NONLINEAR ANALYSIS AND PREDICTION OF BITCOIN RETURN'S VOLATILITY

Tao Yin[1], *Yiming Wang*[2]

[1] *East China University of Political Science and Law, Business School, China, ORCID: 0000-0001-6971-7374, 1701110936@pku.edu.cn;*
[2] *Peking University, School of Economics, China, wangyiming@pku.edu.cn.*

Abstract: *This paper mainly studies the market nonlinearity and the prediction model based on the intrinsic generation mechanism (chaos) of Bitcoin's daily return's volatility from June 27, 2013 to November 7, 2019 with an econophysics perspective, so as to avoid the forecasting model misspecification. Firstly, this paper studies the multifractal and chaotic nonlinear characteristics of Bitcoin volatility by using multifractal detrended fluctuation analysis (MFDFA) and largest Lyapunov exponent (LLE) methods. Then, from the perspective of nonlinearity, the measured values of multifractal and chaos show that the volatility of Bitcoin has short-term predictability. The study of chaos and multifractal dynamics in nonlinear systems is very important in terms of their predictability. The chaos signals may have short-term predictability, while multifractals and self-similarity can increase the likelihood of accurately predicting future sequences of these signals. Finally, we constructed a number of chaotic artificial neural network models to forecast the Bitcoin return's volatility avoiding the model misspecification. The results show that chaotic artificial neural network models have good prediction effect by comparing these models with the existing Artificial Neural Network (ANN) models. This is because the chaotic artificial neural network models can extract hidden patterns and accurately model time series from potential signals, while the benchmark ANN models are based on Gaussian kernel local approximation of non-stationary signals, so they cannot approach the global model with chaotic characteristics. At the same time, the multifractal parameters are further mined to obtain more market information to guide financial practice. These above findings matter for investors (especially for investors in quantitative trading) as well as effective supervision of financial institutions by government.*

Keywords: *Nonlinear, multifractal, chaos, Bitcoin, prediction.*

JEL Classification: *A10, E44, F37.*

APA Style Citation: Yin, T., & Wang, Y.-M. (2022). Nonlinear Analysis and Prediction of Bitcoin Return's Volatility. *E&M Economics and Management, 25*(2), 102–117. https://doi.org/10.15240/tul/001/2022-2-007

Introduction

Since it was proposed by Satoshi Nakamoto (2008) at the end of 2008, Bitcoin, as an alternative to conventional currencies, has quickly gained wide attention from the media, investors and scholars.

This attention is attributed to its transparency, simplicity, increasing popularity, decentralized peer-to-peer system and self-regulation. There is a growing interest in studying the general dynamics of Bitcoin market. For instance, diversification was measured (Brière et al., 2015; Bouri et al., 2017; Urquhart & Zhang, 2019; Chaim & Laurini, 2018; Lahmiri et al., 2018), statistical properties and market efficiency were examined (Bariviera et al., 2017; Carbone et al., 2004; Martinez et al., 2018; McCarthy, 2009; Symitsi & Chalvatzis, 2018), liquidity and microstructure were explored (Koutmos, 2018; Dyhrberg et al., 2018; Donier & Bonart, 2015), speculative bubble and risk were investigated (Osterrieder & Lorenz, 2017; Bouoiyour et al., 2015; Klein et al., 2018), regulation was studied (Dwyer, 2015; Tasca & Liu, 2018; Katsiampa,

2017) whilst optimal trading was scrutinized (Ajaz & Kumar, 2018; Li & Tourin, 2016; Yi et al., 2018).

The nonlinearity of the Bitcoin market is a very important topic in the existing literature. As far as we know, these scholars (Urquhart, 2016; Nadarajah & Chu, 2017) mainly studied the market nonlinearity of the Bitcoin price and return. But, the market nonlinear of Bitcoin volatility has rarely been studied. Volatility, which is also known as return's volatility, plays an important role in risk modeling and evaluation as well as in the pricing of complex financial products. In fact, Bitcoin's return is highly volatile. Its return depends largely on the shortage of the Bitcoin and people's trust in them (Urquhart, 2016), which affects its value, causing return to fluctuate wildly. Therefore, this paper attempts to discuss the nonlinearity of the Bitcoin volatility market, in an attempt to fill the gap in this hot spot.

At present, multifractal theory and chaos theory are mainly used to study the nonlinear characteristics of financial market. Multifractals and chaos theory reveal the nonlinearity of financial market from different perspectives. To be specific, multifractal theory (especially multifractal detrended fluctuation analysis method, MFDFA) reveals the spatial organization process of financial market and the long-term correlation and self-similarity of financial time series. The MFDFA method is widely used in the field of economy and finance (Rizvi et al., 2014; Cao et al., 2013; Bouoiyour et al., 2018; Uddin et al., 2018), which can not only find the multifractality and nonlinearity of the market, but also excavate more market information to guide financial practice. Meanwhile, chaos theory (Lahmiri, 2017; Adrangi & Chatrath, 2001; Ozun et al., 2010) provides the time evolution process of the financial market, reveals that the internal structure of the time series of the financial market is intrinsically deterministic and nonlinear, and shows that the financial time series is intrinsically generative and can be further predicted in a short time.

The study of multifractals and chaos in nonlinear systems is also of great significance in their predictability. On the one hand, a chaotic system (signal) may have limited short-term predictability, while on the other hand, multifractals and self-similarity can increase the likelihood of accurate prediction of future time series (signal).

Another issue in the paper is to forecast the Bitcoin volatility. Predicting volatility of financial time series can help investors avoid risks, which is a hot and challenging topic in the financial field. There are many prediction models for the Bitcoin market in the existing literature. In particular, Artificial Neural Network (ANN) models can deal with both linear and nonlinear data, so many researchers apply ANN models to predict the Bitcoin market (Hung et al., 2020; Tiwari et al., 2019; Seo & Hwang, 2018). To our knowledge, few researchers have used the internal generation mechanism of time series and ANN technologies to build predictive models of Bitcoin volatility.

This paper will mainly focus on the following two aspects: (1) We attempt to assess the predictability of Bitcoin volatility by examining its inherent nonlinear characteristics, including inherent chaos and multifractals. The chaotic and multifractal characteristics of Bitcoin volatility are detected by using the largest Lyapunov exponent (LLE) and MFDFA based on the extracted generalized Hurst exponent of time series. Specifically, the former allows to test the existence of nonlinear deterministic mapping, while the latter reveals the existence of long-term correlation in the case of non-stationarity. (2) Our goal is to use a special artificial neural network to topology the hidden dynamical system and automatically extract the underlying dynamical model to reveal the nonlinear characteristics of its time series. In other words, a chaotic intelligent signal data mining and prediction system (i.e., chaotic artificial neural network) is constructed through the neural network topology. We expect the prediction accuracy of chaotic artificial neural network model to be higher than that of the existing neural network benchmark model. In a word, the results from the nonlinear perspective are expected to show that the predictability of Bitcoin volatility in the short term depends on the measured values of multifractal and chaos, and the results of introducing chaotic artificial neural network are expected to prove the consistency and accuracy of its prediction ability.

