We adpot the associative flow rule, which may be written as

$$\dot{\boldsymbol{\varepsilon}}_{pl} = \lambda \frac{\partial f}{\partial \boldsymbol{\sigma}}. \tag{2}$$

The *MAFTr* KH rule reads

$$\bar{\boldsymbol{\alpha}}_i = a_{1i} \left\| \frac{\partial f}{\partial \boldsymbol{\sigma}} \right\| \left(\boldsymbol{n} - a_{2i} \left[r_i \boldsymbol{\alpha}_i + (1 - r_i) \left(\boldsymbol{\alpha}_i \boldsymbol{n} \right) \boldsymbol{n} \right] \right), i = 1, 2, 3, \tag{3a}$$

$$\bar{\boldsymbol{\alpha}}_4 = a_{14} \left\| \frac{\partial f}{\partial \boldsymbol{\sigma}} \right\| \left(\boldsymbol{n} - a_{24} \left[r_4 \boldsymbol{\alpha}_4 + (1 - r_4) \left(\boldsymbol{\alpha}_4 : \boldsymbol{n} \right) \boldsymbol{n} \right] \left\langle 1 - \frac{\bar{a}}{g\left(\boldsymbol{\alpha}_4\right)} \right\rangle \right), \tag{3b}$$

$$g\left(\boldsymbol{\alpha}_4\right) = \left[\frac{3}{2} \boldsymbol{\alpha}_4 : \boldsymbol{\alpha}_4 \right]^{\frac{1}{2}}, \tag{3c}$$

$$r_i = \sqrt{\frac{3}{2} \boldsymbol{\alpha}_i : \boldsymbol{\alpha}_i} \sqrt{\frac{2}{3}} a_{2i}\,, \tag{3d}$$

where a_{1i}, a_{2i} and $\bar{a}$ are material parameters, and $\boldsymbol{n}$ denotes the unit norm of the yield surface. Parameter r_i is a weighting factor and evolves along the loading history.

The implementation via UMAT follows the scheme published in [5]. This algorithm is built on predictor-corrector method with radial return onto the yield surface. Implementation of *MAFTr* was verified by comparison with closed form solution

$$\alpha_i = \frac{1}{a_{2i}} - \left(\frac{1}{a_{2i}} - \alpha_i^0 \right) \exp \left[-a_{1i} a_{2i} \sqrt{\frac{3}{2}} \left(\varepsilon_{cum} - \varepsilon_{cum}^0 \right) \right], \tag{4a}$$

$$\alpha_4 = \left(\frac{1}{a_{24}} \pm \sqrt{\frac{2}{3}} \bar{a} \right) - \left(\frac{1}{a_{24}} \pm \sqrt{\frac{2}{3}} \bar{a} - \alpha_4^0 \right) \exp \left[-a_{14} a_{24} \sqrt{\frac{3}{2}} \left(\varepsilon_{cum} - \varepsilon_{cum}^0 \right) \right], \tag{4b}$$

derived for the case of uniaxial loading. To test performance of subroutines, two numerical examples were done. First one was planar plate with hole loaded by a pressure in single direction. Second one was simulation of elbow pipe preloaded by a pressure and cyclicly loaded by displacement according to experiment in [4]. These examples are depicted in Fig. 1.

