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Periodical solution of *n*-DOF parametric system vibration

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1. Introduction

The contribution deals with usage of periodic Green´s function (PGF) to periodic solution of linear vibrating systems described by time dependent coefficient matrices. The PGF enables to transfer system of 2nd order ODE to the system of integral [1, 2, 4] and integer-differential equations [3] and its analytical solution. Behaviour of several mechanical objects such as non-symmetrical rotors can be described by system of linear ordinary differential equations of the 2nd order. This system in matrix form is represented by matrices of mass, damping, stiffness and excitation force vector. The mentioned matrices especially for non-symmetrical rotors have time dependent periodic form. Assuming the excitation is periodic too, the equation of motion can take a form

$$\left[\mathbf{M}_0-\tilde{\mathbf{M}}(t)\right]\ddot{\mathbf{q}}(t)+\left[\mathbf{B}_0-\tilde{\mathbf{B}}(t)\right]\dot{\mathbf{q}}(t)+\left[\mathbf{K}_0-\tilde{\mathbf{K}}(t)\right]\mathbf{q}(t)=\mathbf{f}(t), \tag{1}$$

where

$$\tilde{\mathbf{M}}(t)=\tilde{\mathbf{M}}(t+T),\ \tilde{\mathbf{B}}(t)=\tilde{\mathbf{B}}(t+T),\ \tilde{\mathbf{K}}(t)=\tilde{\mathbf{K}}(t+T),\ \mathbf{f}(t)=\mathbf{f}(t+T), \tag{2}$$

T is period and $\omega = 2\pi/T$ is basic angular frequency. For the reason of periodicity the matrices and force vector can be expressed by Fourier series (let us mark $e_k(t)=e^{ik\omega t}$)

$$\tilde{\mathbf{M}}(t)=\sum_{k=-N}^{N}\mathbf{M}_k e_k(t),\ \tilde{\mathbf{B}}(t)=\sum_{k=-N}^{N}\mathbf{B}_k e_k(t),\ \tilde{\mathbf{K}}(t)=\sum_{k=-N}^{N}\mathbf{K}_k e_k(t),\ \mathbf{f}(t)=\sum_{k=-N}^{N}\mathbf{f}_k e_k(t). \tag{3}$$

2. Periodic Green's function

Periodic Green's function is matrix function whose *j*-th column is system response to excitation in the *j*-th place to force in the form of Dirac train with period *T*. This excitation in *j*-th place and Dirac train [1] can be written in form

$$\frac{1}{T}\mathbf{i}_j\delta_T(t)=\frac{1}{T}\mathbf{i}_j\sum_{k=-N}^{N}e_k(t), \tag{4}$$

where $\mathbf{i}_j$ is *j*-th column of identity matrix. The response taken into account corresponds to the stationary part of left hand side of Eq. (1)

$$\mathbf{M}_0\ddot{\mathbf{q}}^{(j)}(t)+\mathbf{B}_0\dot{\mathbf{q}}^{(j)}(t)+\mathbf{K}_0\mathbf{q}^{(j)}(t)=\frac{1}{T}\mathbf{i}_j\sum_{k=-N}^{N}e_k(t). \tag{5}$$

The PGF after some arrangements has form

$$\mathbf{H}_T(t)=\frac{1}{T}\sum_{k=-N}^{N}\mathbf{L}_k e_k(t),\ \ \mathbf{L}_k=\left(-k^2\omega^2\mathbf{M}_0+ik\omega\mathbf{B}_0+\mathbf{K}_0\right)^{-1}\in\mathbf{C}^{n,n}. \tag{6}$$

N corresponds to the number of respected terms in Fourier series. Solution of (1) is identic with solution of integer-differential equation

$$\begin{aligned}\mathbf{q}(t) = &\int_0^T \mathbf{H}_T(t-s)\tilde{\mathbf{M}}(s)\ddot{\mathbf{q}}(s)\,ds + \int_0^T \mathbf{H}_T(t-s)\tilde{\mathbf{B}}(s)\dot{\mathbf{q}}(s)\,ds + \\ &+ \int_0^T \mathbf{H}_T(t-s)\tilde{\mathbf{K}}(s)\mathbf{q}(s)\,ds + \int_0^T \mathbf{H}_T(t-s)\mathbf{f}(s)\,ds.\end{aligned} \tag{7}$$

The last equation can be rewritten after some arrangements into form

$$\mathbf{q}(t) = \mathbf{E}(t)\boldsymbol{\gamma} + \mathbf{E}(t)\boldsymbol{\beta} + \mathbf{E}(t)\boldsymbol{\alpha}, \tag{8}$$

where $\mathbf{E}(t) = \left[e_{-N}(t)\mathbf{I},\quad e_{-N+1}(t)\mathbf{I},\quad \ldots,\quad e_N(t)\mathbf{I}\right] \in \mathbf{C}^{n,(2N+1)n}$, $\mathbf{I} \in \mathbf{R}^{n,n}$ is identity matrix. Vectors of coefficients are result of solution to Eq. (7).

3. Conclusion

The results and detailed procedure of solution of Eq. (7) will be presented during oral presentation. This result relation (8) enables next possibilities for the solution of equation of motion (1), e.g. response to random excitation on the right hand side of Eq. (1) or solution to the same equation whose matrices on the left hand side contain random parameters.

References

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