This paper improves and complements previous literature on Bitcoin in four aspects: (1) Bitcoin and other cryptocurrencies have received much attention in the economic and financial literature. Several related problems are debated. The nonlinearity of cryptocurrencies are very important issues addressed in the existing literature. The present paper attempts to discuss the nonlinearity of the Bitcoin volatility market, in an attempt to fill the gap in this hot spot. (2) This article will demonstrate a new perspective on prediction: econophysics. While using MFDFA method to study the multifractals of Bitcoin volatility, more parameter information is also mined to guide market practice. (3) We test the robustness of the largest Lyapunov exponent by bootstrap method. (4) This paper is the first time to apply chaos theory to the Bitcoin volatility market, excavates the internal generation mechanism of the market, builds prediction models based on its internal generation mechanism, and proves that its prediction effect is better than that of artificial neural network (ANN) model. In the future, our work will further compare other prediction models (such as GARCH model) to show the superiority of the model.

The paper is organized as follows. In the Section 1, the multifractal detrended fluctuation analysis (MFDFA) and the largest Lyapunov exponent methodologies are proposed; data and model settings are introduced in the Section 2; Section 3 presented and analyzed the empirical results; Section 4 further discusses the prediction and comparison; the conclusion and economic implications are outlined in the Section 5.

1. Methodology

1.1 Determining Chaos by Largest Lyapunov Exponent (LLE)

Chaos is determined by largest Lyapunov exponent (LLE) (Rosenstein et al., 1993; Wolf et al., 1985):

1. The time series with length N is $\{x_i : i = 1, 2, \ldots, N\}$. A new m-dimensional phase space sequence $X_i = \{x_i, x_{i+\tau}, \ldots, x_{i+(m-1)\tau}\}$ can be obtained through the phase space reconstruction method. The total number of observed sample data N, the number of phase points is $M = N - (m - 1)\tau$ the delay time is τ and the embedding dimension is m;
2. Construct initial vector:

$$L(t_0) = \min_j \parallel X_1 - X_j \parallel \tag{1}$$

where: t_0 – the initial time; X_1 – the initial phase point; and X_j – the rest of the phase point set;

3. The linear exponential growth rate can be obtained:

$$\lambda_1 = \frac{1}{k} ln \frac{L(t_1)}{L(t_0)} \tag{2}$$

where: λ_1 – linear exponential growth rate; $L(t_1)$ – the time t_1 vector distance; k – the time step;

4. Successively increase the embedding dimension m, repeat (2) and (3) until the largest Lyapunov exponent becomes stable with the change, and the calculated result is the estimated value of the LLE. It should be noted that if LLE > 0, it indicates a time series with chaotic dynamics. On the contrary, if LLE < 0, it indicates that the time series does not have chaotic dynamic characteristics.

1.2 MFDFA Formalism

The multifractal detrended fluctuation analysis (MFDFA), proposed by Kantelhardt et al. (2002), is a useful tool for detecting multifractal behaviors. We can conduct the MFDFA analysis with the following steps:

1. Suppose $\{u_j\}_{j=1}^{N}$ is financial time series of length N; where: u_j – the j^{th} value in the time series; j – the ordering in the time series.
2. Calculate and divide the profile $p(k)$. $p(k) = \sum_{j=1}^{k}[u_j - \bar{u}]$; where $\bar{u} = \frac{1}{N}\sum_{j=1}^{N} u_j$.
3. Determine the variance:

$$w^2(s,v) = \begin{cases} \frac{1}{s}\sum_{j=1}^{s}\{p[(v-1)s+j] - \widehat{p_v(i)}\}^2, & for\ v = 1, \ldots, N_s \\ \frac{1}{s}\sum_{j=1}^{s}\{p[N-(v-N_s)s+j] - \widehat{p_v(i)}\}^2, & for\ v = N_s+1, \ldots, 2N_s \end{cases} \tag{3}$$

where $\widehat{p_v(i)}$ is the fitting polynomial in segment v.

4. Calculate the q^{th} order fluctuation function $w_q(s) = \left\{\frac{1}{2N_s}\sum_{v=1}^{2N_s}[w^2(s,v)]^{q/2}\right\}^{1/q}$; where s – the number in each segment; and the log-log plots $w_q(s)$ versus s for different q can be described by $w_q(s) \sim s^{H(q)}$. When the series is multifractal, a significant dependence of $H(q)$ on q should be observed. $H(2)$ is the classic Hurst exponent. If $H(2) > 0.5$, it indicates that the trend change is persistent (long-range memory); if it is anti-persistent,

$H(2) < 0.5$; and $H(2) = 0.5$ for the random walk process.

In addition, the singularity strength α and the singularity spectrum $f(\alpha)$ can be calculated via Legendre transform $f(\alpha) = q\alpha - \tau(q) = 1 + q[\alpha - H(q)]$. $f(\alpha)$ describes the fractal dimension of the ensemble formed by all the points that share the same singularity exponent α. Fractal dimension $f(\alpha) \sim \alpha$ is shaped like a single-peaked bell. The difference between α_{max} and α_{min}, $\Delta\alpha = \alpha_{max} - \alpha_{min}$, is called the multifractal spectrum width, that represents the interval between the maximum probability and the minimum probability and measures the degree of the multifractality property. $\Delta f = f(\alpha_{min}) - f(\alpha_{max})$ is greater than 0 means that the chances of the sample being at the top are greater than the chances of being at the bottom, and vice versa. It is worth mentioning that investors can look for investment opportunities according to the size of Δf. This paper chooses to apply MFDFA method, which can not only determine the market nonlinearity and multifractality, but also obtain other by-products, such as the discovery of investment chances.

2. Data Description and Model Settings

2.1 Data

In this paper, we use daily price of Bitcoin from May 8, 2013 to November 7, 2019. The data source is https://coinmarketcap.com/. For convenience, we denote the time sequence for each data set as t and the corresponding price sequence as $p(t)$, where $t = 1, 2, ..., 2385$.

2.2 Definition of Volatility

We estimate the annualized volatility σ using 60 datapoints sliding window. This rolling sample approach works as follows: we compute the annualized volatility for the first 60 returns, then we discard the first return and add the following return of the time series, and continue this way until the end of data. Thus, each σ estimate is calculated from data samples of the same size. We obtained an average of 2385 annualized volatility σ from May 8, 2013 to November 7, 2019.