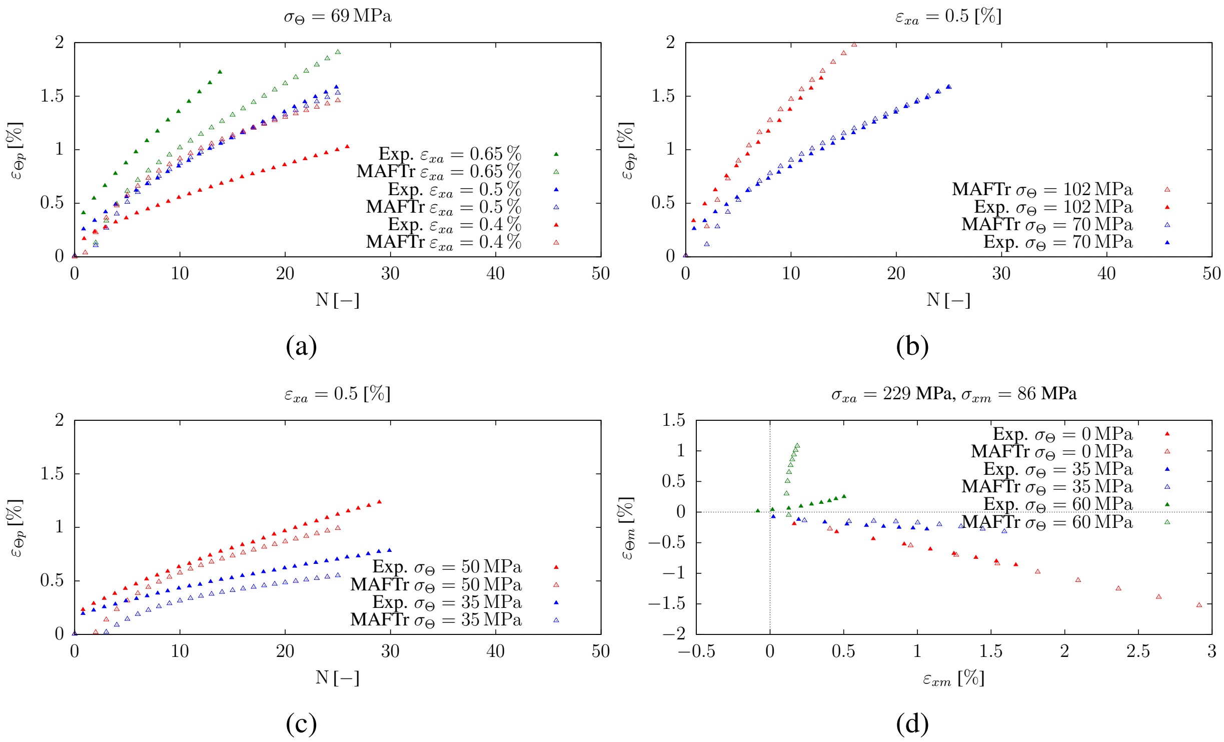


Fig. 2. Biaxial ratcheting predicted by MAFTr model and prediction of ratcheting as a function of number of cycles. Set of experimental data was meassured on CS 1026 published in [3]. Ratcheting simulations with previously estimated stress increment and their comparison with the experimental data are depicted in subfigures (a) – (d)

Prameters of *MAFTr* model were calibrated on experimental MR data published by Hassan et al. [3], reported in [2] and given in Table 1. With calibrated model, range of simulations of MR was performed. Results of some of them are depicted in Fig. 2. In Fig. 2(a), tests differ in amplitude of prescribed axial strain ε_{xa}. The evolution of ratcheting by FEA with implemented *MAFTr* model is underpredicted for $\varepsilon_{xa} = 0.65\,\%$, and overpredicted if $\varepsilon_{xa} = 0.4\,\%$. Accurate results were obtained for case of $\varepsilon_{xa} = 0.5\,\%$. To check importance of prescribed axial strain, tests with different load and preload stress were performed. Results of these set-ups are shown in Figs. 2(b) and 2(c). So far, less promising situation is depicted in Fig. 2(d), which shows evolution of ratio between circumferential mean strain $\varepsilon_{\Theta m}$ and axial mean strain ε_{xm}, when the specimen is preloaded by circumferential stress σ_{xm} and axial stress σ_Θ, and cyclicly loaded by axial stress σ_{xa}.

This work presented the use of advanced KH rule to predict multiaxial ratcheting by FEM. The departure between prediction and experimental data in Fig. 2(a) and its dependency on prescribed axial strain ε_{xa} leads to the need of study of calibration procedure. Hence, in future work, the identification of model parameters will be studied more deeply. Subsequentaly, evolution of ratcheting in complex structures, such as aforementioned elbow pipe [4], and its prediction by FEM in combination with advanced plasticity laws will be modeled.

References

[1] Chaboche, J. L., Dang-Van, K., Cordier, G., Modelization of the strain memory effect on the cyclic hardening of 316 stainless steel, Proceedings of the 5th International Conference on Structural Mechanics in Reactor Technology, Berlin, Germany, 1979, pp. 1-10.

[2] Dafalias, Y. F., Feigenbaum, H. P., Biaxial ratchetting with novel variations of kinematic hardening, International Journal of Plasticity 27 (4) (2011) 479-491.

[3] Hassan, T. Corona, E., Kyriakides, S., Ratcheting in cyclic plasticity, part II: Multiaxial behavior, International Journal of Plasticity 8 (2) (1992) 117-146.

[4] Hassan, T., Rahman, M., Bari, S.,Low-cycle fatigue and ratcheting responses of elbow piping components, Journal of Pressure Vessel Technology 137 (3) (2015) No. 031010.

[5] Marek, R., Plešek, J., Hrubý, Z., Parma, S.,Feigenbaum, H. P., Dafalias, Y. F., Numerical implementation of a model with directional distortional hardening, Journal of Engineering Mechanics 141 (12) (2015) No. 04015048.