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Temperature influence on the acoustic streaming of viscous fluid in a confining layer

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Acoustic streaming (AS) is a secondary effect of acoustic waves propagating in a bulk fluid, or near surfaces. The AS is presented as a quasi-stationary flow propelled by the attenuated energy of the acoustic waves caused by the fluid viscosity. More precisely, AS, results from the inhomogeneities in viscous flow due to non-zero divergence of the Reynolds stress associated with the kinetic energy of the velocity fluctuations. It can be induced by vibrating solid-fluid interface while considering laminar flow in confining channels [3]. In this study, we examine the influence of inhomogeneities in the temperature field on the AS and on the consequent heat transfer from the fluid to the solid, cf. [1, 2].

The AS model can be derived from the mass, momentum and energy conservation laws which yield the coupled set of equations

$$\frac{D\rho}{Dt} = -\rho\nabla\cdot\boldsymbol{v}\;, \quad \rho\frac{D\boldsymbol{v}}{Dt} = \nabla\cdot\boldsymbol{\sigma}\;, \quad \rho C_V\frac{D\Theta}{Dt} = \nabla\cdot(k\nabla\Theta) + \boldsymbol{\sigma} : \nabla\boldsymbol{v}\;. \tag{1}$$

In (1), t is the time, $\boldsymbol{v}$, ρ and Θ denote the fields of velocity, density and temperature, respectively, C_V is the specific heat capacity at constant volume, k is the thermal conductivity, and $\boldsymbol{\sigma} = -p\boldsymbol{I} + \boldsymbol{\tau}$ is the Cauchy stress which splits into a pressure term and a viscous part. In addition, the (ideal gas) state equation closes the system (1),

$$p = R\Theta\rho\;. \tag{2}$$

To distinguish the phenomenon of AS, flow equations (1) can be either solved directly or in a decomposed form obtained from the perturbation analysis [4], i.e., the expansion of any state variable u with respect to a perturbation parameter α,

$$u(\boldsymbol{x},t) = u_0(\boldsymbol{x}) + \alpha u_1(\boldsymbol{x},t) + \alpha^2 u_2(\boldsymbol{x},t) + \ldots\;, \tag{3}$$

with $\alpha \approx v_0/c_0$, where c_0 is the reference sound speed and v_0 is a characteristic flow velocity ($v_0 \ll c_0$). Upon substituting (3) in (1) and pursuing the standard split according to orders in α, two problems for the triplets $(\rho_1, \boldsymbol{v}_1, \Theta_1)$ and $(\rho_2, \boldsymbol{v}_2, \Theta_2)$ are identified being governed by the following linear equations (written in a generic form where $i = 1, 2$ refers to the 1st, or the 2nd order problem)

$$\begin{aligned} \frac{\partial\rho_i}{\partial t} + \rho_0\nabla\cdot\boldsymbol{v}_i &= M_i\;, \\ \rho_0\frac{\partial\boldsymbol{v}_i}{\partial t} + \nabla p_i - \mu_0\nabla^2\boldsymbol{v}_i - \left(\eta_0 + \frac{1}{3}\mu_0\right)\nabla(\nabla\cdot\boldsymbol{v}_i) &= \boldsymbol{F}_i\;, \\ C_V\left(\rho_0\frac{\partial\Theta_i}{\partial t} + \rho_0\boldsymbol{v}_i\cdot\nabla\Theta_0\right) - k_0\nabla^2\Theta_i - \nabla\cdot(k_i\nabla\Theta_0) + p_0(\nabla\cdot\boldsymbol{v}_i) &= Q_i\;, \end{aligned} \tag{4}$$

