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Calculation of the stresses in the tapered FGM beams with varying stiffness

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1. Introduction

In the contribution, the expressions will be presented for semi-analytical calculation of normal and shear stresses in the tapered Functionally Graded Material (FGM) beams with spatially variable stiffness. This variability will be caused by the longitudinal continuous variation of the solid cross-section and spatial variation of material properties. These expressions will be applied to calculate stresses in the cantilever beam with longitudinally variable square cross-section, while the material properties change continuosly in its three main axes. The beam will be loaded in tension, biaxial bending, and pure torsion. The results of the semi-analytical solution will be compared with the results of the numerical solution with 3D solid finite elements. Due to limited scope of this short article, we will present here only the expressions for calculation of the normal stresses caused by the axial force. The complete processing of the solved problem in this contribution will be carried out in the prepared article for scientific journal.

2. Calculation of the normal and shear stresses in the tapered FGM beam.

In Fig. 1a the tapered FGM beam with spatial variability of material properties is shown. Using selected homogenization methods, we obtain homogenized FGM beam with longitudinal variability of effective material properties – Fig. 1b.

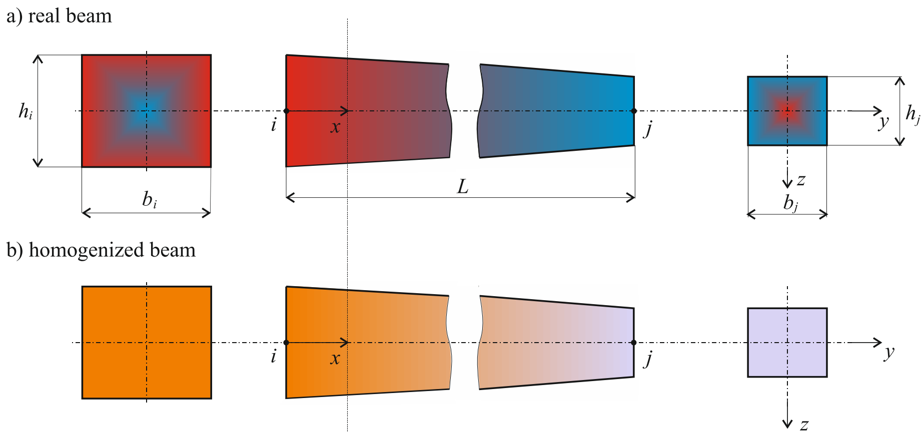
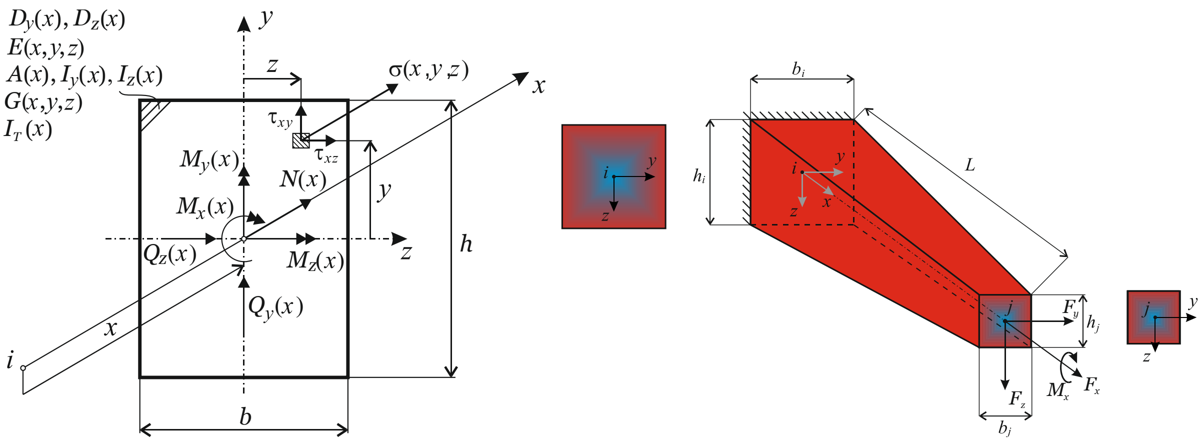


Fig. 1. Tapered FGM beam: a) real beam and b) homogenized beam

Elastostatic deformation analysis of FGM beams (calculation of displacements and rotation angles as well as the internal forces and moments) can be performed on a beam with homogenized stiffnesses [3]. However, the stress calculation must be performed on a real beam [4].

