

Solution of incompressible viscous fluid flow using a physical informed neural network

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A physical informed neural network (PINN) is the novel approach for solving a partial differential equation using a neural network. This concept was firstly introduced in the paper [3], where the PINN was used for the solution of various partial differential equations. Since then, many papers have been published dealing with the solution of a wide range of partial differential equations [1, 2].

The principle of this novel method is quite simple. It considers a solution of an equation as a non-linear function defined by a neural network. In other words, the neural network is used as the mapper from the space-time variables into unknowns. This contrasts sharply with classical methods, where the solution is considered as a linear combination of basis functions. The crucial part of the PINN approach lies in constructing a loss function, see Fig. 1. Firstly, the automatic differentiation process is used to find the exact space and time derivatives of a solution with arbitrary orders, see green layer. Then the solution, with its derivatives, is put into the equations in the classical or weak form, see the red box. If we assume that the exact solution satisfies the equations together with boundary conditions, we can think of the value after substitution directly as the loss function.

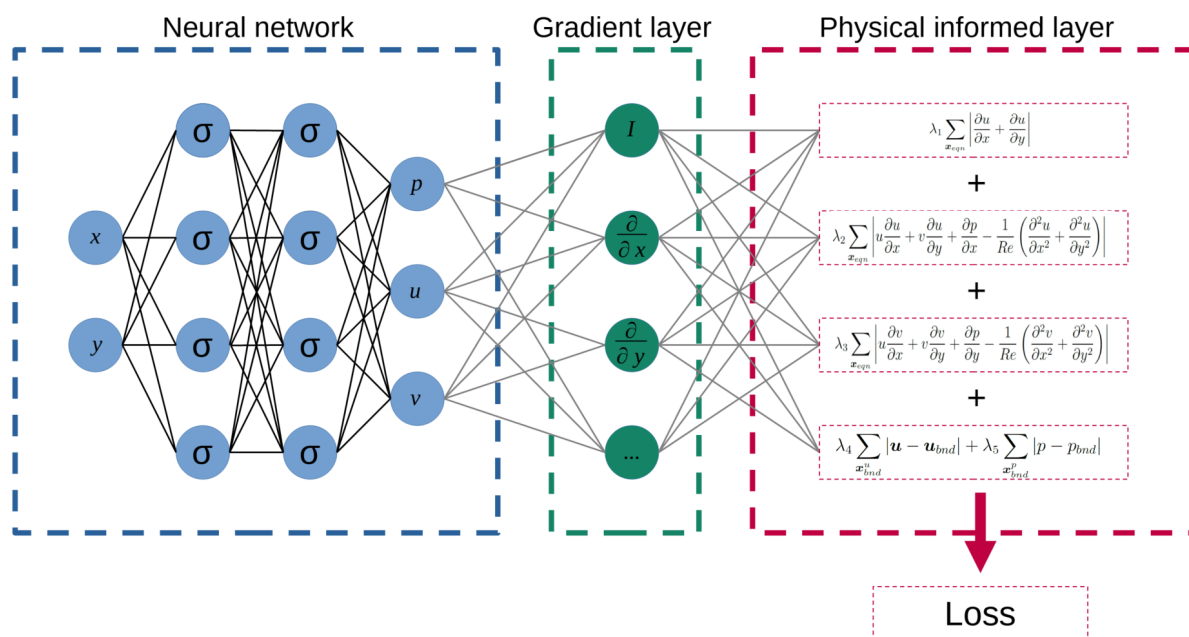


Fig. 1. Example of physical informed neural network for solution of Navier-Stokes equations

The training process starts by choosing points inside and on the boundary of the computational domain. The training algorithm tries to minimize the loss function evaluated for selected points. Suppose the loss function is minimized to zero. In that case, the function described by the neural network will be satisfying boundary conditions at the boundary points and the equation at the inner points.

