

## Comparison of Simulation of a Linear Transmission Line with the Nonlinear Load

R. Kadlec, J. Dědková, M. Steinbauer

Department of Theoretical and Experimental Electrical Engineering,  
Faculty of Electrical Engineering and Communication,  
Brno University of Technology, Kolejní 2906/4, Brno 61200

E-mail: xkadle16@stud.feec.vutbr.cz, dedkova@feec.vutbr.cz, steinbau@feec.vutbr.cz

### Annotation:

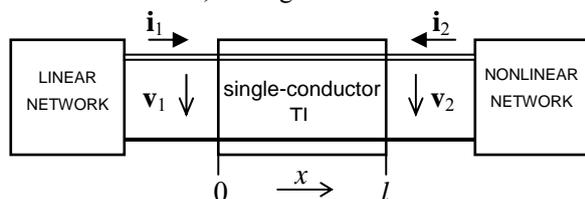
The paper deals with the numerical modeling of voltage and current waveforms on transmission lines with nonlinear loads. It will be considered the linear transmission line only. The propagation of the voltage and current waveforms is solved numerically in the time domain. The new numerical approach based on the Finite Difference method in Time Domain (FDTD) was used and the new algorithm was programmed in Matlab language. There are presented illustrative examples. The results obtained using described simulation demonstrate applicability and suitability of the proposed method and can give as important information for a design of new MTL.

### Anotace:

V článku je prezentována problematika numerického modelování vln proudu a napětí na přenosovém vedení, které je zakončeno zátěží nelineárního charakteru. Časoprostorové rozložení napěťové a proudové vlny je výsledkem numerické analýzy; je uvažováno pouze lineární jednovodičové vedení. Jako nástroj pro tuto analýzu byla použita jednoduchá a dobře známá metoda konečných diferencí. Princip navrženého algoritmu spočívá ve vhodné aproximaci časových a prostorových derivací ve výchozím diferenciálním matematickém popisu. Algoritmus byl naprogramován v jazyku Matlab. Výsledky jsou demonstrovány na ilustrativních příkladech a potvrzují použitelnost a korektnost navrženého algoritmu, který nabízí možnost získat informace důležité například při návrhu nových typů vedení.

## ÚVOD

There are several possibilities how to effectively simulate the voltage and current distributions along the single or multiconductor transmission line. Let us suppose a simple TL linear system consisting of a single-conductor TL terminated at the near end (left, 1) by linear network and at the far end, (right, 2) by the nonlinear load, see Fig. 1.



Obr. 1: Simple model of transmission line

Further let us consider homogenous lossy TL with primary parameters  $R_0$ ,  $L_0$ ,  $G_0$ ,  $C_0$  and length  $l$  (resistance, inductance, conductance, capacitance per-unit-length and total length of TL). The basic TL equations can be expressed as

$$-\frac{\partial \mathbf{v}(t, x)}{\partial x} = R_0 \mathbf{i}(t, x) + L_0 \frac{\partial \mathbf{i}(t, x)}{\partial t}, \quad (1a)$$

$$, -\frac{\partial \mathbf{i}(t, x)}{\partial x} = G_0 \mathbf{v}(t, x) + C_0 \frac{\partial \mathbf{v}(t, x)}{\partial t} \quad (1b)$$

where  $\mathbf{v}(t, x)$  and  $\mathbf{i}(t, x)$  are column vectors of instantaneous voltages and currents of the TL wire at a distance  $x$  from TL's left end.

There are different approaches to solve equations (1a), (1b) in nowadays. These equations will be treated in the  $s$ -domain after the Laplace transform is applied, and then a proper method for numerical inversion of Laplace transform will be used to get the required time-domain solution. In principle, both one- and multi-dimensional Laplace transforms can be utilized for this purpose see e.g. [1], [2]. Unfortunately, the Laplace transform approach does not allow considering a nonlinear multiconductor transmission lines (MTL) in general. On the other hand, it is relatively easy to incorporate boundary conditions defined by terminating networks just in the  $s$ -domain.

One of other approaches how to simulate nonlinear case of TL is described in [3]. This way is based on using of the convolution integral as an interface between the models of a linear TL and its nonlinear termination. This is a very effective method; however, it is not applicable to simulation of nonlinear MTL.

For a numerical solution of the above described wave equation system (1a), (1b) the widely known FDTD method can also be used. The main aim is to approximate the temporal and spatial derivatives by the suitable difference expression, which ensures the

best stability and the highest accuracy of the numerical solution. There are several possibilities how to replace the above mentioned derivatives. One of them is to use the implicit Wendorff formula in which the derivatives of voltages (currents) are replaced by a combination of both forward and backward differences. Some interesting results based on the application of the mentioned formula can be found in [4], where practical examples of a numerical modeling of the surge phenomena on transmission lines caused by the lightning stroke and on  $h\nu$  and  $\nu h\nu$  three phase TLs with earth wire are presented.