The expressions for the stress calculation in the real FGM tapered beam depend among other things also on the cross-sectional area geometry and material properties variation. FGM is created by mixing two or more components so that at each material point the material is isotropic, while its properties change in one, two or three directions.

In the presentation, at the CM2023 conference, we will focus on derivation of the expression for calculation of the normal and shear stresses distribution on an arbitrary doubly symmetric cross-sectional area of the FGM beam with spatial variation of material properties. As an example of such cross-section is the rectangle (Fig. 2a).



a) b)

Fig. 2. To calculating the stresses in the tapered FGM beam

The tapered FGM cantilever beam with doubly symmetric rectangular cross-section at position x with doubly symmetric spatial variation of material properties is shown in Fig. 2b. At the centroid of the cross-sectional area at position x acts the normal force $N(x)$, the bending moments $M_y(x)$ and $M_z(x)$, which cause the normal stresses. The internal torsional moment is $M_x(x)$. Further, $A(x)$ is the cross-sectional area, $I_z(x)$ and $I_y(x)$ are the quadratic moments of the cross-sectional area, and $I_T(x)$ is the torsion constant.

Considered spatial variability of material properties is shown in Fig. 2b using a colour scale. On the neutral axis x of the beam is Aluminium and on the outer longitudinal surfaces of the beam is Tungsten. In each cross-section of the beam, the properties in the transverse and lateral directions change linearly from Aluminium to Tungsten. The longitudinally varying effective stiffnesses for tension-compression, bending, shear, and torsion have been obtained by the Reference Beam Method (RBM) [2].

Due to limited scope of this short article, we will present here only the expressions for calculation of the normal stresses caused by the axial force $N(x) = F_x = 100$ N. The results of their application will be presented as well, while a verification of the semi analytical results will be done by the ones obtained with the SOLID185 finite elements [1].

Considering the variation of material properties as shown in Fig. 2b, and with dimensions of the tapered beam with length $L = 0.1$ m, and $a_i = h_i = b_i = 0.01$ m at node i and $a_j = h_j = b_j = 0.005$ m at node j, the effective axial stiffness has been obtained [2]

$$EA(x) = 3.43 \times 10^7 - 3.43 \times 10^8 x + 8.57 \times 10^8 x^2 - 1.11 \times 10^{-3} x^3 \text{ [N]}. \quad (1)$$

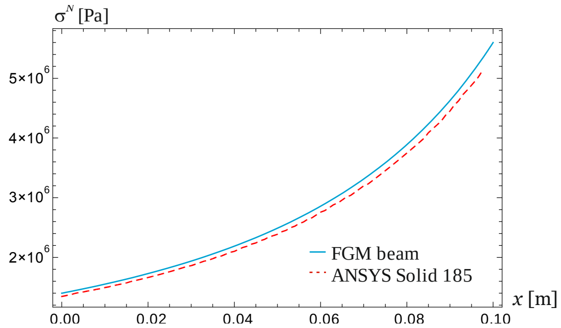
There, the elasticity, and shear modulus for Tungsten is $4.8 \cdot 10^{11}$ Pa, and $2.0 \cdot 10^{11}$ Pa. The elasticity, and shear modulus for Aluminium is $0.69 \cdot 10^{11}$ Pa, and $0.26 \cdot 10^{11}$ Pa.

The normal stresses at the cross-section point (x, y, z) with the elasticity modulus $E(x,y,z)$, is expressed by

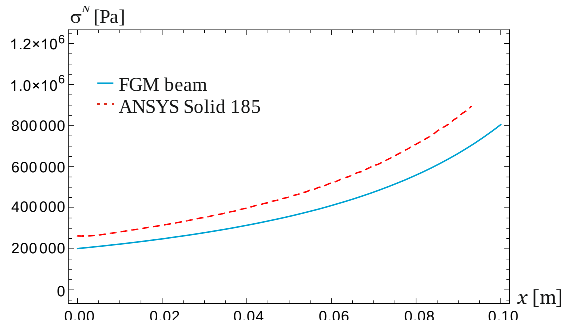
$$\sigma^N(x,y,z) = \varepsilon^N(x)E(x,y,z) = \frac{N(x)}{EA(x)}E(x,y,z), \tag{2}$$

where $\sigma^N(x) = \frac{N(x)}{A(x)}$ is the effective normal stress in the homogenized cross-section while constant effective normal strain $\varepsilon^N(x) = \frac{\sigma^N(x)}{E_L^{NH}(x)}$ over the whole cross-sectional area is assumed. Here, $E_L^{NH}(x)$is the effective elasticity modulus for tension-compression, which is a part of the effective axial stiffness $EA(x)$.

