COMPUTATIONAL MECHANICS 2022
37th conference with international participation
Srní
November 7 - 9, 2022

# Homogenization based two-scale modelling of unilateral contact in micropores of fluid saturated porous media

E. Rohan, J. Heczko

*Faculty of Applied Sciences, University of West Bohemia, Univerzitní 8, 301 00 Plzeň, Czech Republic*

## 1. Introduction

We consider fluid-saturated poroelastic structures characterized by unilateral self-contact at the pore level of the periodic microstructure. The unilateral frictionless contact interaction is considered on matching pore surfaces of the elastic skeleton. Depending on the deformation due to applied macroscopic loads, the self-contact interaction alters the one between the solid and fluid phases. Both the disconnected and connected porosities are treated; in the latter case, quasistatic fluid flow is described by the Stokes model. We derive two-scale models of the homogenized porous media for the two types of porosities using the framework of the periodic unfolding homogenization [2, 4], *cf.* our previous paper [5] where only empty pores were considered. For the closed pore microstructures, a nonlinear elastic model is obtained at the macroscopic scale. For the connected porosity, a regularization is introduced, assuming the contact interaction never close perfectly the pores, which prevents the pore connectivity. The macroscopic model attains the form of a nonlinear Biot continuum, whereby the Darcy flow model governs the fluid redistribution. To respect that the permeability and other poroelastic coefficients depend on the deformation, an approximation based on the sensitivity analysis is employed [6].

We propose and test new modifications of the original two-scale computational algorithm reported [5] which is based alternating micro- and macro-level steps. As a novelty, a dual formulation of the pore-level contact problems in the local representative cells provides actual active contact sets which enables to compute consistent effective elastic coefficients at particular macroscopic points. At the macroscopic level, a sequential linearization leads to an incremental equilibrium problem which is constrained by a projection arising from the homogenized contact constraint, such that the Uzawa algorithm can be used. At the local level, the finite element discretized contact problem attains the form of a nonsmooth equation which which is solved using the semi-smooth Newton method [3] without any regularization, or a problem relaxation. Numerical examples of 2D deforming structures are presented.

## 2. Problem formulation

In the framework of the unfolding method of homogenization, using the asymptotic analysis with respect to heterogeneity scale parameter $\varepsilon \to 0$, we derived limit two-scale models of the unilateral contact in porous structures with disconnected and connected porosity. Below we present the variational formulations of the contact problems for heterogeneous structures with disconnected, or connected pores. An open bounded domain $\Omega \subset \mathbb{R}^d$, with the dimension $d = 2, 3$, is constituted by the solid elastic skeleton $\Omega_s^\varepsilon$ and by the fractures (fissures) $\Omega_f^\varepsilon$ which

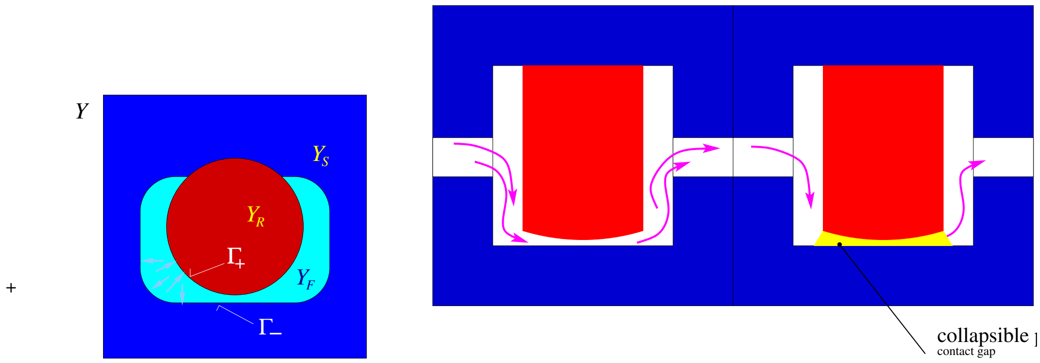


Fig. 1. Representative cells $Y$ for the two types of porous structures. Closed pores (left), connected pores allowing for flow (right). Contact surfaces $\Gamma_c^{+/-}$, subparts on the pore surface $\Gamma_{fs}$