## FDTD ANALYSIS OF LOSSY TRANSMISSION LINE

The aim of our investigation was to find an effective algorithm for numerical simulation of the current and voltage wave propagation on linear TL with nonlinear load. The best solution was obtained by using so-called leapfrog method, when the spatial and temporal derivatives were replaced by the combination of both central and forward differences; this approach was introduced in [5]. This modification of FDTD was discussed for example in [6] and it was applied to the numerical simulation of electromagnetic wave propagations in a free space.

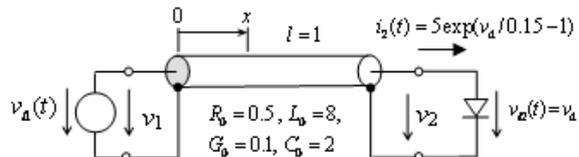
To simulate the current and voltage distributions  $\mathbf{v}(t, x)$  and  $\mathbf{i}(t, x)$  along linear TL with nonlinear load in arbitrary time  $t = n\Delta t$ , the implicit formula can be expressed in a compact matrix form

$$\begin{bmatrix} \mathbf{v}(x_1, \dots, x_{M+1}) \\ \mathbf{i}(x_1, \dots, x_{M+1}) \end{bmatrix}^{n+1} = \mathbf{A}^{-1} \left( \mathbf{B}(V_2, I_2) \begin{bmatrix} \mathbf{v}(x_1, \dots, x_{M+1}) \\ \mathbf{i}(x_1, \dots, x_{M+1}) \end{bmatrix}^n + \mathbf{D}(V_2, I_2) \right) \quad (2)$$

The coefficients of matrices  $\mathbf{A}$  and  $\mathbf{B}$  are given by the per-unit-length matrices, the matrix  $\mathbf{D}$  is given by sources. For the nonlinear case the matrices  $\mathbf{B}$  and  $\mathbf{D}$  are dependent on the instantaneous current and voltage values at the far end of TL,  $V_2 = \mathbf{v}(x_{M+1})$  and  $I_2 = \mathbf{i}(x_{M+1})$ .

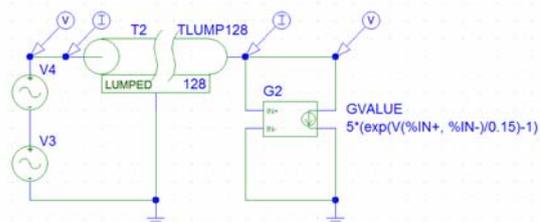
## EXAMPLES OF NONLINEAR TL SIMULATIONS

As an example a section of lossy TL supplied at the near end from voltage source by one period of sinusoidal voltage is shown. The TL is terminated by nonlinear resistor with an exponential current-voltage characteristic, see Fig. 2. The boundary conditions were introduced to be in accordance with voltage source and nonlinear load at both ends of TL.



Obr. 2: Transmission line with nonlinear load

In order to show the correctness of above mentioned FDTD approach, the simulation has been done in Orcad 9.2 PSpice according to the schema in Fig. 3.

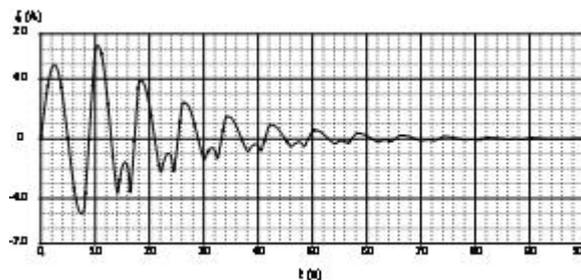
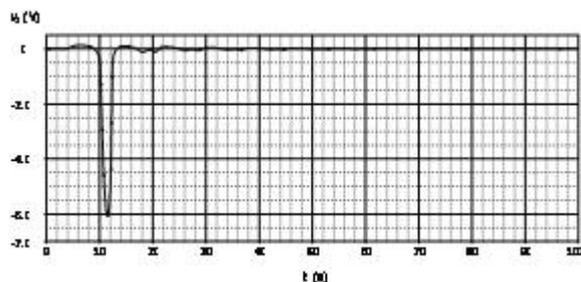
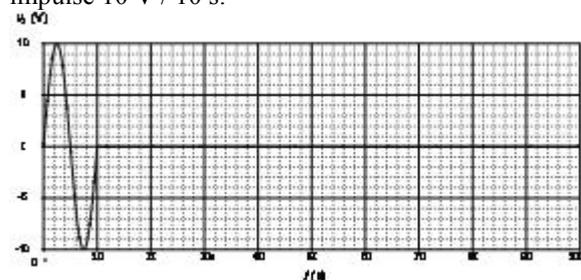