In Fig. 3 and Table 1, results of the normal stresses calculation by the FGM beam using the expression (2) is drawn: a) on the outer surfaces, and b) on the neutral axis of the beam, for $N(x) = F_x = 100$ N. Figs. 3c and 3d show a map of the normal stresses in the cross-section of the beam at a distance of $x = 0.001$ and $x = 0.05$ m obtained from the solution using SOLID 185 finite elements.The normal stresses obtained by SOLID185 finite elements, in MPa, agree very well with the ones obtained with the author's semi-analytical solution. This agreement was also achieved in cross-sections along the entire length of the beam. Some dicrepancies achieved at the free end of the beam affected by the different way of the load insertion.



a)



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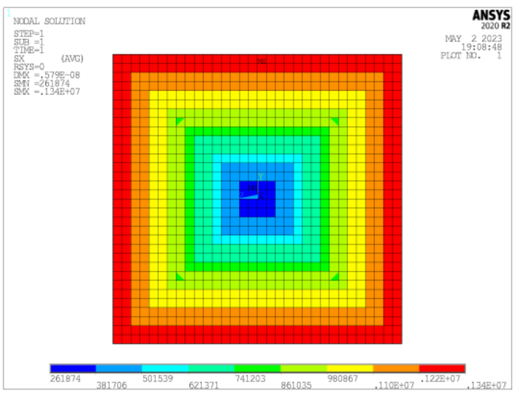

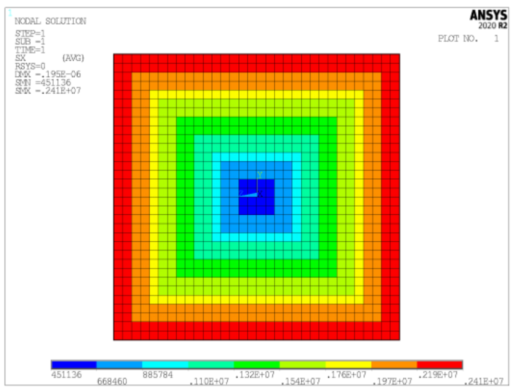

c) d)

Fig. 3. Longitudinal distribution of the tensile normal stresses, in [Pa], on the outer surfaces: a) and on neutral axis, b) calculated semianalytically (FGM beam), and the maps of the stresses at distances of x = 0.001 m, c) and x = 0.05 m, d) calculated by SOLID185 finite elements

Table 1. Comparison of the normal stresses on the outer surfaces, $\sigma^N(x,y,z)out$, and on the neutral axis, $\sigma^N(x,y,z)nax$, of the beam calculated by semianalytical method and with ANSYS [1] – red numbers

x [m]	0	0.05	0.1
$\sigma^N(x,y,z)out$/SOLID 185 [Mpa]	1.40/1.34	2.49/2.41	5.6
$\sigma^N(x,y,z)nax$/SOLID 185 [Mpa]	0.20/0.26	0.36/0.45	0.80

The expressions for calculation of the stresses caused by the bending moments, shear forces, and torsion moment have been established by a similar way. In all the load cases, a very well agreement of the normal s shear stresses calculated by the author´s method and SOLID185 finite elements has been obtained.

3. Conclusions

The elastostatic analysis of FGM beams using 3D solid finite elements is very demanding in terms of the need for their fine mesh and processing of the auxiliary program for assigning real functionally graded material properties to individual finite elements. However, these procedures are suitable for comparative calculations used to assess the effectiveness and accuracy of other, more effective solutions. Presented original semi-analytical relations for calculation of normal and shear stresses in FGM tapered beams represent a sufficiently accurate and effective tool. They are also part of our new tapered FGM beam finite element with variable stiffness, which was subjected to deformation analysis of the beam shown in Fig. 2b). This cantilever in Fig. 2b) was modeled by a single FGM beam finite element. The complete processing of the solved problem in this contribution will be carried out in the prepared article for scientific journal.

Acknowledgements

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