are saturated by a viscous fluid, so that

$$\Omega = \Omega_s^\varepsilon \cup \Omega_f^\varepsilon \cup \Gamma^\varepsilon\;, \quad \Omega_s^\varepsilon \cap \Omega_f^\varepsilon = \emptyset\;, \quad \overline{\Omega_f^\varepsilon} \subset \Omega\;, \tag{1}$$

where $\Gamma^\varepsilon = \overline{\Omega_s^\varepsilon} \cap \overline{\Omega_f^\varepsilon}$ is the interface; the contact is possible on $\Gamma_c^\varepsilon \subset \Gamma^\varepsilon$. The pores $\Omega_f^\varepsilon$ and the skeleton are constituted as periodic lattices using domains $Y_f$ and $Y_s$, respectively, where $Y = Y_s \cup Y_f \cup \Gamma$ is the representative unit cell.

*2.1 Disconnected pores – static problems*

The problem is described by a variational inequality governing the displacements $\boldsymbol{u}^\varepsilon$ and pore pressure $p^\varepsilon$ which is defined by constants in each closed pore $\Omega^{k,\varepsilon} \subset \Omega_f^\varepsilon$. The following sets are employed:

$$\begin{aligned} \textit{kinematic constraint:} \quad & \mathcal{K}^\varepsilon = \{\boldsymbol{v} \in \mathbf{H}^1(\Omega_s^\varepsilon) |\; \boldsymbol{v} = 0 \text{ on } \partial_u\Omega_s^\varepsilon\;,\; g_c^\varepsilon(\boldsymbol{v}) \leq 0 \;\text{ on } \Gamma_c^\varepsilon\}\;, \\ \textit{admissible pressure field:} \quad & \mathcal{Q}^\varepsilon = \{q \in L^2(\Omega) |\; q \text{ is constant in each } \Omega^{k,\varepsilon}\;,\; k \in I_f^\varepsilon\}\;, \end{aligned}$$

where $g_c^\varepsilon$ is the contact gap function. The variational formulation reads: Find $\boldsymbol{u}^\varepsilon \in \mathcal{K}^\varepsilon$ and the pressure $p^\varepsilon \in \mathcal{Q}^\varepsilon$ such that (given volume forces $\boldsymbol{f}^\varepsilon$)

$$\begin{aligned} \int_{\Omega_s^\varepsilon} \mathbb{D}\boldsymbol{e}(\boldsymbol{u}^\varepsilon) : \boldsymbol{e}(\boldsymbol{v}^\varepsilon - \boldsymbol{u}^\varepsilon) + \int_{\partial\Omega_f^\varepsilon} p^\varepsilon \boldsymbol{n}^{[s]} \cdot (\boldsymbol{v}^\varepsilon - \boldsymbol{u}^\varepsilon) \geq \int_{\Omega_s^\varepsilon} \boldsymbol{f}^\varepsilon \cdot (\boldsymbol{v}^\varepsilon - \boldsymbol{u}^\varepsilon)\;, \quad \forall \boldsymbol{v}^\varepsilon \in \mathcal{K}^\varepsilon\;, \\ \int_{\partial\Omega_f^\varepsilon} q^\varepsilon \boldsymbol{u}^\varepsilon \cdot \boldsymbol{n}^{[s]} - \gamma \int_{\Omega_f^\varepsilon} p^\varepsilon q^\varepsilon = 0 \quad \forall q^\varepsilon \in \mathcal{Q}^\varepsilon\;, \end{aligned} \tag{2}$$

where $\boldsymbol{e}(\boldsymbol{v}) = (e_{ij}(\boldsymbol{v}))$ is the small strain tensor, $\gamma$ is the fluid compressibility, and $\mathbb{D} = (D_{ijkl})$ is the elasticity. $\boldsymbol{n}^{[s]}$ designates the unit normal vector outward to $\Omega_s^\varepsilon$.