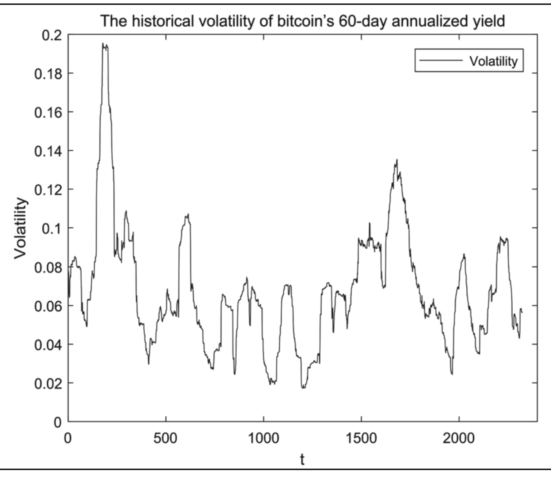
For each time series, we define the daily return $\mathrm{r(t)}$ as follows: $\mathrm{r(t)} = \mathrm{lnp(t)} - \mathrm{lnp(t-1)}$.

We then calculate the 60-day standard deviation as follows:

$$\mathrm{s(i)} = \sqrt{\frac{1}{60-1}\sum_{t=1+i}^{60+i}(\mathrm{r(t)} - \overline{\mathrm{r}})^2},\ i = 1, 2, \ldots,\ 2325;$$,

where $\overline{\mathrm{r}} = \frac{\sum_{t=1+i}^{60+i}\mathrm{r(t)}}{60}$. At last, the 60-day actual

Fig. 1: The historical volatility of Bitcoin's 60-day annualized yield



Source: own

Note: The horizontal axis represents time and the vertical axis is the volatility of Bitcoin.

annualized volatility can be obtained as follows: $\sigma(i) = \frac{s(i)}{\sqrt{2\times 60}} \times \sqrt{365}$.

Fig. 1 exhibits the historical volatility of Bitcoin's 60-day annualized yield time series ranging from June 27, 2013 to November 7, 2019 including 2325 data. The horizontal axis represents the time axis, and the vertical axis is the volatility per 60-day levels. These estimates are obtained as described in the above with a window width of 60-day. It shows that there are considerable variability and irregularity in historical volatility in Fig. 1.

What follows, in order to examine the nonlinearity, multifractality and predictability of the Bitcoin market, we will respectively discuss the multifractal properties and chaos properties of the historical volatility time series.

2.3 MFDFA Model Settings

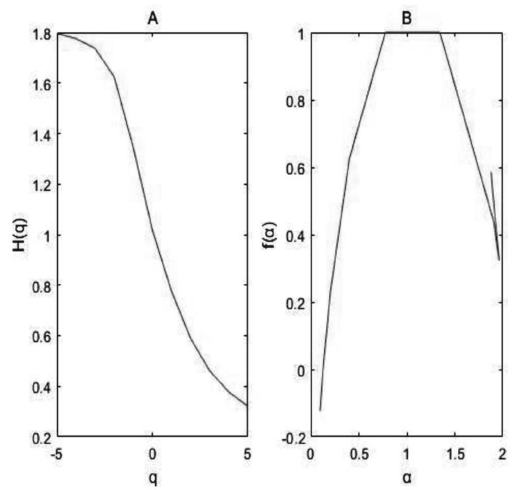
The first step to set the MFDFA model, which means that it is necessary to set the input parameters m, q, and scale for MFDFA analysis. Normally, the value of m should be between 1 and 3 when the smallest segment sizes contain 10–20 samples. After comparison of the multifractal spectrum with different m values, we choose $m = 1$ in the MFDFA model, in order to prevent overfitting of polynomial trend. As noted by Lashermes et al. (2004), q-orders between −5 and 5 are sufficient in most cases. According to Zhou (2009) and Ihlen (2012), we set 8 as the minimum segment size, 23 as the maximum segment size in MFDFA model.

3. Empirical Results

3.1 The Multifractal of the Bitcoin Volatility

The Hurst exponent and multifractal spectrum of the Bitcoin volatility are shown in Fig. 2. The line in Fig. 2A refers to the q-order Hurst exponent $H(q)$ of Bitcoin volatility. Considering the preceding model, q-order based on generalized Hurst exponent $H(q)$ is an indicator for multifractal properties. The figure shows that the value of $H(q)$ is apparently dependent on q values. The decreasing $H(q)$ indicates that the volatility series of the Bitcoin has significant multifractal properties. When $q = -5$, $H(q)$ is 1.8, higher than 0.5, and decreases smoothly with a rising q value between -5 and 5. This indicates significant persistent properties of small fluctuations. When the value of becomes positive, $H(q)$ stays slightly above 0.5. In particular, $H(2) > 0.5$, which implies persistence and long-range memory structure.

Fig. 2: The Hurst exponent and multifractal spectrum of the Bitcoin volatility



Source: own

Note: $H(q)$ – the generalized Hurst exponent; q – the order of fluctuation function; α – the singularity strength; $f(\alpha)$ – the singularity spectrum.

Tab. 1: Multifractal test results for the time series of Bitcoin volatility

$H(2) > 0.5$	$\Delta f = -0.7 < 0$	$\Delta\alpha = 1.78 > 0$
Long-range memory	The chances of the Bitcoin volatility being at the bottom are greater than the chances of being at the top	The Bitcoin volatility market with higher multifractality

Source: own

Note: $H(2)$ – the classic Hurst exponent; $\Delta f = f(\alpha_{min}) - f(\alpha_{max})$; $\Delta\alpha = \alpha_{max} - \alpha_{min}$.

By looking into the multifractal spectrum, the line in Fig. 2B gives more information about multifractal features. The shape of the multifractal spectrum has a long-left tail, meaning the Bitcoin volatility has a multifractal structure sensitive to the local fluctuations with large magnitudes, but insensitive to the local fluctuations with small magnitudes. And through calculation, $\Delta f = -0.7$ is less than 0, indicating the chances of the Bitcoin volatility being at the bottom are greater than the chances of being at the top. By calculating the width (e.g., the value $\Delta\alpha = \alpha_{max} - \alpha_{min}$), we learn that the multifractality degree is 1.78. The $\Delta\alpha$ value is far from 0, indicating the Bitcoin volatility market with higher multifractality. These characteristics fully show that the market is nonlinear and the main conclusions are listed in Tab. 1.

Businesses, banks and institutions can hold Bitcoin, because Bitcoin provides users with lower legitimate transaction costs (Kim, 2017). From the perspective of the policy makers who regulate the Bitcoin market, there are potential reasons for the existence of long-range memory behavior: the lack of clear regulatory laws and regulatory authorities. Therefore, comparing with the traditional financial and commodity markets, policy makers should strengthen market supervision, formulate relevant laws and regulations and establish reform measures to reduce the long-range memory level.

The long-range memory property means that the Bitcoin volatility market can be predicted in short term to capture speculative profits. The higher multifractality shows that the volatility market will change greatly, and the market is very complex. These results have implications for economic entity. Investors can predict the future volatility to analyze price fluctuations and carry out risk control.