The methodology is demonstrated in the solution of the flow field of incompressible viscous fluid in the channel for various Reynolds numbers. Figs. 2 and 3 show the comparison between PINN predicted solution and solution computed by discontinuous Galerkin finite element method (DGFEM).

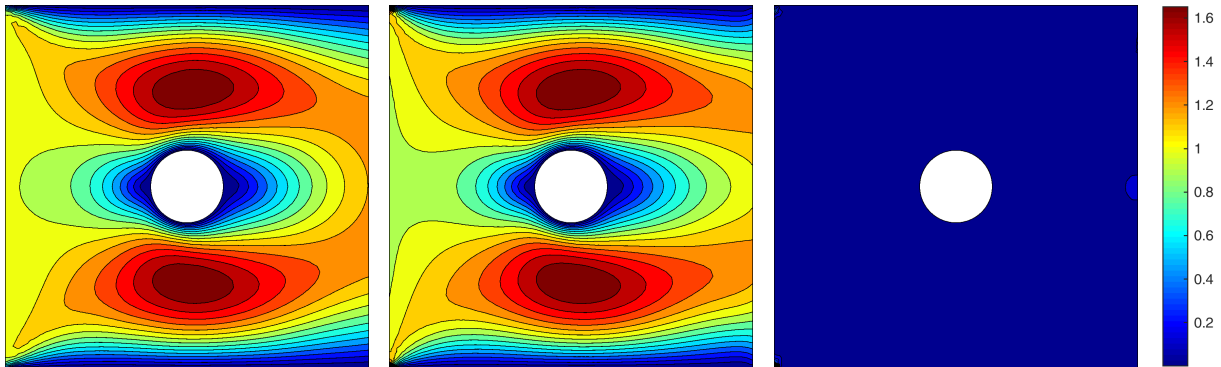


Fig. 2. Comparison of PINN predicted solution (*left*) with a solution computed by DGFEM (*middle*) for Reynolds number 10. The figure on the right shows the error between solutions

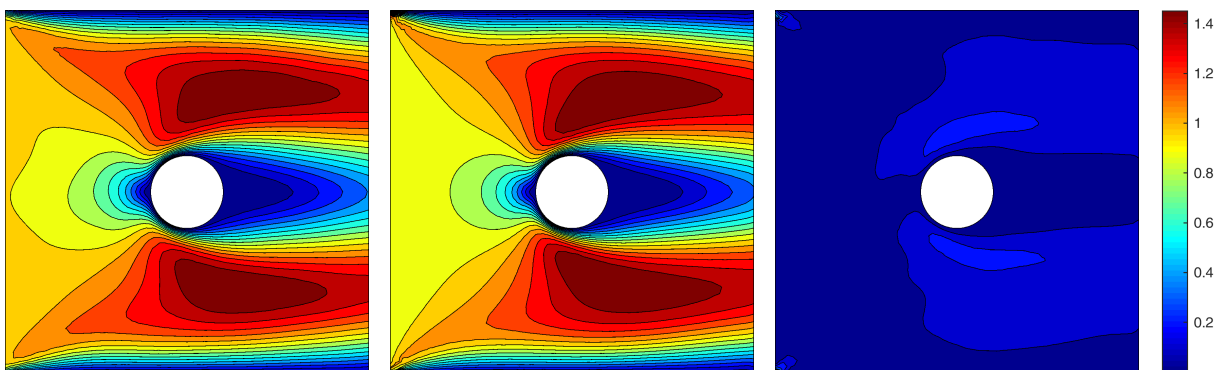


Fig. 3. Comparison of PINN predicted solution (*left*) with a solution computed by DGFEM (*middle*) for Reynolds number 100. The figure on the right shows the error between solutions

References

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- [3] Raissi, M., Perdikaris, P., Karniadakis, G. E., Physics-informed neural networks: A deep learning framework for solving forward and inverse problems involving nonlinear partial differential equations, *Journal of Computational Physics* (378) (2019) 686-707.