*2.2 Quasistatic flow in collapsible connected pores*

We consider the Stokes slow flow of an incompressible fluid in collapsible pores $\tilde{\Omega}_f^\varepsilon$ of a deformable porous structure. While the solid skeleton small deformations are described in the fixed (initial) configuration $\Omega_s^\varepsilon$, the flow in the deformed pores $\tilde{\Omega}_f^\varepsilon(\boldsymbol{u}^\varepsilon)$,

$$\tilde{\Omega}_f^\varepsilon(\boldsymbol{u}^\varepsilon) = \{\boldsymbol{z} \in \mathbb{R}^3 | \boldsymbol{z} = \boldsymbol{x} + \boldsymbol{u}^\varepsilon(\boldsymbol{x}),\; \boldsymbol{x} \in \Omega_f^\varepsilon\}\;, \tag{3}$$

must be respected to comply with the unilateral contact on $\Gamma_c^\varepsilon$. The variational formulation reads, as follows: Find $(\boldsymbol{u}^\varepsilon, p^\varepsilon, \boldsymbol{w}^\varepsilon) \in \mathcal{K}^\varepsilon \times L^2(\tilde{\Omega}_f^\varepsilon) \times W(\tilde{\Omega}_f^\varepsilon)$ satisfying

$$
\begin{aligned}
a_\Omega^\varepsilon(\boldsymbol{u}^\varepsilon, \boldsymbol{v}^\varepsilon - \boldsymbol{u}^\varepsilon) + \mathcal{I}^\varepsilon(\boldsymbol{\sigma}_f^\varepsilon, \boldsymbol{v}^\varepsilon - \boldsymbol{u}^\varepsilon) \geq \int_{\Omega_s^\varepsilon} \boldsymbol{f}^\varepsilon \cdot (\boldsymbol{v}^\varepsilon - \boldsymbol{u}^\varepsilon) \;, \quad \forall \boldsymbol{v}^\varepsilon \in \mathcal{K}^\varepsilon \;, \\
\varepsilon^2 \int_{\tilde{\Omega}_f^\varepsilon} \bar{\mu} \nabla \boldsymbol{w}^\varepsilon \cdot \nabla \boldsymbol{\vartheta}^\varepsilon - \int_{\tilde{\Omega}_f^\varepsilon} (\nabla p^\varepsilon - \boldsymbol{f}^\varepsilon) \cdot \boldsymbol{\vartheta}^\varepsilon = 0 \;, \quad \forall \boldsymbol{\vartheta}^\varepsilon \in W(\tilde{\Omega}_f^\varepsilon) \;, \\
\nabla \cdot \boldsymbol{w}^\varepsilon = 0 \quad \text{a. e. in } \tilde{\Omega}_f^\varepsilon \;,
\end{aligned}
\tag{4}
$$

where $a_\Omega^\varepsilon(,)$ is the elastic bilinear form and the interaction integral is established using the stress in fluid $\boldsymbol{\sigma}_f^\varepsilon = -p^\varepsilon \boldsymbol{I} + \varepsilon^2 2\bar{\mu} \boldsymbol{e}(\boldsymbol{w}^\varepsilon)$,

$$
a_\Omega^\varepsilon(\boldsymbol{u}, \boldsymbol{v}) = \int_{\Omega_s^\varepsilon} \mathbb{D}^\varepsilon \boldsymbol{e}(\boldsymbol{u}) : \boldsymbol{e}(\boldsymbol{v}) \;, \quad \mathcal{I}^\varepsilon(\boldsymbol{\sigma}_f^\varepsilon, \boldsymbol{v}^\varepsilon) = \int_{\Gamma_{fs}^\varepsilon} \boldsymbol{n}^{[\mathrm{s}]} \cdot \boldsymbol{\sigma}_f^\varepsilon \cdot \boldsymbol{v}^\varepsilon \;. \tag{5}
$$