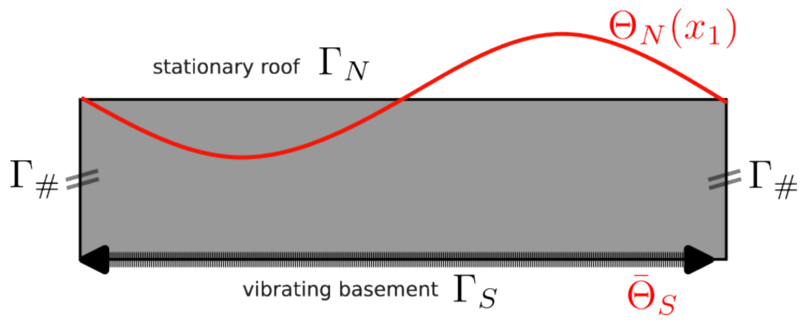

Fig. 1. The domain Ω with vibrating basement and prescribed walls temperature

supplemented by the corresponding state equations obtained from (2) using the expansions (3). The second order problem involves functions $(\rho_2, \boldsymbol{v}_2, \Theta_2)$ which are considered as time-averaged over a time-period T which equals the one of the acoustic waves; this makes disappear all T-periodic fluctuations present in the linear form and time derivatives of expressions involving the 1st order problem responses. In (4), the r.h.s. terms of the 1st order problem $i = 1$, vanish ($M_1 = 0$, $\boldsymbol{F}_1 = \boldsymbol{0}$, $Q_1 = 0$), whereas the following holds for driving forces of the acoustic streaming governed by the 2nd order problem ($\langle\cdot,\cdot\rangle$ denotes the time average),

$$
\begin{aligned}
M_2 &= -\left\langle \nabla \cdot (\rho_1 \boldsymbol{v}_1) \right\rangle, \\
\boldsymbol{F}_2 &= -\left\langle \nabla \rho_0 (\boldsymbol{v}_1 \otimes \boldsymbol{v}_1) \right\rangle + \left\langle \nabla \cdot \left(\Theta_1 \left(d_\mu (\nabla \boldsymbol{v}_1 + (\nabla \boldsymbol{v}_1)^T) + \left(d_\eta - \frac{2}{3} d_\mu \right) (\nabla \cdot \boldsymbol{v}_1) \boldsymbol{I} \right) \right) \right\rangle, \\
Q_2 &= -C_V \left\langle \nabla \cdot (\rho_0 \boldsymbol{v}_1 \Theta_1) + \rho_1 \boldsymbol{v}_1 \cdot \nabla \Theta_0 \right\rangle + \left\langle d_k \nabla \cdot (\Theta_1 \nabla \Theta_1) \right\rangle - \left\langle p_1 (\nabla \cdot \boldsymbol{v}_1) \right\rangle + \left\langle \boldsymbol{\tau}_1 : \nabla \boldsymbol{v}_1 \right\rangle.
\end{aligned}
\tag{5}
$$

It is worth to emphasize that the material parameters are temperature dependent except of the specific heat capacity which is assumed to be constant [1]. In (5), the Taylor expansion has been used and $d_u = \frac{\partial u}{\partial \Theta}|_{\Theta_0}$ is applied with $u := \mu, \eta$.

The flow equations (1) (or (4)) are solved in a two-dimensional rectangular domain $\Omega =]0, L[\times]0, h[\subset \mathbb{R}^2$ representing a section of the infinite layer $]-\infty, +\infty[\times]0, h[$ and shown in Fig. 1. The domain Ω is bounded by $\partial\Omega$ consisting of four parts Γ_N, Γ_S and $\Gamma_\#$. Periodic conditions are prescribed on the vertical boundary segments $\Gamma_\#$. The flow is induced by harmonic oscillations of the south wall Γ_S whereas fixed north wall Γ_N is considered. The temperature at both walls is prescribed; they are stationary. Hence,

$$
\begin{aligned}
\boldsymbol{v}|_{x_2=0} &= \boldsymbol{v}_S(x_1) \sin\left(\frac{2\pi t}{T}\right), \quad & \Theta|_{x_2=0} &= \Theta_S(x_1)\,, \\
\boldsymbol{v}|_{x_2=h} &= 0\,, & \Theta|_{x_2=h} &= \Theta_N(x_1)\,,
\end{aligned}
\tag{6}
$$

where $\boldsymbol{v}_S(x_1)$, $\Theta_S(x_1)$ and $\Theta_N(x_1)$ are given amplitudes. For illustration, in Fig. 2, $\boldsymbol{v}_S(x_1) = \bar{\boldsymbol{v}}_s \sin(2\pi(x_1 - x_0)/L))$, $\Theta_S(x_1) = \bar{\Theta}_S$ and (a) $\Theta_N(x_1) = \bar{\Theta}_S + \Delta\bar{\Theta}$ or (b) $\Theta_N(x_1) = \bar{\Theta}_S(1 + 1/2 \sin(2\pi(x_1 - x_0)/L))$. The influence of the the temperature gradient on the distribution of the AS is tested performing numerical simulation with (a) various values of the difference $\Delta\bar{\Theta}$ or (b) various positions x_0. The distribution (AS magnitude and streamlines) are shown is the whole domain Ω (left figure) and also distribution along a horizontal line probe $x_2 = h/2$ are plotted (right figures). In Fig. 2a), one may recognize in the middle figure which corresponds to the zero gradient $\Delta\bar{\Theta} = 0$, the solution observed for barotropic material [3]. The temperature gradient can accentuate or mitigate the AS effects.