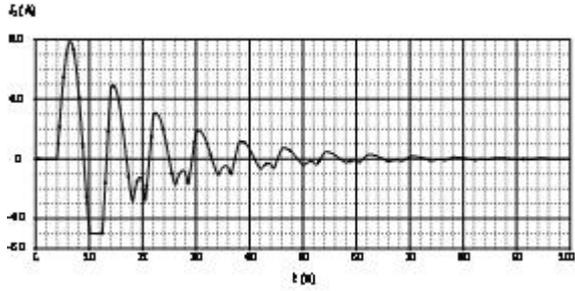
Obr. 3: Circuit schema for simulation in Orcad 9.2 PSpice

In Fig. 4 the waveforms of voltage and current at the near and at the far end of the TL are shown. The results obtained from Orcad 9.2 PSpice correspond very well to results obtained using FDTD method.

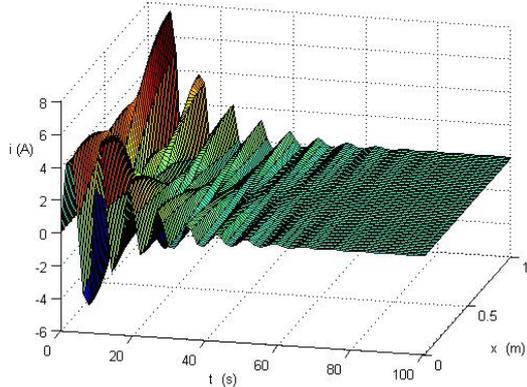
In Fig. 5 you can see the spatial distribution of the waveform of the current along transmission line which was obtained using FDTD method.

Other example of FDTD analysis of TL with nonlinear load is presented in Fig. 6. In this case the TL from Fig. 2 is supplied at the near end with square impulse 10 V / 10 s.