## 3. Homogenized porous medium with self-contact at pore level

For the structures with fluid saturated disconnected pores, the homogenized limit problem attains the same form as the one derived for the structures without fluid (empty pores), although the effective tangent stiffness modulus involved in the incremental formulation reflects the fluid action. Henceforth, we focus on the model describing the quasistatic response of the homogenized medium with connected pores. We denote by $\boldsymbol{u}^0$ and $p^0$ the macroscopic displacement and pressure fields, respectively, and by $\boldsymbol{u}^1$ and $p^1$ the two scale counterparts of these fields, being $Y$-periodic functions in the micro-variable $y \in Y$. These constitute the truncated asymptotic expansions introduced using the unfolding operator $\mathcal{T}_\varepsilon()$, see [1], for $x \in \Omega$ and $y \in Y$,

$$
\begin{aligned}
\mathcal{T}_\varepsilon(\boldsymbol{u}^\varepsilon(x)) &= \boldsymbol{u}^0(x) + \varepsilon \boldsymbol{u}^1(x,y) + \varepsilon^2(\dots \;, \\
\mathcal{T}_\varepsilon(p^\varepsilon(x)) &= p^0(x) + \varepsilon p^1(x,y) + \varepsilon^2(\dots \;, \\
\mathcal{T}_\varepsilon(\boldsymbol{w}^\varepsilon(x)) &= \hat{\boldsymbol{w}}(x,y) + \varepsilon(\dots \;, \qquad \text{where } \hat{\boldsymbol{w}}(x,\cdot) = 0 \text{ in } \overline{Y_s} \;.
\end{aligned}
$$

Admissible two-scale displacements must satisfy $\boldsymbol{u}^1 \in \mathcal{K}_Y(\nabla \tilde{\boldsymbol{u}}^0)$ where the set $\mathcal{K}_Y$ is defined using the gap function $g_c^Y(\boldsymbol{u}^1, \nabla \boldsymbol{u}^0) = [\nabla \boldsymbol{u}^0 \widehat{y} + \boldsymbol{u}^1 - \widehat{y}]_n^Y \leq 0$ with $\widehat{y} \in \Gamma_c$, where $\Gamma_c \subset \Gamma_{fs}$ is the contact surface, a part of the pore wall $\Gamma_{fs}$. The limit two-scale problem with quasistatic flow is derived from Problem (3)-(4). It involves Local problems defined in $Y$ for a.a. $x \in \Omega$, and the Global problem defined in $\Omega$.

The Local problem describes the FSI problem with the unilateral contact and with the Stokes flow in deformed pores $\tilde{Y}_f$,

$$
\begin{aligned}
\fint_{Y_s} a_{Y_S}\left(\boldsymbol{u}^1 + \boldsymbol{\Pi}^{ij} e_{ij}^x(\boldsymbol{u}^0),\, \boldsymbol{v} - \boldsymbol{u}^1\right) + p^0 \fint_{Y_s} \nabla_y \cdot (\boldsymbol{v} - \boldsymbol{u}^1) \geq 0 \;, \quad \forall \boldsymbol{v} \in \mathcal{K}_Y(\nabla \boldsymbol{u}^0) \;, \\
\bar{\mu} \fint_{\tilde{Y}_f} \nabla_y \hat{\boldsymbol{w}} \cdot \nabla_y \hat{\boldsymbol{v}} + \fint_{\tilde{Y}} (\nabla_y p^1 + \nabla_x p^0 - \boldsymbol{f}^f) \cdot \hat{\boldsymbol{v}} = 0 \;, \forall \hat{\boldsymbol{v}} \in \mathbf{H}_{\#0}^1(\tilde{Y}_f) \;, \\
\fint_{\tilde{Y}_f} q \nabla_y \cdot \hat{\boldsymbol{w}} = 0 \;, \forall q \in L^2(\tilde{Y}_f) \;,
\end{aligned}
\tag{6}
$$

where $\bar{\boldsymbol{u}} = \boldsymbol{\Pi}^{ij} e_{ij}^x(\boldsymbol{u}^0)$ is the displacement field in $Y_s$ produced by the homogeneous strain $\boldsymbol{e}_x(()\boldsymbol{u}^0)$ with $\Pi_k^{ij} = \delta_{ik} y_j$ and the Sobolev space $\mathbf{H}_{\#0}^1$ contains $Y$-periodic functions with zero traces on the pore wall $\Gamma_{fs}$. The elastic bilinear form $a_{Y_S}(,)$ is defined in analogy with the one introduced in (5), but using strains $\boldsymbol{e}_y(\;)$ and domain $Y_s$ in the periodic cell $Y$. The Global problem is constituted by the static equilibrium and by the Darcy flow involving the permeability $\tilde{\boldsymbol{K}}$, thus,