There are also important practical implications of Bitcoin fluctuations being more likely to be at the bottom than at the top: Risk averse investors will continue to hold positions or increase positions appropriately to maximize profits based on the higher probability that the volatility of Bitcoin is low. However, the risk appetite investors tend to have a strong risk tolerance, in the hope of higher expected return on investment, will reduce the appropriate positions. These, which are also important by-products of the MFDFA approach, can help investors (especially for investors in quantitative trading) capture the arbitrage opportunity and manage risk. All these characteristics will provide good judgment for investment decision makers, risk control managers and government regulators.

3.2 The Chaos of the Bitcoin Volatility

As the Bitcoin market is predictable, in order to make a more accurate prediction, we want to know whether the internal generation mechanism of Bitcoin volatility time series is chaotic. In order to identify the chaos of the Bitcoin volatility time series, the first step is to reconstruct the phase space, which requires the determination of two parameters: the embedding dimension m and the delay time τ. Firstly, we obtain the delay time $\tau = 6$ with the mutual information function method (Fraser & Swinney, 1986), and can determine the embedding dimension $m = 3$ by Cao method (Gao & Zheng, 1993). Then, the largest Lyapunov exponent LLE = 0.0091 is greater than 0 but very close to zero according to the Wolf algorithm. At the last, we apply ***the Model-based bootstrap method*** (Davison & Hinkley, 1997; Yin & Wang, 2019) to verify the robustness of LLE. The idea of this method is shown in the Appendix A1. When the confidence value is 95%, the confidence interval [0.1279; 0.1733] can be obtained. Since the quantiles for the empirical distribution are all larger than

Tab. 2: Chaos test results for the time series of Bitcoin volatility

Delay time	Embedding dimension	Largest Lyapunov exponent (LLE)	Confidence interval of 95% for LLE	Chaos
$\tau = 6$	$m = 3$	LLE = 0.0091 > 0	[0.1279; 0.1733]	Yes

Source: own

the largest Lyapunov exponents LLE = 0.0091, this means that the value of LLE is significantly greater than zero. It can be determined that the time series of Bitcoin volatility is chaotic using the chaos theory, as shown in the following Tab. 2.

This shows that the volatility time series of Bitcoin has an intrinsic deterministic generation mechanism, namely chaos. Therefore, in the following section, we will focus on whether the prediction model based on intrinsic generation mechanism (chaos) can significantly improve the existing prediction model with chaos.

4. Prediction and Comparison

Accurate prediction of Bitcoin volatility means high returns for investors, risk management and control and effective regulation of the financial market by government departments, so we then build multiple prediction models based on the endogenous structure (chaos) of the time series of Bitcoin volatility and compare with the two existing models without chaos. It is hoped that the prediction accuracy of the chaotic prediction models will be higher, avoiding the forecasting model misspecification. Specifically, we introduce three kinds of prediction models, namely, chaos + ANN (artificial neural network)-type, chaos-type and ANN-type. The chaos + ANN-type contains a hybrid based RBF neural network model with chaos (RBF-CHAOS model) and a hybrid based BP neural network with chaos (BP-CHAOS model); the chaos-type is the weighted first-order Local Region (LR-CHAOS model), which is a model based on chaos without ANN; the ANN-type includes two existing prediction models only with ANN, namely, RBF model and BP model. For the construction of these prediction models, please refer to Appendix A2–A4 or literature.

In the simulation experiment, the original samples and the predicted samples are $\mathrm{x(n)}$ and $\mathrm{xp(n)}$ espectively, and the absolute error $\mathrm{e(n) = xp(n) - x(n)}$, the mean absolute error (MAE) and the percentage error (Perr) are used as the evaluation criteria for the prediction accuracy. The smaller MAE and Perr values are, the more predictive effect of the model is, where the MAE and the Perr are respectively defined as: $\mathrm{MAE} = \frac{1}{\mathrm{N}}\sum_{\mathrm{n=1}}^{\mathrm{N_P}}\left|\mathrm{x(n)} - \mathrm{x_p(n)}\right|$,

$$\mathrm{Perr} = \sum_{\mathrm{n=1}}^{\mathrm{N_P}}(\mathrm{x(n)} - \mathrm{x_p(n)})^2 \Big/ \sum_{\mathrm{n=1}}^{\mathrm{N_P}}\mathrm{x^2(n)},$$

where Np represents the number of predicted samples.

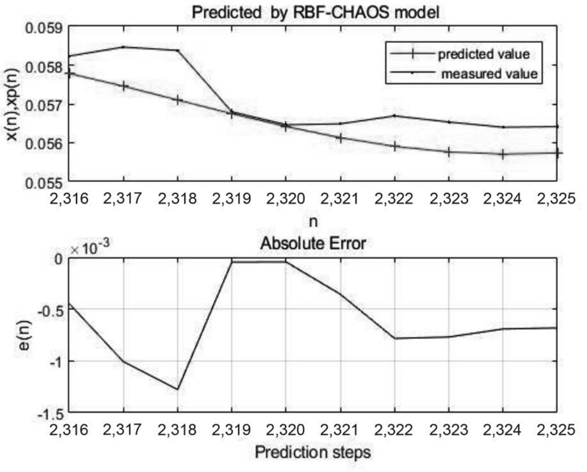
In this section, the first 2,315 data of Bitcoin volatility time series are used as training samples to predict the Bitcoin volatility in the next 10 days. Fig. 4–8 show the prediction images of the five models respectively. The horizontal axis represents the predicted days, and the vertical axis is the volatility per 60-day levels. The red line represents the predicted value and the blue line is the measured value. The closer these two lines are, the closer the predicted value and the measured value are. The absolute value of $\mathrm{e(n)}$ of these five figures does not exceed 1.5×10^{-3}. It follows from Fig. 3–7 that these five models can predict prices of the Bitcoin volatility, and being more accurate in the short term and larger errors in the long term. It should be emphasized that LR-CHAOS model can simulate the measured value well.

The MAE value and Perr value are listing in Tab. 3, and we sort these 5 models based on MAE and Perr value. The smaller the MAE value and Perr value are, the higher the order of the corresponding model is, as shown in Tab. 3.

We can see from Tab. 3 that:

(1) It can be concluded that the ordering of all prediction models is LR-CHAOS > RBF-CHAOS > BP-CHAOS > RBF > BP, indicating that the optimal prediction model is LR-CHAOS among these models. It follows from the above that the model which really improves the prediction accuracy is based on the internal generation mechanism of time series, and this matters for the relevant investment institutions (investors).