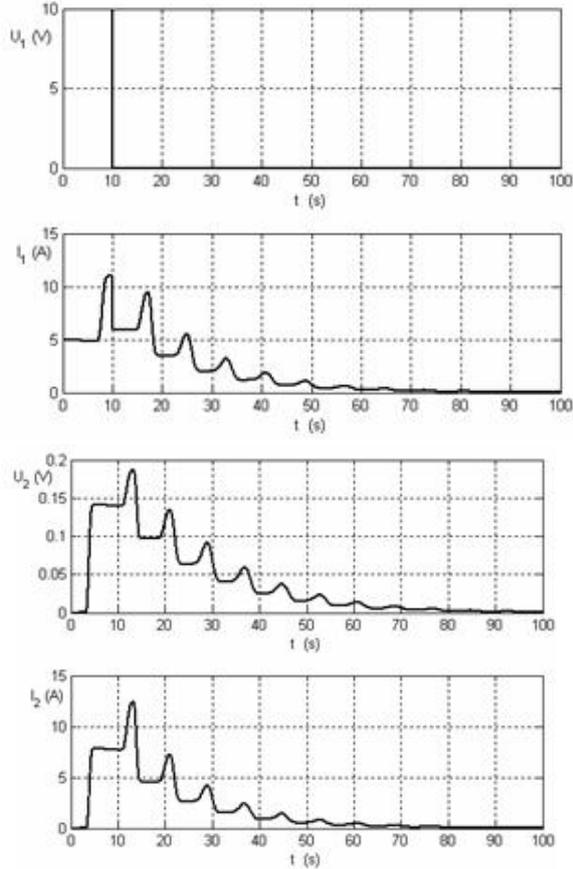




Obr. 4: Numerical results obtained by both Spice simulation and FDTD



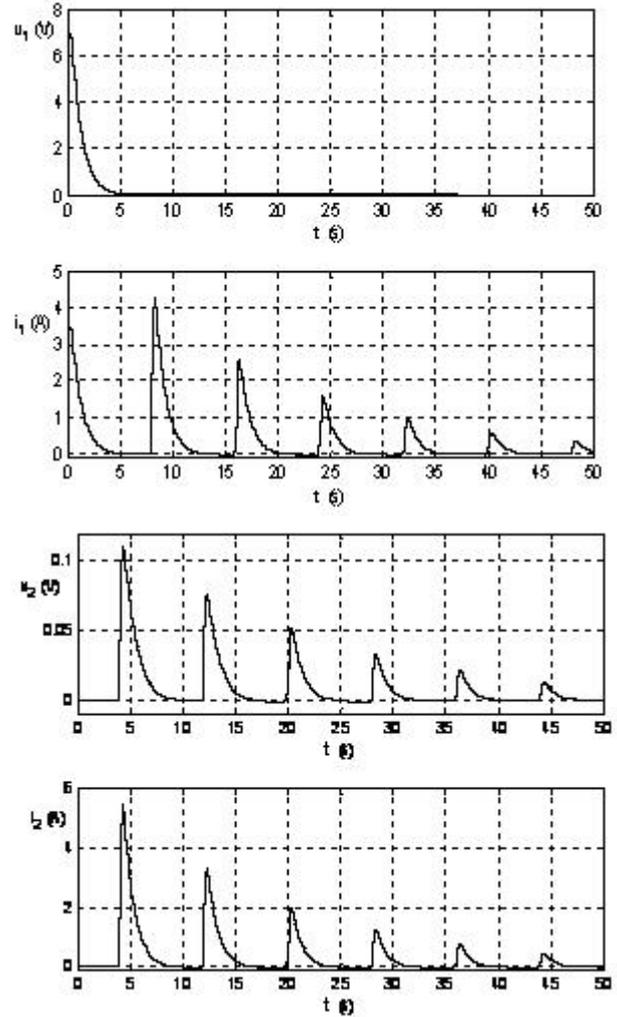
Obr. 5: Spatial waveform of current obtained using FDTD



Obr. 6: Current and voltage distributions obtained by FDTD  
Other example of FDTD analysis is presented by the waveforms of voltage and current at the near and at

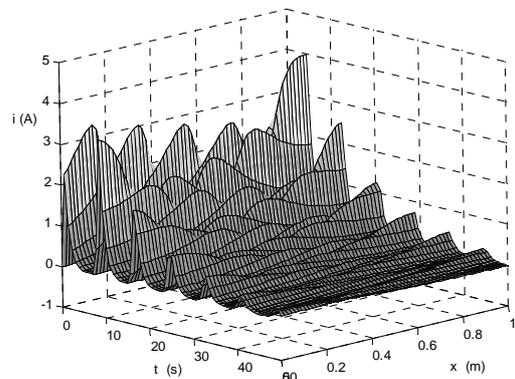
the far end of the TL in Fig. 7. In this case the TL is actuated by voltage with impulse wave

$$v_1(t) = 10[\exp(-t) - \exp(-10t)] \text{ V} \quad (3)$$



Obr. 7: Current and voltage distributions actuated by impulse wave

In Fig. 8 you can see the spatial distribution of the waveform of the current along transmission line actuated by voltage with impulse wave.



Obr. 8: Spatial waveform of current obtained using FDTD

The results obtained from Orcad 9.2 PSpice correspond very well to results obtained using FDTD method.

## CONCLUSION

The FDTD approach can handle both linear and nonlinear cases of transmission line. It is possible also to enlarge this numerical method to the simulation of nonlinear multiconductor transmission line. Finally, among many other methods, the FDTD approach seems to be very well applicable in general cases.

## ACKNOWLEDGMENTS

The research described in the paper was financially supported by the research programs MSM 0021630513.

## REFERENCES

- [1] Valsa J., Brančík L. Approximate Formulae for Numerical Inversion of Laplace Transform. *International Journal of Numerical Modeling. Electronic Networks, Devices and Fields*, Vol. 11, 1998.
- [2] Griffith J. R., Nakhla M. S. Time-domain Analysis of Lossy Coupled Transmission Lines. *IEEE Transaction on Microwave Theory and Techniques*. Vol. 38, No. 10, Oct. 1990.
- [3] Orel T., Valsa J. An efficient numerical calculation of the convolution integral as an interface between the models of a linear transmission line and its nonlinear terminations. *21st seminar on fundamentals of electrotechnics and circuit theory*. Gliwice, 1998.
- [4] Benešová Z., Kotlan V. Propagation of surge waves on interconnected transmission lines induced by lightning stroke. *Acta Technica IEE CSAV*, Vol. 51, No. 3, 2006.
- [5] Dědková J., Brančík L. Laplace Transform and FDTD Approach Applied to MTL Simulation. *Proceedings of International Conference PIERS, Hangzhou, China, March 2008*.
- [6] Sullivan D. M. *Electromagnetic simulation using the FDTD method*. IEEE Press, New York, 2000.