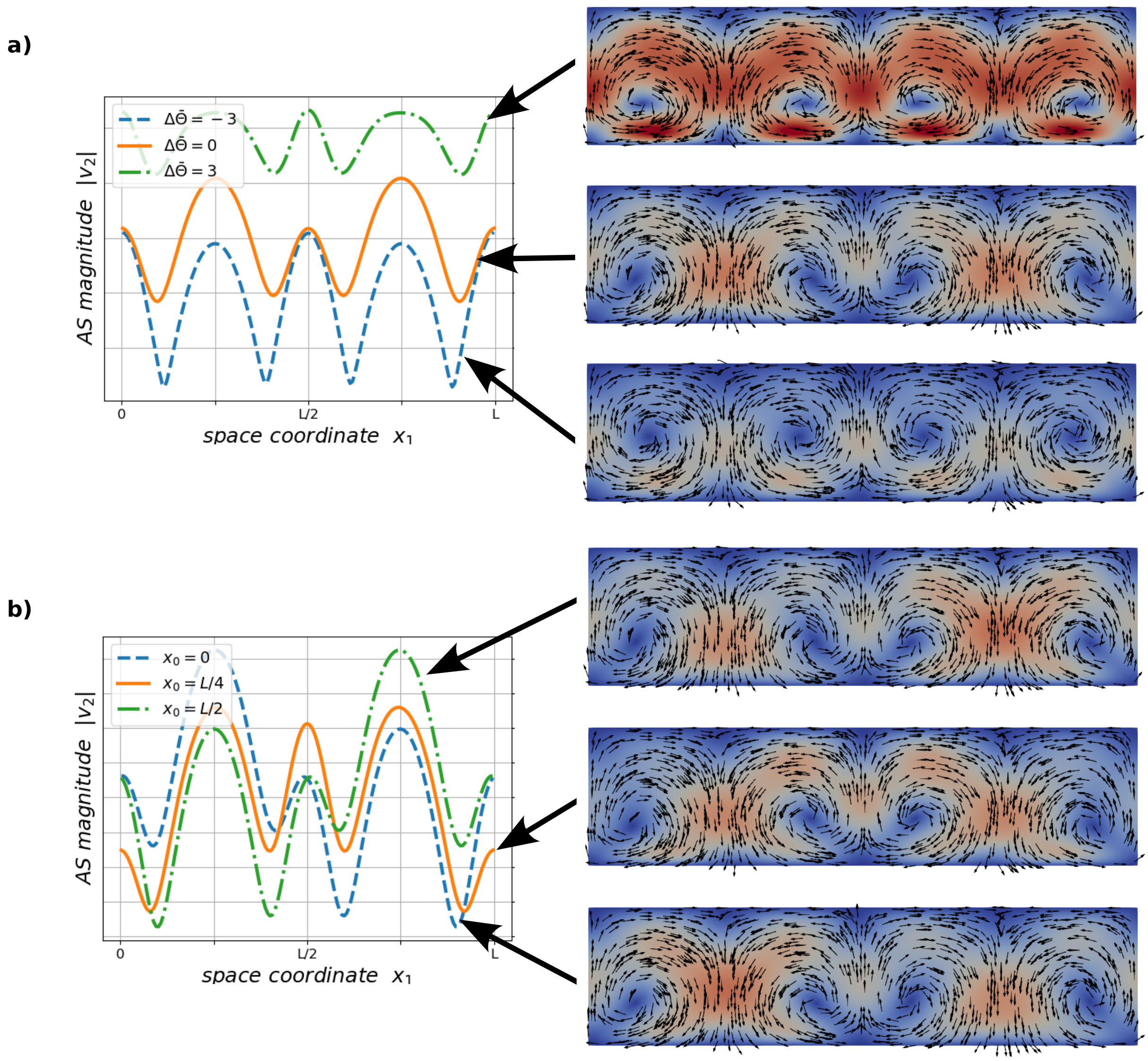

Fig. 2. Distribution of the streaming velocity $|\boldsymbol{v}_2|$ along a horizontal line ($x_2 = h/2$) and in the whole domain Ω, two cases considered with the temperature prescribed on Γ_N (a) by constant different from the one on Γ_S, (b) by a sinus function

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