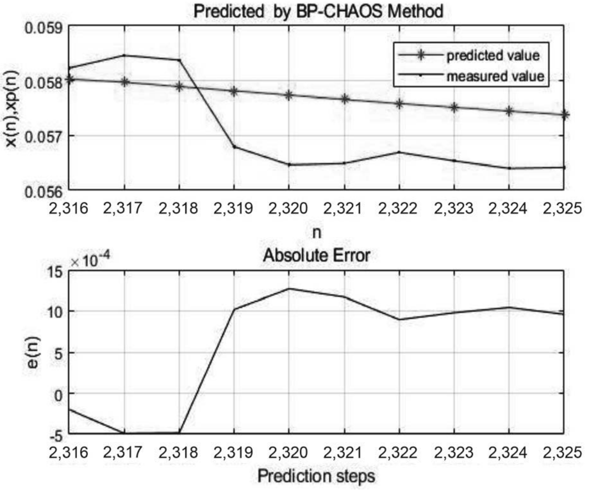
Fig. 3: Predicted by RBF-CHAOS model



Source: own

Note: $x(n)$ – the original price; $x_p(n)$ – the predicted price; $e(n) = x_p(n) - x(n)$.

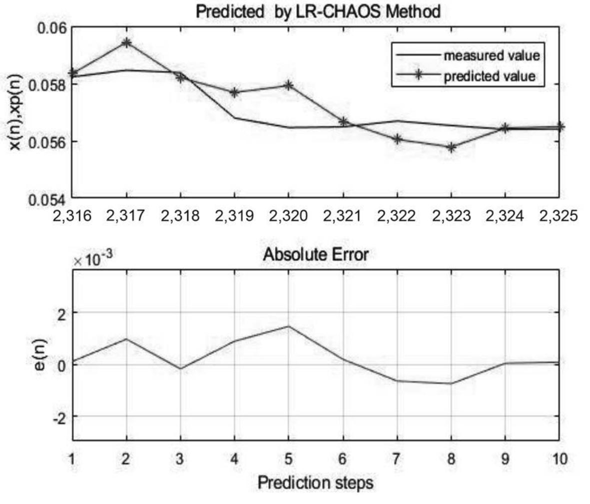
Fig. 4: Predicted by BP-CHAOS model



Source: own

Note: $x(n)$ – the original price; $x_p(n)$ – the predicted price; $e(n) = x_p(n) - x(n)$.

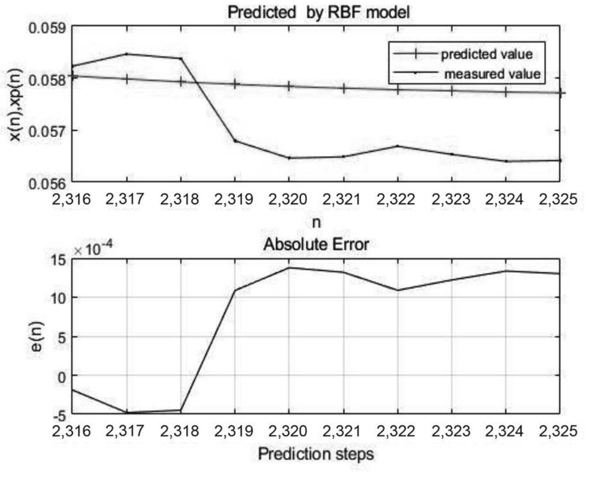
Fig. 5: Predicted by LR-CHAOS model



Source: own

Note: $x(n)$ – the original price; $x_p(n)$ – the predicted price; $e(n) = x_p(n) - x(n)$.

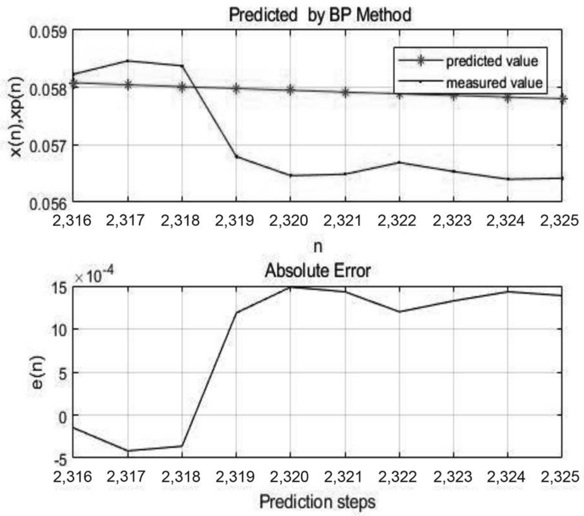
Fig. 6: Predicted by RBF model



Source: own

Note: $x(n)$ – the original price; $x_p(n)$ – the predicted price; $e(n) = x_p(n) - x(n)$.

Fig. 7: Predicted by BP model



Source: own

Note: $x(n)$ – the original price; $x_p(n)$ – the predicted price; $e(n) = x_p(n) - x(n)$.

Tab. 3: Prediction errors of these models

	MAE	Rank	Perr	Rank
RBF-CHAOS	0.0006	2	0.02%	2
BP-CHAOS	0.0009	3	0.03%	3
LR-CHAOS	0.0005	1	0.01%	1
RBF	0.001	4	0.04%	4
BP	0.0013	5	0.05%	5

Source: own

Note: MAE – Mean absolute error; Perr – Percentage error; BP – Back propagation; RBF – Radial basis function; BP-CHAOS – Hybrid back propagation neural network and chaos; RBF-CHAOS – Hybrid radial basis function neural network and chaos; LR-CHAOS – Local region chaos; 'RBF-CHAOS, BP-CHAOS, LR-CHAOS, RBF and BP' represent the data predicted by the corresponding model (for details on the four models see the Appendix).

(2) Furthermore, the results show that the prediction ability of the model is from high to low successively: RBF-CHAOS > RBF, BP-CHAOS > BP, which means that the prediction accuracy of the prediction model combining chaos and ANN is higher than that of the ANN model. At the same time, it is also found that LR-CHAOS > RBF, LR-CHAOS > BP, indicating that the prediction accuracy based only on chaotic characteristics of Bitcoin volatility time series is higher than that based solely on ANN. Furthermore, models based on intrinsic generation mechanism have better predictive power than existing models. This fully shows

that it is very meaningful to build prediction models based on the endogenous structure (chaos) of the time series of Bitcoin volatility.

(3) The order of prediction accuracy of RBF-CHAOS, BP-CHAOS and LR-CHAOS is LR-CHAOS > RBF-CHAOS > BP-CHAOS from high to low. As we all known, RBF-CHAOS and BP-CHAOS are the prediction model combining chaos and ANN, while LR-CHAOS is the model based solely on chaos. This result also shows that adding ANN technology cannot improve the prediction accuracy of chaotic model.

Conclusion and Economic Implications

Bitcoin has received a special attention since its emerging. Thus, it is crucial to understand the stylized facts of this digital currency for investors and academicians. This paper mainly studies the market nonlinearity and the prediction model based on the intrinsic generation mechanism (chaos) of Bitcoin's daily return's volatility from June 27, 2013 to November 7, 2019 with an econophysics perspective, so as to avoid the forecasting model misspecification.

