

## Trajectory planning for tensegrity structures

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Tensegrities are structures composed of tensile and compressive elements or other objects. Tensegrities are usually composed of cables and struts, which can also transmit the force in both directions [4]. These elements are connected to each other and may be joined by more complex objects. The position is also related to the distribution of bodies in trajectory planning [1].

When considering a planar structure, each body has three degrees of freedom. The position of the end-effector, and possibly its orientation is required. For a three-body structure (three stages, Fig. 1) including the end-effector body, we get in total 9 degrees of freedom. Determining the three degrees of freedom for end-effector leaves 6 coordinates of the bodies that we can choose. There are thus an infinite number of configurations to choose from. Determining the positions of the remaining bodies is often approached by choosing a trajectory to place the bodies between the end-effector and the base frame uniformly. The limitation may then be the available space or the collision of the bodies and cables. However, this approach does not necessarily lead to a good solution, and so the design is often supplemented by optimizing some of the key features. The optimization criterion is often the stiffness at the end-effector or dexterity [5]. Closely related to these properties is the distribution of the cables and their prestressing. Since a redundant number of cables is used, it is also possible to work with the choice of cable prestressing. By prestressing cables it is possible to influence the resulting stiffness of the structure [2], [3].

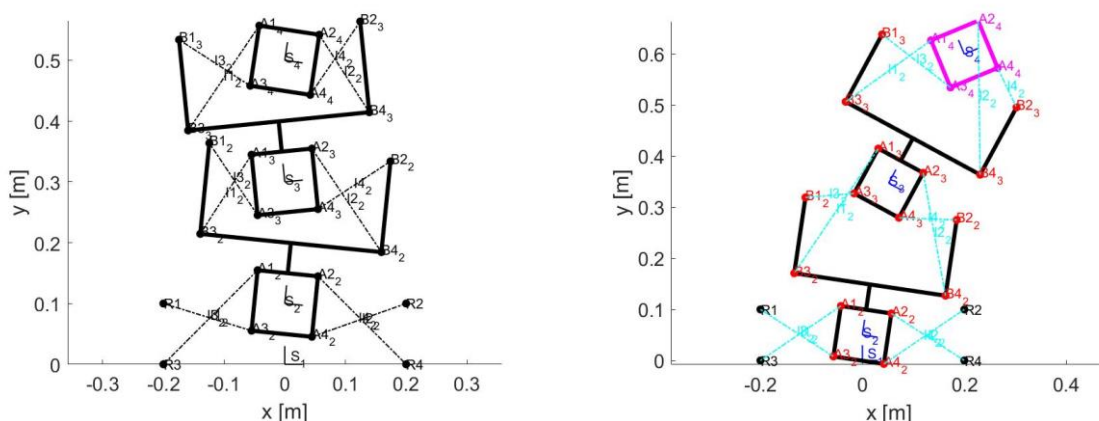


Fig. 1. The initial position of the tensegrity structure (left) and the common position (right)

To find the forces in the cables in a defined position, the dynamic equations of the bodies are used, from which the required forces for the dynamic equilibrium of the system are solved. Due to the redundant number of cables, this problem is also ambiguous. One approach to solving the problem is to choose the forces in the form of prestresses in the redundant cables and calculate the forces of the other cables from the dynamic equilibrium of the system. The

best option is then determined for different combinations of prestresses in the cables, for example by finding the configuration with the minimum of maximum of all cable forces. However, the equilibrium may also yield negative forces in the cables where a such configurations are not allowed. Another possibility for designing forces in cables is to use constrained optimization. However, this optimization is not suitable for a higher number of parameters due to its time consuming nature. Another option is to use SVD decomposition for the dynamic equilibrium equations of the whole tensegrity system. The number of equations is redundant and we have the possibility of using a redundant number of parameters to influence the result.

The modelled tensegrity structure (Fig. 1) consists of three stages, where one is an end-effector whose position is prescribed. The system contains a total of 12 cables at 9 degrees of freedom. Thus, there are a total of 3 redundant cables. A heuristic method is used to distribute the bodies as they move along the trajectory, with optimization of the forces in the cables along with analysis of the system dynamics equations using singular value decomposition method

$$\begin{aligned} m_i \ddot{x}_{1i} &= \sum_{j=1}^8 F_{ji} \vec{e}_{jl} \cdot \vec{e}_x , \\ m_i \ddot{y}_{1i} &= \sum_{j=1}^8 F_{ji} \vec{e}_{jl} \cdot \vec{e}_y - m_i g , \\ I_{S_i} \ddot{\varphi}_i &= \sum_{j=1}^8 \vec{r}_{jl} \times F_{ji} \vec{e}_{jl} , \end{aligned}$$

where the  $m_i$  is the mass of the  $i$ -th body,  $I_{S_i}$  is the moment of inertia,  $\ddot{x}_{1i}, \ddot{y}_{1i}$  accelerations of the center of gravity and  $\ddot{\varphi}_i$  the angular accelerations,  $F_{ji}$  the corresponding force in the cable,  $\vec{e}_{jl}, \vec{e}_x$  and  $\vec{e}_y$  unit vectors of cables and world frame axes. The dynamics equations for the  $i$ -th body are based on the free body diagram (Fig. 2). The resulting forces in the cables determined by the different methods are strongly dependent on the given structure configuration and the chosen prestress in the cables. The results of the different approaches verified the possibility of using the methods to design the move along the trajectory for redundantly actuated tensegrity structures.

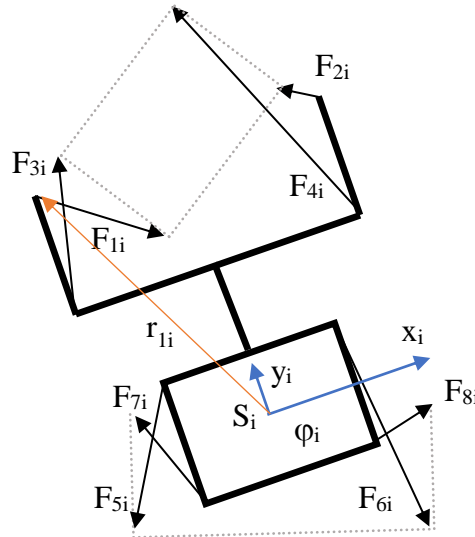


Fig. 2. The action of forces on the  $i$ -th stage

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