$$
\begin{aligned}
\int_\Omega \tilde{\boldsymbol{K}}(\nabla_x p^0 - \boldsymbol{f}^f) \cdot \nabla q = 0 \;, \forall q \in Q_0(\Omega) \;, \\
\int_\Omega a_{Y_S} \left( \boldsymbol{u}^1 + \boldsymbol{\Pi}^{ij} e^x_{ij}(\boldsymbol{u}^0),\, \tilde{\boldsymbol{v}}(\boldsymbol{v}^0) + \boldsymbol{\Pi}^{ij} e^x_{ij}(\boldsymbol{v}^0) \right) \\
- \int_\Omega p^0 \left( \phi_f \nabla_x \cdot \boldsymbol{v}^0 - \fint_s \nabla_y \cdot \tilde{\boldsymbol{v}}(\boldsymbol{v}^0) \right) = \int_\Omega \bar{\boldsymbol{f}} \cdot \boldsymbol{v}^0 \;, \forall \boldsymbol{v}^0 \in U_0(\Omega) \;,
\end{aligned}
\tag{7}
$$

whereby the test displacement field $\tilde{\boldsymbol{v}}(\boldsymbol{v}^0)$ must satisfy $g_c^Y(\tilde{\boldsymbol{v}}, \nabla \boldsymbol{v}^0) = 0$ on the actual contact set $\Gamma^*$ defined a.e. in $\Omega$.

The permeability tensor $\tilde{\boldsymbol{K}}$ depends on the deformation by virtue of the deformed pores $\tilde{Y}_f$ in the local reference cell $Y(x)$. A regularization is considered to prevent a complete closing of the pore at the vicinity of the active contact, *i.e.* where $g_c^Y = 0$. This enables to preserve the well posedness of the local flow problem $(6)_{2,3}$ and, by the consequence, to rely on a strict posive definiteness of $\tilde{\boldsymbol{K}}$, though possibly very small. The permeability dependence on the deformation of $Y_f$ is treated approximately using the sensitivity analysis approach [6], thus $\tilde{\boldsymbol{K}} \approx \boldsymbol{K}^0 + \delta \boldsymbol{K} = \partial_p \boldsymbol{K} \delta p^0 + \partial_{\boldsymbol{e}} \boldsymbol{K} : \boldsymbol{e}_x(\delta \boldsymbol{u}^0)$ at $x \in \Omega$. The two-scale algorithm proposed in [5] can be adapted. The macroscopic increments $(\delta \boldsymbol{u}^0, \delta p^0)$ driven by the out of balance are computed with the "fixed sliding contact" due to active contact sets (local true contact surfaces identified), which modifies the effective macroscopic tangent stiffness.


## Acknowledgement

The research has been supported by the grant project GA 22-00863K of the Czech Science Foundation.


## References


[1] Cioranescu, D., Damlamian, A., Griso, G., The periodic unfolding method in homogenization, SIAM Journal on Mathematical Analysis 40 (4) (2008) 1585-1620.

[2] Cioranescu, D., Damlamian, A., Orlik, J., Homogenization via unfolding in periodic elasticity with contact on closed and open cracks, Asymptotic Analysis 82 (3-4) (2013) 201-232.

[3] De Luca, T., Facchinei, F., Kanzow, C., A semismooth equation approach to the solution of nonlinear complementarity problems, Mathematical Programming 75 (3) (1996) 407-439.

[4] Griso, G., Migunova, A., Orlik, J., Homogenization via unfolding in periodic layer with contact, Asymptotic Analysis 99 (1-2) (2016) 23-52.

[5] Rohan, E., Heczko, J., Homogenization and numerical modelling of poroelastic materials with self-contact in the microstructure, Computers & Structures 230 (2020) No. 106086.

[6] Rohan, E., Lukeš, V., Modeling nonlinear phenomena in deforming fluid-saturated porous media using homogenization and sensitivity analysis concepts, Applied Mathematics and Computation 267 (2015) 583-595.