Firstly, this paper studies the multifractal and chaotic nonlinear characteristics of Bitcoin volatility by using MFDFA and LLE methods.

Then, from the perspective of nonlinearity, the measured values of multifractal and chaos show that the volatility of Bitcoin has short-term predictability. The study of chaos and multifractal dynamics in nonlinear systems is very important in terms of their predictability. An unstable or noisy system (signals) may have short-term predictability, and multifractals and self-similarity can increase the likelihood of accurately predicting future sequences of these signals.

Finally, we constructed a number of chaotic artificial neural network models to forecast the Bitcoin return's volatility avoiding the misspecification of the prediction models. The results show that chaotic artificial neural network models have good prediction effect by comparing these models with the existing artificial neural network (ANN) models. This is because the chaotic artificial neural network model can extract hidden patterns and accurately model time series from potential signals, while the benchmark artificial neural network is based on Gaussian kernel local approximation of non-stationary signals, so it cannot approach the global model with chaotic characteristics.

The paper also points out that from a technical point of view, it is feasible to predict and analyze the volatility of the Bitcoin market, which makes market arbitrage possible. Arbitrage opportunities can increase risk in the Bitcoin market by attracting speculative capital. Investors can use the predictive model to design risk control strategies in the Bitcoin market and effectively manage the risks in the Bitcoin market.

At the same time, this paper also fully explores the system parameters, which can also help investors (especially those in quantitative trading) seize arbitrage opportunities and manage risks. These characteristics will provide good judgment for investment decision makers, risk control managers and government regulators. Specifically: the $\Delta\alpha$ value is far from 0, indicating the Bitcoin volatility market with higher multifractality, this shows that the volatility market will change greatly, and the market is very complex. The reason may be the conversion of heterogeneous trading strategies or insufficient information transmission. Therefore, the government regulators should strengthen the education of traders and disclose information in time to avoid information asymmetry. Then, $H(2) > 0.5$ shows that the Bitcoin volatility market has a long-range memory, which also indicates that the market can make predictions based on past data. This provides strong evidence against the efficient market hypothesis (EMH). There are potential reasons for the existence of long-range memory behavior: the lack of clear regulatory laws and regulatory authorities. Therefore, comparing with the traditional financial and commodity markets, policy makers should strengthen market supervision, formulate relevant laws and regulations and establish reform measures to reduce the long-range memory level. As well as $\Delta f < 0$ indicates the chances of the Bitcoin volatility being at the bottom are greater than the chances of being at the top. This is a forecast of the direction of the market and indicates that the market will go down, which will help investors and regulators in the market. There are also important practical implications of that: Risk averse investors will continue to hold positions or increase positions appropriately to maximize profits based on the higher probability that the volatility of Bitcoin is low. However, the risk appetite investors tend to have a strong risk tolerance, in the hope of higher expected return on investment, will reduce the appropriate positions.

References

Adrangi, B., & Chatrath, C. (2001). Chaos in oil prices? Evidence from futures market. *Energy Economics, 23*(4), 405–425. https://doi.org/10.1016/S0140-9883(00)00079-7

Ajaz, T., & Kumar, A. (2018). Herding in cryptocurrency markets. *Annals of Financial Economics, 13*(02), 1850006. https://doi.org/10.1142/S2010495218500069

Bariviera, A., Basgall, M., Hasperué, W., & Naiouf, M. (2017). Some stylized facts of the Bitcoin market. *Physica A: Statistical Mechanics and its Applications, 484,* 82–90. https://doi.org/10.1016/j.physa.2017.04.159

Bouoiyour, J., Selmi, R., & Wohar, M. (2018). Are Islamic stock markets efficient? A multifractal detrended fluctuation analysis. *Finance Research Letters, 26,* 100–105. https://doi.org/10.1016/j.frl.2017.12.008

Bouoiyour, J., Selmi, R., & Tiwari, A. K. (2015). Is Bitcoin Business Income or Speculative Foolery? New Ideas through an Improved Frequency Domain Analysis. *Annals of Financial Economics, 10*(1), 1550002. https://doi.org/10.1142/S2010495215500025

Bouri, E., Molnar, P., Azzi, G., Roubaud, D., & Hagfors, L. I. (2017). On the hedge and safe haven properties of Bitcoin: Is it really more than a diversifier? *Finance Research Letters, 20,* 192–198. https://doi.org/10.1016/j.frl.2016.09.025

Brière, M., Oosterlinck, K., & Szafarz, A. (2015). Virtual currency, tangible return: Portfolio diversification with Bitcoin. *Journal of Asset Management, 16*(6), 365–373. https://doi.org/10.1057/jam.2015.5

Cao, G., Cao, J., & Xu, L. (2013). Asymmetric multifractal scaling behavior in the Chinese stock market: Based on asymmetric MF-DFA. *Physica A: Statistical Mechanics and its Applications, 392*(4), 797–807. https://doi.org/10.1016/j.physa.2012.10.042

Carbone, A., Castelli, G., & Stanley, H. (2004). Time-dependent Hurst exponent in financial time series. *Physica A: Statistical Mechanics and its Applications, 344*(1–2), 267–271. https://doi.org/10.1016/j.physa.2004.06.130

Davison, A., & Hinkley, D. (1997). *Bootstrap Methods and their Application*. Cambridge University Press. https://doi.org/10.1017/CBO9780511802843

Donier, J., & Bonart, J. (2015). A Million Metaorder Analysis of Market Impact on the Bitcoin. *Market Microstructure and Liquidity, 1*(02), 1550008. https://doi.org/10.1142/S2382626615500082

Dwyer, G. (2015). The economics of Bitcoin and similar private digital currencies. *Journal of Financial Stability, 17,* 81–91. https://doi.org/10.1016/j.jfs.2014.11.006

Dyhrberg, A., Foley, S., & Svec, J. (2018). How investible is Bitcoin? Analyzing the liquidity and transaction costs of Bitcoin markets. *Economics Letters, 171,* 140–143. https://doi.org/10.1016/j.econlet.2018.07.032

Fraser, A. M., & Swinney, H. L. (1986). Independent coordinates for strange attractors from mutual Information. *Physical Review A, 33*(2), 1134. https://doi.org/10.1103/PhysRevA.33.1134

Gao, J., & Zheng, Z. (1993). Local exponential divergence plot and optimal embedding of a chaotic time series. *Physics Letters A, 181*(2), 153–158. https://doi.org/10.1016/0375-9601(93)90913-K

Hung, J.-C., Liu, H.-C., & Yang, J. J. (2020). Improving the realized GARCH's volatility forecast for Bitcoin with jump-robust estimators. *The North American Journal of Economics and Finance, 52,* 101165. https://doi.org/10.1016/j.najef.2020.101165

Ihlen, E. A. F. (2012). Introduction to multifractal detrended fluctuation analysis in Matlab. *Frontiers in Physiology, 3,* 141. https://doi.org/10.3389/fphys.2012.00141

Kantelhardt, J. W., Zschiegner, S. A., Koscielny-Bunde, E., Havlin, S., Bunde, A., & Stanley, H. E. (2002). Multifractal detrended fluctuation analysis of nonstationary time series. *Physica A: Statistical Mechanics and its Applications, 316*(1–4), 87–114. https://doi.org/10.1016/S0378-4371(02)01383-3

Katsiampa, P. (2017). Volatility estimation for Bitcoin: A comparison of GARCH models. *Economics Letters, 158,* 3–6. https://doi.org/10.1016/j.econlet.2017.06.023

Kim, T. (2017). On the transaction cost of Bitcoin. *Finance Research Letters, 23,* 300–305. https://doi.org/10.1016/j.frl.2017.07.014

Klein, T., Thu, H. P., & Walther, T. (2018). Bitcoin is not the New Gold – A comparison of volatility, correlation, and portfolio performance. *International Review of Financial Analysis, 59,* 105–116. https://doi.org/10.1016/j.irfa.2018.07.010

Koutmos, D. (2018). Liquidity uncertainty and Bitcoin's market microstructure. *Economics*

Letters, 172, 97–101. https://doi.org/10.1016/j.econlet.2018.08.041

Lahmiri, S. (2017). On fractality and chaos in Moroccan family business stock returns and volatility. *Physica A: Statistical Mechanics and Its Applications, 473,* 29–39. https://doi.org/10.1016/j.physa.2017.01.033

Lahmiri, S., Bekiros, S., & Salvi, A. (2018). Long-range memory, distributional variation and randomness of bitcoin volatility. *Chaos Solitons & Fractals, 107,* 43–48. https://doi.org/10.1016/j.chaos.2017.12.018

Lashermes, B., Abry, P., & Chainais, P. (2004). New insights into the estimation of scaling exponents. *International Journal of Wavelets, Multiresolution and Information Processing, 02*(04), 497–523. https://doi.org/10.1142/S0219691304000597

Li, T., & Tourin, A. (2016). Optimal pairs trading with time-varying volatility. *International Journal of Financial Engineering, 03*(03), 1650023. https://doi.org/10.1142/S2424786316500237

Martinez, L. B., Guercio, M. B., Bariviera, A. F., & Terceño, A. (2018). The impact of the financial crisis on the long-range memory of European corporate bond and stock markets. *Empirica, 45*(1), 1–15. https://doi.org/10.1007/s10663-016-9340-8

McCarthy, J., Pantalone, C., & Li, H. C. (2009). Investigating Long Memory in Yield Spreads. *The Journal of Fixed Income, 19*(1), 73–81. https://doi.org/10.3905/JFI.2009.19.1.073

Moody, J., & Darken, C. J. (1989). Fast Learning in Networks of Locally-Tuned Processing Units. *Neural Computation, 1*(2), 281–294. https://doi.org/10.1162/neco.1989.1.2.281

Nadarajah, S., & Chu, J. (2017). On the inefficiency of bitcoin. *Economics Letters, 150,* 6–9. https://doi.org/10.1016/j.econlet.2016.10.033

Nakamoto, S. (2008). *Bitcoin: A Peer-to-Peer Electronic Cash System.* https://bitcoin.org/bitcoin.pdf

Osterrieder, J., & Lorenz, J. (2017). A Statistical Risk Assessment of Bitcoin and Its Extreme Tail Behavior. *Annals of Financial Economics, 12*(1), 1750003. https://doi.org/10.1142/S2010495217500038

Ozun, A., Hanias, M. P., & Curtis, P. G. (2010). A chaos analysis for Greek and Turkish equity markets. *EuroMed Journal of Business, 5*(1), 101–118. https://doi.org/10.1108/14502191011043189

Rizvi, S. A. R., Dewandaru, G., Bacha, O. I., & Masih, M. (2014). An analysis of stock market efficiency: Developed vs Islamic stock markets using MF-DFA. *Physica A: Statistical Mechanics and its Applications, 407,* 86–99. https://doi.org/10.1016/j.physa.2014.03.091

Rosenstein, M. T., Collins, J. J., & De Luca, C. J. (1993). A practical method for calculating largest Lyapunov exponents from small data sets. *Physica D: Nonlinear Phenomena, 65*(1–2), 117–34. https://doi.org/10.1016/0167-2789(93)90009-P

Rumelhart, D. E., & McCleland, J. L. (1986). Learning Internal Representations by Error Propagation. In *Parallel Distributed Processing: Explorations in the Microstructure of Cognition: Foundations* (pp. 318–362). Cambridge, MA: MIT Press. Retrieved from https://ieeexplore.ieee.org/document/6302929

Seo, Y., & Hwang, C. (2018). Predicting Bitcoin market Market Trend with Deep Learning Models. *Quantitative Bio-Science, 37*(1), 65–71. https://doi.org/10.22283/qbs.2018.37.1.65

Symitsi, E., & Chalvatzis, K. (2018). Return, volatility and shock spillovers of Bitcoin with energy and technology companies. *Economics Letters, 170,* 127–130. https://doi.org/10.1016/j.econlet.2018.06.012

Tasca, P., Hayes, A., & Liu, S. (2018). The Evolution of the Bitcoin Economy: Extracting and Analyzing the Network of Payment Relationships. *Journal of Risk Finance, 19*(2), 94–126. https://doi.org/10.1108/JRF-03-2017-0059

Tiwari, A. K., Kumar, S., & Pathak, R. (2019). Modelling the dynamics of Bitcoin and Litecoin: GARCH versus stochastic volatility models. *Applied Economics, 51*(37), 4073–4082. https://doi.org/10.1080/00036846.2019.1588951

Uddin, G. S., Hernandez, J. A., Shahzad, S. J. H., & Yoon, S.-M. (2018). Time-varying evidence of efficiency, decoupling, and diversification of conventional and Islamic stocks. *International Review of Financial Analysis, 56,* 167–180. https://doi.org/10.1016/j.irfa.2018.01.008

Urquhart, A. (2016). The inefficiency of Bitcoin. *Economics Letters, 148,* 80–82. https://doi.org/10.1016/j.econlet.2016.09.019

Urquhart, A., & Zhang, H. (2019). Is Bitcoin a Hedge or Safe-Haven for Currencies? An

Intraday Analysis. *International Review of Financial Analysis, 63,* 49–57. https://doi.org/10.2139/ssrn.3114108

Wolf, A., Swift, J. B., Swinney, H. L., & Vastano, J. A. (1985). Determining Lyapunov exponents from a time series. *Physica D: Nonlinear Phenomena, 16*(3), 285–317. https://doi.org/10.1016/0167-2789(85)90011-9

Yang, S.-S., & Tseng, C.-S. (1996). An orthogonal neural network for function approximation. *IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics), 26*(5), 779–785. https://doi.org/10.1109/3477.537319

Yi, S., Xu, Z., & Wang, G.-J. (2018). Volatility connectedness in the cryptocurrency market: Is Bitcoin a dominant cryptocurrency? *International Review of Financial Analysis, 60,* 98–114. https://doi.org/10.1016/j.irfa.2018.08.012

Yin, T., & Wang, Y. (2019). Predicting the Price of WTI Crude Oil Using ANN and Chaos. *Sustainability, 11*(21), 5980. https://doi.org/10.3390/su11215980

Zhang, Q.-Y., Pan, L., & Zi, X.-C. (2010). A Shared Congestion Detection Technique Based on Weighted First Order Local-Region Method. *Journal of Shanghai Jiaotong University, 44*(2), 286–287. https://doi.org/10.3724/SP.J.1187.2010.00953

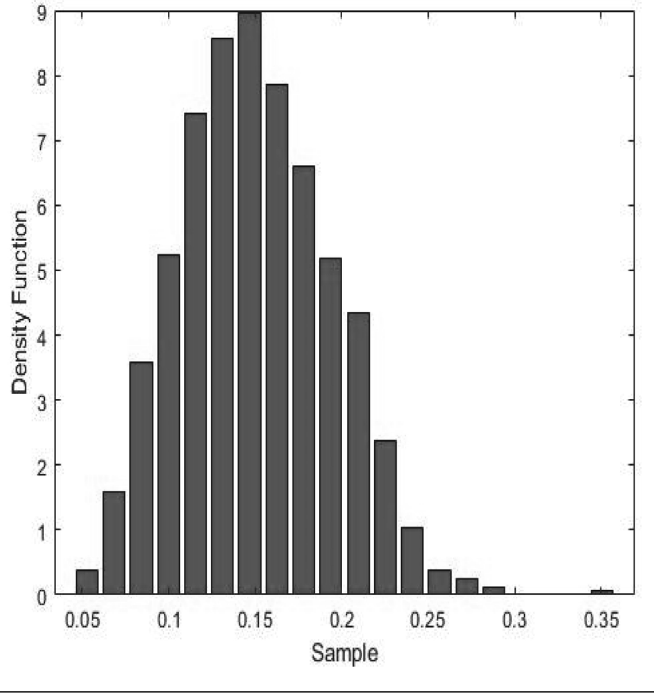
Zhou, W.-X. (2009). The components of empirical multifractality in financial returns. *Europhysics Letters, 88*(2), 28004. https://doi.org/10.1209/0295-5075/88/28004

Appendix:

A1. Model-based Bootstrap

The idea of Model-based bootstrap method is to fit a suitable model for the data, construct the residuals from the fitted model, and then bring the random samples in the residual into the fitting model to generate a new sequence. When we use the AR (1) model to fit the time series $x_1, x_2, \ldots, x_n$, giving estimated autoregressive coefficient $e_j = x_j - \widetilde{\alpha_1} x_{j-1}$, j = 2, …, n; where e_j represents the residuals, it is found that the correlation of $\{e_j\}_{j=2}^{n}$ sequence was very weak. So we choose AR (1) as the based model. The estimated largest Lyapunov exponent is denoted $\widetilde{\lambda_1}$, and then the empirical distribution for $\widetilde{\lambda_1}$ is constructed by bootstrap samples. When the confidence value is 95%, the confidence interval [0.1279; 0.1733] can be obtained. Since the quantiles for the empirical distribution are all larger than the largest Lyapunov exponents LLE = 0.0091, this means that the value of LE is significantly greater than zero.

Fig. A1: The histogram of the estimated largest Lyapunov exponent by bootstrap method



Source: own

A2. RBF Model and RBF-CHAOS Model

RBF neural network (RBF model), which is also called Radial Basis Function neural network and proposed by Moody (1989), is a three-layer feed-forward network with a similar structure of multiple layers of forward networks with a single hidden layer. RBF neural network is a kind of local approximation network with three typical network structures: input layer, hidden layer and output layer. The specific structure of RBF model has been mentioned in many literatures, so it will not be repeated here.

A hybrid based RBF neural network model with Chaos (RBF-CHAOS model) can be constructed by following steps:

Step 1: Take the embedded dimension as the input number of RBF network, and let the output number as 1;

Step 2: Take the radial basis function form as:

$$w_j\left(\left\|\overrightarrow{h(t)}-\overrightarrow{k_j}\right\|\right)=\exp\left(-\frac{\left\|\overrightarrow{h(t)}-\overrightarrow{k_j}\right\|}{c^2}\right), j=1,2,\ldots,n,$$

where c is called the width value, the input vector of the network is $\overrightarrow{h(t)} \in R^m$; $w_j(.)$ is called a radial basis function; $||.||$ represents norm; $\overrightarrow{k_j}$ signifies the center of the radial basis function.

A3. BP Model and BP-CHAOS Model

Back Propagation neural network (BP model), including input layer, hidden layer and output layer, is using minimum variance learning method (Rumelhart et al., 1986; Yang, 1996). At the same time, it is a kind of supervised learning neural network, which contains three or more layers of neural networks.

The hybrid based BP neural network model with Chaos (BP-CHAOS model) can be constructed by following steps:

Step 1: Take the embedded dimension as the input number of BP network, and let the output number as 1;

Step 2: Take input and output of layer nodes, respectively as $L=\sum_{j=1}^{n} u_j c_j - b$, $x_{i+1}=\frac{1}{1+exp(-\sum_{j=1}^{n} u_j c_j + b)}$, $j = 1, 2, \ldots, n$; where $c_j=\frac{1}{1+exp(-\sum_{i}^{m} w_{ij} x_i + h_j)}$ is the output of hidden layer, the connection weight from the input layer to the hidden layer is w_{ij}, the threshold value of hidden layer node is represented by h_j.

The connection weight from the hidden layer to the output layer is u_j, and b is the threshold of the output layer.

Note that the two models (RBF-CHAOS and BP-CHAOS) are based on ANN and chaos, while the other two models (RBF and BP) are only based on ANN.

A4. LR-CHAOS Model

The LR-CHAOS model (Zhang, 2010) can be constructed by following steps:

Step 1: Set the neighboring point M_{ki} of the center point M_k, $i = 1,2, \ldots, q$, and let $d_i = ||M_{ki} - M_k||$, where $||.||$ represents norm.

Step 2: Set $d_m = \min\{d_i\}$, and define the weight π_i of the point M_{ki} as:

$$\pi_i=\frac{\exp(-(d_i-d_m))}{\sum_{k=1}^{q}\exp(-(d_i-d_m))}.$$

Step 3: Make the linear fit as $M_{ki+1} = ae + bM_{ki}$ to estimate the coefficients a and b, where $e = (1, \ldots, 1)^T$.

Note that the LR-CHAOS model is only based on chaos.